Credit Constraints and Growth in a Global Economy†

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We show that in an open-economy OLG model, the interaction between growth differentials and household credit constraints—more severe in fast-growing countries—can explain three prominent global trends: a divergence in private saving rates between advanced and emerging economies, large net capital outflows from the latter, and a sustained decline in the world interest rate. Micro-level evidence on the evolution of age-saving profiles in the US and China corroborates our mechanism. Quantitatively, our model explains about a third of the divergence in aggregate saving rates, and a significant portion of the variations in age-saving profiles across countries and over time. (JEL E21, E22, F21, F32, F41, O16, P24)

Two of the most important developments in the global economy of the recent decades are the integration of emerging markets into world capital markets and their rapid growth, particularly in certain parts of Asia. Alongside these events are three striking and unprecedented macroeconomic trends: (i) a large and persistent increase in the private saving rate in emerging Asia against a steady decline in the private saving rate in advanced economies; (ii) the emergence of global imbalances, with developing countries running a large current account surplus and advanced economies a current account deficit; (iii) a sustained fall in the world long-term interest rate.

These global patterns challenge standard open-economy growth models. Fast-growing emerging economies should, according to neoclassical theory, borrow

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against their higher future income to augment consumption and investment—and experience a fall in saving rate. At the same time, their fast growth should exert upward pressure on the world interest rate. And in the face of high domestic investment needs, they should become net capital importers rather than net exporters. This sharp discrepancy between theory and evidence is forcefully pointed out by Gourinchas and Jeanne (2013), who refer to it as the “allocation puzzle.”

In principle, the observed “upstream” capital flows could stem from either low investment, or high savings (or both) in emerging markets. The data seems to point to differences in savings. An immediate observation is the striking divergence in private saving rates between advanced economies and emerging Asia that coincided with a period of widening imbalances (Figure 1). Interestingly, the differences in the level of saving rates across these regions were rather small at the time of their integration around 1990 (panel A). The large divergence in the subsequent 20 years culminated into a marked disparity in recent years. The pattern is even more obvious for household saving rates, particularly between countries such as the US and China (panel B). In the late 1980s, China’s household saving rate was a mere 2–3 percentage points higher than that of the US. By 2008, it had reached almost 30 percent while the US household saving rate had declined to about 2.5 percent—leading to the popular caricature of a “debt-ridden” US put into sharp contrast against a “thrifty” Asia.

The experience of the US between 1970–2008 also makes a compelling case: while the current account exhibits a strong correlation with household saving over this period, there is hardly any relationship with investment (Figure 2, panel A). The pattern is echoed in the case of China (panel B). Gourinchas and Jeanne (2013) provide further support to this view, showing that “saving wedges,” rather than “investment wedges,” are necessary for the standard neoclassical model to replicate the observed patterns of international capital flows.

Against this background, the paper offers a theory of saving wedges—focusing specifically on heterogeneous household credit constraints across countries and their interaction with growth differentials. Our baseline theoretical framework, analyzed in Section I, consists of large open economies populated by agents living for three periods. This structure provides scope for both international and intergenerational borrowing. Young borrowers are subject to borrowing constraints, but the severity of the constraint differs between advanced and emerging economies. Faster growth in emerging economies, where credit constraints are tighter, exerts downward pressure on the world interest rate as greater weight is placed on their (lower) long-run autarkic interest rate.

This fall in the interest rate induces greater borrowing by the young—through a loosening of constraints, and greater savings of the middle-aged—through a

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1 Our baseline framework is an extension of Jappelli and Pagano’s (1994) closed-economy, three-period OLG model with household credit constraints. Our environment differs from theirs in several dimensions: (i) the multi-country, open-economy aspect of our setup; (ii) the asymmetry in household credit constraints across countries; (iii) more general preferences; (iv) more general income profiles.

2 What matters for the long-run dynamics of the world interest rate is that emerging economies have a lower autarkic steady-state interest rate. If they are capital scarce initially, however, their interest rate can be higher than that of advanced economies at the time of opening.
dominant income effect. The young’s saving rate falls by more in less constrained economies, while the rise in the middle-aged’s saving rate is larger in more constrained ones. The asymmetry in the tightness of the constraint across economies thus leads to different responses of aggregate saving rates, and a divergence in saving rates in the long run. The interaction of growth and heterogeneous credit constraints is key: without growth differentials, or with symmetric constraints across countries, the world interest rate would not permanently decline—critical for the saving divergence. Moreover, in the transition, tighter credit constraints in emerging countries serve to limit the impact of the positive wealth effect caused by fast productivity growth for young consumers.

\footnote{In our baseline model, the income effect dominates if the elasticity of intertemporal substitution is smaller than one, as usually assumed and in line with most of the empirical evidence (see Campbell 2003).}
A natural question arising from our theory is: how did different age groups contribute to the divergence in household savings observed in the data? To address this question, we dissect household survey data to provide new micro-evidence on saving behavior by age groups (Section II). The two exemplary economies selected, the US and China, arguably occupy opposite positions in the spectrum of credit constraint tightness, and are also the two most important contributors to global imbalances. The empirical challenge is to accurately measure age-saving profiles in the presence of potentially large biases inherent to household surveys in both countries—distinct problems yet equally taxing. The US consumption survey data suffers from significant underreporting biases that can, in addition, be time-varying (Slesnick...
The Chinese household survey suffers from limited data availability at the individual level. A common practice to circumvent this problem is to use the age of the household head in constructing age-saving profiles. We demonstrate that two biases arise in the presence of multi-generational households which is typical in China: a *selection* bias which tends to overestimate the saving rate of the young and its change over time, and an *aggregation bias* which tends to underestimate those of the middle-aged (the Deaton and Paxson 2000 critique). We attempt to remove these biases to the best of our efforts and estimate age-saving behavior for both economies over two decades. The corrected age-saving profiles generally conform better with standard life-cycle hypotheses and lend broad support to the qualitative implications of our theory.

The following stylized micro facts emerge: (i) the saving rate of young individuals fell significantly in the US over 1988–2008—by about 13 percentage points—while increasing slightly in China; (ii) the saving rate of the middle-aged rose in both countries, but by about 15 percentage points more in China than in the US; (iii) there is a marked divergence of saving rates for the retirees—with China’s elderly seeing a sharp rise and the US’s seeing a large drop. The elderly, however, contribute less to aggregate saving than the other age groups.

Equipped with both macroeconomic and microeconomic facts, we assess in Section III the quantitative relevance of the model. An extended, quantitative version of the theoretical model is calibrated to the experiences of the US and China over the period 1968–2008, incorporating in particular the evolution of demographics and income profiles in both countries. The model can explain about 30 percent of the divergence in aggregate saving rates between the US and China, and a significant portion of the evolution in the shape of the age-saving profile in both economies. However, it does fall short of explaining the very large increase in household savings in China, especially for the elderly. The model captures well the dynamics of the current account observed in the data, with China experiencing a small current account deficit at the time of opening, before building up a large current account surplus. Finally, the model predicts a significant drop in the world interest rate.

While the cross-country asymmetry in credit constraints is essential for our results, our analysis indicates that the sharp aging of the population in China and differences in income profiles across countries, in interacting with credit constraints, also contribute to the divergence in saving rates. The data reveals that the age-income profile in China reaches its peak at an earlier age than in the US and falls more steeply in old age, especially in the more recent period. This particular feature reduces the strength of positive wealth effects on middle age consumption implied by faster growth and a falling interest rate—thus contributing to the large increase in the saving rate in China generated by the model (see also Guo and Perri 2012). The role of demographics matters insofar as the rapid aging of the Chinese population, mostly a result of the one-child policy, implies an increase in the share of the middle-aged savers—a composition effect which also amplifies the increase in household savings in China.

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4 Gourinchas and Rey (2013) also point out the role of the shape of income profiles in generating differences in savings and autarky interest rates across countries. Note that wealth effects on middle-aged consumers do not operate in the three-period model of Section I since agents receive zero labor income in old age.
Combining the macro- and micro-level approaches is a distinctive feature of this paper. Past papers on international capital flows between developed and developing economies have usually taken up the former. Among these, theories relying on market imperfections are most closely related to our work (see Gourinchas and Rey 2013 for a recent survey). Frictions that impact savings include asset scarcity in developing countries (Caballero, Farhi, and Gourinchas 2008), incomplete financial markets and uninsurable risk in these economies (Mendoza, Quadrini, and Ríos-Rull 2009), lack of firm’s access to liquidity to finance investment in periods of rapid growth (Bacchetta and Benhima forthcoming), and international borrowing constraints (Benigno and Fornaro 2012). Financial frictions on investment are analyzed in Song, Storesletten, and Zilibotti (2011); Buera and Shin (2009); Benhima (2013); and Broner and Ventura (2013). Aguiar and Amador (2011) provide a political economy perspective with contracting frictions, where fast growing countries tend to experience net capital outflows.

There are three distinguishing elements that mark our theory from the aforementioned. The first is the emphasis on growth in emerging economies as a key driver of these aggregate phenomena—as opposed to capital market integration or shocks to financial markets in developing countries that are typically analyzed.

The second aspect is the ability of our model to explain the saving rate divergence across countries (a time-series effect)—as opposed to mere differences in levels. Third, we emphasize household saving divergence as the main driver of global imbalances, in contrast to investment-based or corporate-saving-based explanations.

Our quantitative findings are also related to previous papers highlighting the role of demographics, combined with life-cycle saving behavior, in explaining international capital flows. These include empirical studies such as Lane and Milesi-Ferretti (2002), and quantitative analyses focusing on OECD countries such as Domeij and Flodén (2006) and Ferrero (2010).

The decline in the household saving rate in the US and its rise in China have, independently, garnered a lot of attention. The particular stance we take in this paper is that global forces shaped these patterns simultaneously. That is not to say that there are no separate, country-specific, reasons why the US saving rate may have declined and why China’s saving rate may have risen. As our theory relies on one single global mechanism, unsurprisingly, it falls short of explaining the full divergence of

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5 See also Carroll and Jeanne (2009); Sandri (2010); and Angeletos and Panouzi (2011).

6 Exceptions are Caballero, Farhi, and Gourinchas (2008); Buera and Shin (2009); and Bacchetta and Benhima (forthcoming) who also analyze the impact of faster growth in developing countries.

7 Buera and Shin (2009); Song, Storesletten, and Zilibotti (2011); and Benhima (2013) show that financial frictions on firms can limit the rise in investment during a phase of growth acceleration, leading to net capital outflows from developing countries. Sandri (2010) and Bacchetta and Benhima (forthcoming) emphasize the role of corporate savings in the presence of liquidity constraints on firms.

8 Though important, corporate savings have risen uniformly in developing and advanced economies (Karabarbounis and Neiman 2012), and thus may not be able to account for the observed pattern of capital flows. Using firm-level data, Bayoumi, Tong, and Wei (2012) show that the corporate saving rate in China is not significantly higher than the global average and did not increase faster than the global trend. In 2009, Chinese corporate savings amounted to 21 percent of GDP, against 25 percent for the household sector and 5 percent for the public sector (Laffargue and Yu 2014). Over the period 1992–2009, the household saving rate increased by 15 percentage points. Despite the fact that its income share of GDP declined from 70 percent to 61 percent, the household sector contributed more to the increase in the national saving rate than the government sector, whose savings as a share of GDP increased by 6 percentage points over the period (Yang, Zhang, and Zhou 2011).
saving rates across countries. We thus view the alternative explanations relevant to each of these economies as complementary to ours in accounting for the full dynamics of savings. Our work is therefore partly related to a series of papers attempting to explain the large decline in the US household saving rate, summarized in Parker (2000) and Guidolin and La Jeunesse (2007); as well as to a large literature tackling the “Chinese saving puzzle” (Modigliani and Cao 2004), recently surveyed in Yang, Zhang, and Zhou (2011); and Yang (2012). In a nutshell, our work provides a micro-founded explanation for the emergence of a “global saving glut” (Bernanke 2005) that induced a decline in the world interest rate and the subsequent saving divergence.

The paper proceeds as follows. Section I develops the theoretical framework and provides some key intuitions. Analytical results are derived, shedding light on the mechanisms through which fast growth and integration of emerging markets impinge on the global economy in our model. Section II investigates micro-level evidence on saving behavior by age groups in China and the US. Section III examines the quantitative performance of a fully-calibrated model for these two economies. Section IV concludes.

I. Theory

The world economy consists of large open economies, populated by overlapping generations of consumers who live for three periods. Let $\gamma \in \{y, m, o\}$ denote a generation. Consumers supply one unit of labor when young ($\gamma = y$) and in middle age ($\gamma = m$), and retire when old ($\gamma = o$). In youth, consumers are credit constrained, but the severity of that constraint differs across countries. In all other aspects our framework is standard: all countries use the same technology to produce one homogeneous good, which is used for consumption and investment, and is traded freely and costlessly. Preferences and production technologies have the same structure across countries. Labor is immobile across countries, and firms are subject to changes in country-specific productivity and labor force.

A. Production

Let $K_{i,t}^\gamma$ denote the aggregate capital stock at the beginning of period $t$ in country $i$, and $e_t^i L_{y,t}^i + L_{m,t}^i$ the total labor input employed in period $t$, where $L_{\gamma,t}^i$ denotes the size of generation $\gamma$ and $e_t^i$ the relative productivity of young workers ($e_t^i < 1$). The gross output in country $i$ is

$$Y_t^i = (K_i^\gamma)^\alpha \left[ A_i^\gamma (e_t^i L_{y,t}^i + L_{m,t}^i) \right]^{1-\alpha},$$

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9 The decline in the US saving rate has been attributed to positive wealth effects (Poterba 2000; Juster et al. 2006; Carroll, Otsuka, and Slacalek 2011); financial innovation and relaxation of borrowing constraints (Parker 2000; Boz and Mendoza 2014; and Ferrero 2012); changes in social security and redistribution schemes (Gokhale, Kotlikoff, and Sabelhaus 1996; Huggett and Ventura 2000).

10 Some compelling explanations emphasize the role of precautionary savings (Blanchard and Giavazzi 2006; Chamon, Liu, and Prasad 2010; and Chamon and Prasad 2010); structural demographic changes (Curtis, Lugauer, and Mark 2011; Ge, Yang, and Zhang 2012; and Choukhmane, Coeurdacier, and Jin 2013); changes in life-income profiles and pension reforms (Song and Yang 2010; Guo and Perri 2012); gender imbalances and competition in the marriage market (Wei and Zhang 2011).
where $0 < \alpha < 1$, and $A_i^t$ is country-specific productivity. The capital stock in country $i$ depreciates at rate $\delta$ and is augmented by investment goods, $I_i^t$, with law of motion

$$K_{t+1}^i = (1 - \delta) K_t^i + I_t^i.$$  

Factor markets are competitive so that each factor, capital and labor, earns its marginal product. Thus, the wage rates per unit of labor in youth and middle age for country $i$ are

$$w_{y,t}^i = e_t^i (1 - \alpha) A_t^i (k_t^i)^\alpha, \quad w_{m,t}^i = (1 - \alpha) A_t^i (k_t^i)^\alpha,$$

where $k_t^i \equiv K_t^i / [A_t^i (e_t^i L_t^i + L_{m,t}^i)]$ denotes the capital-effective-labor ratio. The rental rate earned by capital in production equals the marginal product of capital, $r_{k,t}^i = \alpha (k_t^i)^{\alpha - 1}$, and the gross rate of return earned between period $t - 1$ and $t$ in country $i$ is $R_t^i = 1 - \delta + r_{k,t}^i$. Productivity and the size of consecutive cohorts grow at rates $g_{A,t}^i$ and $g_{L,t}^i$, respectively, so that $A_t^i = (1 + g_{A,t}^i) A_{t-1}^i$ and $L_{y,t}^i = (1 + g_{L,t}^i) L_{y,t-1}^i$.

B. Households

A consumer born in period $t$ earns the competitive wage rate $w_{y,t}^i$ when young and $w_{m,t+1}^i$ in the following period. Let $c_{y,t}^i$ denote the consumption of an agent in country $i$ belonging to generation $\gamma$. The lifetime utility of a consumer born in period $t$ in country $i$ is

$$U_t^i = u(c_{y,t}^i) + \beta u(c_{m,t+1}^i) + \beta^2 u(c_{o,t+2}^i),$$

with standard isoelastic preferences $u(c) = (c^{1-\sigma} - 1) / (1 - \frac{1}{\sigma})$. The discount factor $\beta$ satisfies $0 < \beta < 1$ and the intertemporal elasticity of substitution coefficient satisfies $\sigma \leq 1$.\footnote{Our analytical expressions are still valid when $\sigma > 1$, but some of our mechanisms rely on a sufficiently low e.i.s. coefficient. Most of the empirical literature since the seminal paper of Hall (1988) finds estimates of the elasticity of intertemporal substitution below 0.5 (see Ogaki and Reinhart 1998; Vissing-Jørgensen 2002; and Yogo 2004 among others). The macro and asset pricing literature (discussed in Guvenen 2006) typically assumes higher values between 0.5 and 1.}

Let $a_{s,t+1}^i$ denote the net asset holdings at the end of period $t$ of an agent belonging to generation $\gamma$. An agent born in period $t$ faces the following sequence of budget constraints:

$$c_{y,t}^i + a_{y,t+1}^i = w_{y,t}^i,$$

$$c_{m,t+1}^i + a_{m,t+2}^i = w_{m,t+1}^i + R_{t+1}^i a_{y,t+1}^i,$$

$$c_{o,t+2}^i = R_{t+2} a_{m,t+2}^i.$$
When young, individuals can borrow in order to consume \( a_{y,t+1}^i < 0 \). When middle-aged, they earn the competitive wage, repay their loans, consume, and save for retirement. When old, they consume all available resources. A bequest motive is omitted for convenience but is introduced later in the quantitative analysis (Section III).

We assume that young agents are subject to credit constraints: they can only borrow up to a fraction \( \theta^i \) of the present value of their future labor income,

\[
a_{y,t+1}^i \geq -\theta^i \frac{w_{m,t+1}^i}{R_{t+1}}.
\]

The tightness of credit conditions, captured by \( \theta^i \), can differ across countries but is assumed constant over time.\(^{12}\) We analyze the case in which (8) is binding for all countries.\(^{13}\)

**ASSUMPTION 1:** Credit constraints for the young are binding at all times in all countries.

This assumption is satisfied if two conditions hold: (i) \( \theta^i \) is small enough—smaller than the fraction of intertemporal wealth that the young would consume in the absence of credit constraints; (ii) the wage profile is steep enough.\(^{14}\) When credit constraints are binding, the net asset position of the young is

\[
a_{y,t+1}^i = -\theta^i \frac{w_{m,t+1}^i}{R_{t+1}}.
\]

The net asset position of a middle-aged agent at the end of period \( t \) is obtained from the Euler condition that links \( c_{m,t}^i \) and \( c_{o,t+1}^i \), yielding

\[
a_{m,t+1}^i = \frac{1}{1 + \beta^{-\sigma} (R_{t+1}^i)^{1-\sigma}} (1 - \theta^i) w_{m,t}^i.
\]

Changes in \( R_{t+1}^i \) affects middle-aged asset holdings through a substitution and income effect, the latter dominating when \( \sigma < 1 \).

\(^{12}\) We are interested in a scenario where financial development lags economic development, so that household credit constraints remain significantly more severe in emerging countries than in advanced economies.

\(^{13}\) This assumption is made for analytical convenience but our mechanism goes through as long as the credit constraint is binding in the more constrained economies. The exact nature of the credit constraint matters only insofar as a fall in interest rate leads to a greater fall in the saving rate of the young in less constrained economies. If the credit constraint was independent of the interest rate (e.g., a function of current wages only), the saving rate divergence would be weaker, unless the constraint does not bind in advanced economies.

\(^{14}\) The conditions are \( \theta^i < \eta^*_t \) and \( w_{m,t+1}^i / (R_{t+1}^i w_{y,t}^i) > (1 - \eta^*_t) / (\eta^*_t - \theta^i) \), for all \( t \), where \( \eta^*_t \equiv [\beta^{-2\sigma} (R_{t+1}^i R_{t+2}^i)^{1-\sigma}] / \{1 + \beta^{-\sigma} (R_{t+2}^i)^{1-\sigma} [1 + \beta^{-\sigma} (R_{t+1}^i)^{1-\sigma}] \} \). In the case of log utility, these conditions amount to \( \theta^i < (1 + \beta + \beta^2)^{-1} \), and \( w_{m,t+1}^i / (R_{t+1}^i w_{y,t}^i) > \beta(1 + \beta) / [1 - \theta^i(1 + \beta + \beta^2)] \).
C. Autarky Equilibrium

Under financial autarky, market clearing requires that the total capital stock accumulated at the end of period $t$ is equal to aggregate country wealth,

$$K_{t+1}^i = L^i_{y,t}a^i_{y,t+1} + L^i_{m,t}a^i_{m,t+1}.$$  

Along with (9) and (10), this gives the law of motion for $k^i_t$, the capital-effective-labor ratio in country $i$. In the full depreciation case ($\delta = 1$), the dynamic of $k^i_t$ is given implicitly by

$$\left(1 + g^i_{A,t+1}\right)\left(1 + g^i_{L,t}\right)\left[1 + e^i_{t+1}\left(1 + g^i_{L,t+1}\right) + \theta^i \frac{1 - \alpha}{\alpha}\right]k_{t+1}^i = \frac{\left(1 - \theta^i\right)\left(1 - \alpha\right)}{1 + \beta^{-\sigma}\left\{\alpha\left(k^i_{t+1}\right)^{\alpha-1}\right\}^{1-\sigma}}\left(k^i_t\right)^{\alpha}.$$

Figure 3 depicts the autarkic law of motion for capital for two different values of the credit constraint parameter, $\theta_L$ and $\theta_H > \theta_L$. We now characterize the impact of $\theta^i$ on the steady state of the economy. To zero in on the effect of differences in credit constraints, we assume constant and identical productivity and labor force growth rates $g_A$ and $g_L$ across countries, and a fixed relative productivity of young workers $e$.

**THEOREM 1:** Suppose that $\delta = 1$. There exists a unique, stable, autarky steady state. All else equal, more constrained economies have a higher capital-to-efficient-labor ratio ($dk^i_t/d\theta^i < 0$) and a lower interest rate ($dR^i_t/d\theta^i > 0$).

The proof of Theorem 1 and all other proofs are relegated to Appendix A. More constrained economies accumulate more capital as a result of less dissaving of the young and lower debt repayment of the middle-aged, and hence feature a lower rate of return in the long run. In the case $\sigma = 1$, the autarky steady-state interest rate in country $i$ is

$$R^i = (1 + g_A)(1 + g_L)^{\frac{1 + \beta\alpha\left[1 + e(1 + g_L)\right] + \theta^i(1 - \alpha)}{(1 - \alpha)(1 - \theta^i)}}.$$

This expression shows that the rate of return is also increasing in productivity and labor growth rates, $g_A$ and $g_L$, and in the relative efficiency of young workers $e$—all of which raise the marginal productivity of capital. Demographics matter not only through the impact on labor force growth, but also on the population composition: a higher proportion of young agents relative to middle-aged agents due to high $g_L$.

15 Most of our theoretical results are derived for $\delta = 1$, but they hold more generally.

16 The impact of productivity growth differentials and effects related to cross-country differences in demographics and income profiles on the transition path are discussed in Section III.
increases the proportion of borrowers relative to savers and hence puts upward pressure on the rate of return to capital.

D. Integrated Equilibrium

Under financial integration, capital flows across borders until rates of return are equalized across countries. Financial integration in period $t$ implies that $R^i_{t+1} = R_{t+1}$ and $k^i_{t+1} = k_{t+1}$, for all $i$. The capital market equilibrium condition becomes

\begin{equation}
\sum_i K^i_{t+1} = \sum_i (L^i_{y,t} a^i_{y,t+1} + L^i_{m,t} a^i_{m,t+1}),
\end{equation}

which, along with (9) and (10), gives the law of motion for $k_t$. Next, we characterize the integrated steady state where the growth rates of productivity and labor, as well as the relative efficiency of young workers, are identical across countries.

PROPOSITION 1: Suppose that $\delta = 1$. Let $\theta_L \equiv \min_i \{\theta^i\}$, $\theta_H \equiv \max_i \{\theta^i\}$, with $\theta_L \neq \theta_H$. The steady-state world interest rate $R$ satisfies

\begin{equation}
R(\theta_L) < R < R(\theta_H),
\end{equation}

Notes: Parameter values are $\sigma = 0.5$, $\beta = 0.97$ (annual), $\alpha = 0.28$, $\delta = 10$ percent (annual), $\theta_H = 0.2$, $\theta_L = 0.02$, $g_A = 1.5$ percent (annual), $g_L = 1$ percent, $e = 0.33$. A period lasts 20 years.
where \( R(\theta) \) denotes the autarky steady-state interest rate for credit constraint parameter \( \theta \).

Proposition 1 points to the first factor that can cause a fall in the rate of return faced by less constrained economies: financial integration with more constrained ones. Figure 3 illustrates this effect in a two-country case, assuming that the less constrained country starts at its autarkic steady state \( k(\theta_H) \) whereas the more constrained one is initially capital scarce—so that the two economies have identical capital-effective-labor ratios at the time of opening. On integration, the transition path of capital is determined by the integrated law of motion, which lies in between the autarkic ones. Effectively, the world economy behaves like a closed-economy with credit constraint parameter \( \bar{\theta} \equiv \sum_i \lambda_i \theta_i \), where \( \lambda_i \) denotes the relative size of country \( i \) measured by its share in world effective labor.

Along the convergence to the integrated steady state \( k(\bar{\theta}) \) depicted in Figure 3, the world interest rate experiences a sustained decline.

The second factor that can lead to such a decline is faster growth in more constrained economies. Indeed in the long run, the world interest rate is determined (up to a monotonously increasing transformation) as a weighted average of the autarky steady-state interest rates of all countries, with weight on country \( i \) increasing in \( \lambda_i \). Hence as the more constrained economies grow faster and account for a greater share of the world economy over time, the world interest rate falls.

PROPOSITION 2: A relative expansion of the more constrained economies (i.e., an increase in the share \( \lambda_i \) of a country with low \( \theta_i \)) causes a fall in the world steady-state interest rate. A relative expansion of less constrained economies has the opposite effect.

E. Saving and Investment

We now show that asymmetric credit constraints lead to heterogeneous responses of saving rates to the endogenous fall in the world interest rate across countries, both at the aggregate level and for each generation. In the integrated steady state, the aggregate net saving to GDP ratio of country \( i \) is

\[
\frac{S_i}{Y_i} = -\frac{g}{1 + e(1 + g_L)} (1 - \alpha) \frac{\theta_i}{R} + \frac{g}{1 + g} \frac{1}{1 + e(1 + g_L)} (1 - \alpha) \frac{1 - \theta_i}{1 + \beta^{-\sigma} R^{1-\alpha}},
\]

17 This assumption is made for the ease of graphical representation. One way to think about it is that the more constrained economy experiences an episode of fast productivity growth before integration, which drives its capital-effective-labor ratio down at the time of opening.

18 This statement follows directly from the proof of Proposition 1. In the special case where \( \sigma = 1 \), an alternative representation of the long-run world interest rate is given by equation (12), substituting the world average credit constraint parameter \( \bar{\theta} \) in place of \( \theta \).

19 Formal definitions of savings, at the aggregate level and for each generation, are given in the online Appendix.
where $R$ is at its steady-state value, and $g \equiv (1 + g_A)(1 + g_L) - 1 > 0$. The equation shows that more constrained economies (lower $\theta^i$) place a greater weight on the middle-aged savers and less weight on young borrowers, resulting in a higher saving rate. Moreover, it implies that in response to a fall in the world interest rate $R$, the saving rate increases by more in the more constrained economy, $\partial^2 (S/Y)/\partial \theta \partial R > 0$. These slope differences, combined with differences in levels, imply that a fall in $R$ induces a divergence in saving rates across countries. Given the fall in interest rate caused by an increase in the relative size of the more constrained economies (Proposition 2), the next proposition follows.

**PROPOSITION 3:** A relative expansion of the more constrained economies (i.e., an increase in the share $\lambda^i$ of a country with low $\theta^i$) causes a greater dispersion of steady-state saving rates across countries.

Away from the steady state, it is useful to decompose the response of the saving rate into the response of each generation’s saving rate (expressed as a share of GDP for the purpose of aggregation). We show in Appendix A that

\[
S_{s,t}^i = -(1 + g_{A,t+1}^i) \frac{1 + g_{L,t}^i}{1 + e_t^i(1 + g_{L,t}^i)} \frac{1 - \alpha}{k_t^\alpha} \frac{\theta^i}{R_{t+1}} \left( \frac{\alpha}{R_{t+1} - 1 + \delta} \right)^{\frac{\alpha}{1-\alpha}},
\]

\[
\frac{S_{m,t}^i}{Y_t^i} = \frac{1 - \alpha}{1 + e_t^i(1 + g_{L,t}^i)} \left[ \frac{1 - \theta^i}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}} + \frac{\theta^i}{R_{t+1}} \right],
\]

\[
\frac{S_{o,t}^i}{Y_t^i} = -\frac{1}{1 + g_{A,t+1}^i} \frac{1}{1 + g_{L,t}^{i-1}} \frac{1 - \alpha}{1 + e_t^i(1 + g_{L,t}^i)} \frac{1 - \theta^i}{1 + \beta^{-\sigma} R_{t+1}^{1-\sigma}} \left( \frac{k_{t-1}}{k_t} \right)^{\alpha}.
\]

These expressions indicate that the response of savings to the interest rate $R_{t+1}$ varies across generations, and that the strength of the response varies across countries. The following proposition characterizes the partial effects of a drop in $R_{t+1}$ on the savings of the young and middle-aged, abstracting from the direct effect of factors causing the interest rate to fall.

**PROPOSITION 4:** All else constant, in response to a fall in the interest rate $R_{t+1}$, the young borrow more while the middle-aged save more under the condition that $\sigma < 1$. The increase in borrowing by the young is larger in less constrained economies (high $\theta^i$), while the increase in saving of the middle-aged is larger in more constrained economies (low $\theta^i$).

Proposition 4 implies that the net response of the aggregate saving rate to a fall in interest rate depends on $\theta^i$: a high $\theta^i$ gives more importance to the young borrowers’ larger dissavings, whereas a low $\theta^i$ gives more importance to the rise in middle-aged’s savings.

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20 Normalizing by each generation’s factor income yields similar expressions, up to some multiplicative terms common across countries.
Also worthy of note is that the presence of credit constraints limits the negative impact of future growth \( g_{A,t+1} \) on the saving rate: the dissavings of the young can only increase up to the extent permitted by the binding credit constraints. Thus, the standard wealth effect of growth on saving is mitigated when growth is experienced by a country with tight credit constraints. In addition, the wealth effect of growth does not operate on middle-aged consumers when the old have no wage income. In the more general case, this wealth effect is weaker when the income profile falls in old age.

Investment is governed by the same forces that underlie the neoclassical growth model. Under financial integration, differences in investment-output ratios across countries are largely determined by their relative growth prospects. With full depreciation \((\delta = 1)\), investment to GDP ratios obey

\[
\frac{I_t^i}{Y_t^i} = \frac{1 + \tilde{g}_{t+1}^i}{1 + \tilde{g}_{t+1}^j},
\]

where \( 1 + \tilde{g}_{t+1}^i \equiv (1 + g_{A,t+1}^i)(1 + e_{t+1}^i + (1 + g_{L,t+1}^i)^{-1}) \) denotes the combined growth rate in productivity and effective labor input in country \( i \).

### F. Discussion

The model can be used to shed light on how financial integration of emerging markets and their faster growth impinge on the world economy. Consider the following experiment where a fast-growing developing country with tight constraints, integrates with an advanced economy. If the developing country starts out by being capital scarce, it can feature a higher autarkic interest rate than the advanced economy. After opening, the rapid decline of its (shadow) autarkic interest rate owing to capital accumulation, along with its increasing weight in the integrated global economy, leads the world interest rate to decline (Proposition 2). Saving rates diverge across countries due to their asymmetric responses to the fall in interest rates (Proposition 3). The rise in saving rate in the developing economy is driven by the middle-aged, while the decline in the advanced economy is driven by the young. Although the investment rate also rises in the fast-growing developing country, the rise in its saving rate soon dominates, leading to a current account surplus.

By contrast, if credit constraints were absent (or not binding), the aggregate saving rate would fall in the fast-growing economy as the young borrow more against their higher future income. Investment would rise and the country would run a large

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21 The illustrative results from a numerical experiment are presented in detail in Section 2.6 of the longer working paper version (Coeurdacier, Guibaud, and Jin 2015a). A comprehensive quantitative analysis is deferred until Section III.

22 Three factors determine the dynamics of interest rates. The first two factors pin down the paths of interest rates that would prevail if both economies remained in autarky throughout. The “growth effect” tends to raise the interest rate in the developing country due to higher marginal productivity of capital, while the “convergence effect” tends to lower it as the country rapidly accumulates capital from a capital-scarce starting point. After the opening of capital markets, the “integration effect” determines the world interest rate according to the relative size of each economy. The interest rate falls throughout the transition if the last two effects dominate.
current account deficit. The fall in the world interest rate would be mitigated, and the interest rate would not experience a prolonged decline. Saving rates would tend to converge across economies as agents respond similarly to changes in the interest rate in all countries. A model with binding but equally loose credit constraints in both economies would generate qualitatively similar results. Thus, both the presence of credit constraints and their asymmetry are essential for our results. Growth is also key since in the case of mere financial integration, the world interest rate would barely fall, and the divergence in saving rates would be much smaller.

II. Micro Evidence on Savings by Age Groups

Motivated by the predictions of our theory at the micro level, we now provide direct evidence on savings by age groups in advanced and emerging economies and their evolution over the last two decades. Because of limited data availability, we focus on two exemplary countries—the US and China. These two economies are the most important contributors to global imbalances, and arguably occupy opposite positions in the spectrum of household credit constraint tightness. A number of complex issues arise when using household survey data to construct age-saving profiles. This section describes a careful treatment of these issues and the way we attempt to deal with potential biases. These micro findings are used subsequently to calibrate the quantitative model and evaluate its performance. Readers interested only in the quantitative implications can proceed directly to Section III.

A. Evidence for the United States

The Consumer Expenditure Survey (CEX) provides the most comprehensive data on disaggregated consumption, and is therefore our primary data source for the US. Annual data from 1986 to 2008 are available for six age groups: under 25, 25–34, 35–44, 45–54, 55–64, and above 65. Details of the data are provided in Appendix B.

Underreporting Biases.—The main issue involved in using CEX data is their sharp discrepancy with the National Income and Product Account (NIPA) data. This discrepancy is well-documented in Slesnick (1992); Laitner and Silverman (2005); Heathcote, Perri, and Violante (2010); and Aguiar and Hurst (2013); and arises from underreporting of both consumption and income in the CEX data. The degree of underreporting has become more severe over time for consumption but not for income, the consequence of which is a stark rise in the aggregate saving rate as computed from CEX data, compared to an actual decline as measured in NIPA data (Figure 4). Some important corrections of the CEX are therefore needed to estimate reasonable age-saving profiles for the US.

---

23 The interest rate could even rise temporarily if the growth effect dominates the convergence effect.
24 The shape of the age-income profile, typical of an OLG model, is also important for the savings divergence. Credit constraints are binding for the young because they start with a lower labor income. Moreover as noted above, the positive wealth effect of growth and falling interest rates on middle-aged consumers is strongly mitigated when their income in old age is low. A flatter age-income profile would bring the model closer to a standard representative agent model without constraints.
Correction Method.—Following previous works (Parker, Vissing-Jørgensen, and Ziebarth 2009 among others), we assume that NIPA data is well measured, and propose a correction method to bring about consistency between CEX and NIPA data. Our correction method adjusts income uniformly across all age groups so as to match NIPA data. On the consumption side, we take into account the fact that the degree of underreporting may vary across goods, which becomes a concern if the composition of the consumption basket differs across age groups (see Aguiar and Hurst 2013 for recent evidence). While allowing the degree of underreporting in CEX to vary over time and across consumption goods, the correction method relies on the assumption that it is constant across age groups.

In practice, to correct for underreporting in consumption, we use CEX and NIPA data on aggregate consumption for 15 sectors to construct time-varying, sector-specific adjustment factors $\chi_{kt} = \frac{C_{kt}^{NIPA}}{C_{kt}^{CEX}}$, where $C_{kt}^{D}$ denotes aggregate consumption of good $k$ in dataset $D$. For all sectors, $\chi_{kt}$ is greater than 1, and rises over time as the underreporting bias in CEX consumption becomes more severe. We use the sector-specific factors to adjust CEX sectoral consumption data by age: given $c_{jkt}^{CEX}$ the average consumption of goods of sector $k$ by individuals of age $j$ as reported in CEX, we define $\hat{c}_{jkt} = \chi_{kt} c_{jkt}^{CEX}$. The adjusted consumption expenditure for age $j$ is then obtained as $\hat{c}_{jt} = \sum_k \hat{c}_{jkt}$. Similarly, our adjusted

\[ \hat{c}_{jt} = \sum_k \hat{c}_{jkt} \]

\[ \hat{c}_{jt} = \sum_k \hat{c}_{jkt} \]

25 The 15 sectors matched between NIPA and CEX are: Food and alcoholic beverages, Shelter, Utilities and public services, Household expenses, Clothing and apparel, Vehicles purchases, Gas and motor oil, Other vehicle expenses, Public transportation, Health, Entertainment, Education, Tobacco, Miscellaneous and cash contributions, Life/personal insurance.

26 Another issue is that health expenditures are treated differently in NIPA and CEX. Health expenditures in CEX are restricted to “out-of-pocket” expenses, but NIPA also includes health contributions (Medicare and Medicaid), leading to very large adjustment factor $\chi_{health}$. This mostly affects our consumption estimates for the
measure of income for age \( j \) is 
\[
\hat{y}_{j,t} = \left( \frac{Y_t^{\text{NIPA}}}{Y_t^{\text{CEX}}} \right) y_{j,t}^{\text{CEX}},
\]
where \( y_{j,t}^{\text{CEX}} \) denotes the average income reported in CEX for age \( j \) in year \( t \), and \( Y_t^{\text{D}} \) the aggregate income in dataset \( \mathcal{D} \). By construction, the corrected consumption and income measures match NIPA in the aggregate. \(^{27}\) Finally, the estimated saving rate for age \( j \) in period \( t \) is 
\[
\hat{s}_{j,t} = \left( \hat{y}_{j,t} - \hat{c}_{j,t} \right) / \hat{y}_{j,t}.
\]

**Corrected US Age-Saving Profiles.**—Figure 5 displays the estimated saving rates by age groups for the years 1988 and 2008 using our correction method. Age-saving profiles are in line with the life-cycle theory, and their shapes show some interesting evolution. In two decades, the group of young people (under 25) saw a decline of 12.7 percentage points in their saving rate, while those between 35–54 a small increase of about 2.3 percentage points, and the eldest group a large decline of about 19 percentage points.

### B. Evidence for China

The main data source for China is the Urban Household Survey (UHS) conducted by the National Bureau of Statistics, available for the year 1986 and annually over the period 1992–2009. We use the sample of urban households which covers 112 prefectures across 9 representative provinces, with an overall coverage of about 5,500 households in the 1992 to 2001 surveys and 16,000 households in the 2002 to 2009 surveys. \(^{28}\) The UHS data records detailed information on income,

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\(^{27}\) A small discrepancy remains for consumption since NIPA includes expenditure types (e.g., “Net foreign travel and expenditures abroad by US residents” and “Final consumption expenditures of nonprofit institutions serving households”) which cannot be matched with CEX categories.

\(^{28}\) The 1986 survey covers a different sample of 12,185 households across 31 provinces.
consumption expenditures, and demographic characteristics of households. It also provides employment, wages, and other characteristics of individuals in the household. Further information about the data can be found in Appendix B.

The main issue that arises with UHS data is that, while income is available at the individual level, consumption is only available at the household level. For this reason, previous studies analyzing age-specific saving behavior in China use household-level data. That is, the saving rate they impute to a certain age is the average household saving rate computed over all households whose head is of this age. Following this approach, Song and Yang (2010); Chamon and Prasad (2010); and Chamon, Liu, and Prasad (2010) find evidence against standard life-cycle motives of saving in China. In particular, they find that the traditional hump-shaped age-saving profile is replaced by a U-shaped profile in recent years, with saving rates being highest for the young and close to retirement age, and lowest for the middle-aged. This would run counter to our prediction that the middle-aged savers in China should have contributed the most to the rise in household saving rate in the last two decades. However, the “household approach” is subject to potential measurement errors, which we now examine.

Aggregation and Selection Biases.—Deaton and Paxson (2000) have forcefully shown the problems associated with using the household approach to construct age-saving profiles in the presence of multi-generational households. If a large fraction of households comprise members that are at very different life-cycle stages, the age-saving profile obtained from household data will be obscured by an aggregation bias. For instance, suppose that middle-aged individuals have a high saving rate as they save for retirement, but middle-aged household heads live with younger adults or elderly members who have much lower saving rates. In this case, the household approach would lead to an under-estimation of the saving rate of the middle-aged. More generally, the aggregation bias tends to flatten the true age-saving profile. A second potential bias arises from the possibility that household headship is not random. If being a head at a certain age is correlated with certain characteristics (such as income) that affect saving behavior, the age-saving profile estimated by the household approach would suffer from a selection bias. Moreover, any time-variation in these two biases would affect the estimated change in age-specific saving behavior over time.

A multi-generational household is the norm in the case of China, thus making the aggregation bias a serious concern (Table 1). In urban households, more than 50 percent of individuals live in multi-generational households (defined as

<table>
<thead>
<tr>
<th>Year</th>
<th>Two-generation</th>
<th>Three-generation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992</td>
<td>41</td>
<td>15</td>
</tr>
<tr>
<td>2009</td>
<td>37</td>
<td>18</td>
</tr>
</tbody>
</table>

Note: Share of individuals living in households comprising two or three generations (in percent).

Source: Authors’ calculations based on UHS data.
households in which the maximum age difference between two adults is above 18 years), and roughly one out of six in households with three different generations. Multi-generational households are observed when young adults (typically in their twenties) stay in their parents’ household or when older individuals (typically in their seventies) live with their children. A closer look at the data shows that, toward the end of the sample period, young adults tend to stay longer with their parents, while the elderly tend to join their children’s household at a later age as a result of an increase in life expectancy. These evolutions are likely to introduce some bias in the estimates of changes in age-specific saving rates obtained from the household approach.

Figure 6 offers suggestive evidence of a potential bias arising from the fact that household heads are not selected randomly. The figure displays the income premium of household heads as a function age, with the average income of heads of a given age expressed as the log ratio of the average income of all individuals of that age. Both young and elderly household heads are significantly richer than their non-household head counterparts. This is of no surprise—only the richer individuals can afford to live independently when young or in old age. If high individual income is correlated with high individual saving rate, the household approach would therefore tend to overestimate the saving rates of the young and of the elderly. The evolution of the income premium over time, apparent in the figure, suggests that the selection bias is likely to be more severe for the elderly in 1992, and more severe for the young in 2009.

**Projection Method.**—To improve upon the household approach, the key challenge is to identify individual consumption. Our approach applies a projection method proposed by Chesher (1997, 1998) and Deaton and Paxson (2000) to disaggregate household consumption into individual consumption, from which we estimate new age-saving profiles. Essentially, the idea is to recover the consumption of each individual member of the household using cross-sectional variations in the composition of households as a source of identification. In practice, this is done by projecting household consumption on the number of household members belonging to various age groups, controlling for observable household characteristics. Following Chesher (1997), we conduct a nonlinear least squares estimation of the following model for each year:

$$C_h = \exp(\gamma \cdot Z_h) \left( \sum_{j \geq 19} c_j N_{h,j} \right) + \epsilon_h,$$

where $C_h$ is the aggregate consumption of household $h$, $N_{h,j}$ is the number of members of age $j$ in household $h$, and $Z_h$ denotes a set of household-specific controls (income group, number of adults, number of children, uni- versus multi-generational, etc.).

---

29 Any household with one adult or several adults belonging to the same generation, possibly with a child, is considered as unigenerational.

30 This assumes that individual consumption can be written as multiplicatively separable functions of individual age and household characteristics. The identification therefore relies on the restriction that the effect of household characteristics on individual consumption is independent of age.
The estimated consumption of an individual of age $j$ living in a household with characteristics $Z_{h}$ is then equal to $\exp(\hat{\gamma} \cdot Z_{h} \hat{c}_{j})$. Details of the methodology are given in the online Appendix.

**Limitations and Robustness Checks.**—Our estimation method is not without potential issues. One possible concern is that, if intergenerational transfers within the household are important, our estimated age-saving profiles could be biased. As a robustness check, we implement an alternative methodology in which age-saving profiles are estimated on the restricted sample of unigenerational households, which constitute more than 40 percent of the entire sample. The estimated profiles are similar to the ones produced by Chesher’s projection method, albeit using a different sample of households and a different identification strategy.

Another potential issue comes from the fact that household composition is treated as exogenous by Chesher’s method, although households may not be formed randomly. For instance, if the decision made by young people to live alone is positively correlated with their propensity to save, one might be concerned that the projection method artificially increases the young’s saving rate. The online Appendix provides a number of robustness checks to address this type of selection issue. In

---

31 This alternative approach, including the way observations are reweighted to match the characteristics of the whole sample along some important observable characteristics (income in particular), is described in detail in the online Appendix, where the issue of transfers is also discussed more generally.

32 Such selection bias would be a concern if the projection method identifies individual consumption by age mostly based on unigenerational observations, rather than based on variations in the composition of multigenerational households.
particular, when implemented on a restricted sample which excludes unigenerational households containing at least one individual under 30 or above 65, the projection method is found to produce very similar results.33

Estimated Age-Saving Profiles for China.—Figure 7 exhibits the estimated age-saving profiles, at the beginning and at the end of the sample period.34 In the online Appendix, we show that our estimates differ substantially from the ones produced by the household approach based on the age of the household head. Echoing the results of Deaton and Paxson (2000) for Taiwan and Thailand, we find that the age-saving profiles computed by the individual approach are more in accord with the life-cycle theory of saving.35 In particular, the young do save less than the middle-aged, especially so in the most recent period. Over time, we observe a large increase in the saving rate of the middle-aged, between 15–20 percentage points. The saving rate of the youngest also increases, but significantly less. The striking increase in the saving rate of the elderly (above 65) is quite peculiar and seems at odds with standard life-cycle motives. However, it is important to recognize that,

33 Note also that in identifying individual consumption, Chesher’s method already controls for household income and household composition— in particular, if individuals of a certain age living alone consumed differently from those living with N other adults, this effect would at least partly be captured by the method. The question is whether there could remain other unobservable characteristics, correlated with both household formation and propensity to consume, which we do not account for. Our results, however, suggest that, once controlling for income, individuals of a given age living in unigenerational households do not have different saving behavior than those living in multigenerational households.

34 For the beginning of the sample period, due to the lack of observations in 1988, we show the estimated profile for 1992 along with averages over the first three available years to minimize issues related to the smaller size of our sample in early years.

35 We also estimated profiles from the Chinese Household Income Project (CHIP) data, available for 1995 and 2002, finding consistent results across methods and across surveys.
because of their modest income share, the old’s contribution to aggregate savings remains small.36

C. Summary of Micro Evidence

Our baseline three-period model predicts that in the face of a fall in the world interest rate (caused by capital markets integration and fast growth in Asia), (i) the saving rate of the young falls by more in developed countries; (ii) the saving rate of the middle-aged increases by more in emerging markets. As a result, age-saving profiles across countries become more distant from each other over time. In the data, the change in the young’s saving rate over time is markedly different across the US and China, leading to a widening of the cross-country difference in their saving rates. Meanwhile, the saving rate of the middle-aged (35–54) in China rose by about 15 percentage points more in China than in the US. Within countries, the evolution of saving rates across age groups makes the profiles more hump-shaped, both in theory and in the data. Overall, apart from the large increase in the saving rate of the elderly in China, our empirical findings are broadly supportive of the qualitative predictions of our theory.

III. Quantitative Analysis

Equipped with facts on the macro and micro level, we now assess the ability of the model to match the evolution of saving rates in the US and China over the period 1988–2008—both on the aggregate level and by age groups. The quantitative model enriches the baseline model of Section I along several dimensions, and is fully calibrated to the experiences of these two economies. First, we increase the number of periods/generations in order to yield more refined micro and aggregate predictions. Having more periods allows us to incorporate the exact shapes of age-income profiles across countries, and their variations over time. Second, we introduce a bequest motive to allow for a savings initiative by the old. The demographic evolution in each country is also calibrated to the data—thus incorporating the aging of population in both countries. Model parameters that are not directly observable are calibrated to micro and macro data for the US and China at the beginning of the sample period.

A. A Multi-Period OLG Model with Asymmetric Constraints

A brief description of the quantitative model follows. Unless specified otherwise, the notations are retained from Section I.

36 Although the reasons explaining the increase in Chinese elderly’s saving rate lie outside of the model and are beyond the scope of this paper, we can speculate that it is large part driven by the rise in life expectancy and increased out-of-pocket medical expenditures (De Nardi, French, and Jones 2010 argue that these factors are key to explain the elderly’s saving behavior in the US). Life expectancy in China rose from 68.9 in 1985 to 75.2 in 2010 (UN population prospects 2012 revision). Out-of-pocket expenditures for people over 65 increased by 22 percent per year between 1995–2002 (Meng and Yeo 2005).
Preferences and Bequests.—We consider agents whose economic life runs for \( J + 1 \) periods. Age is indexed by \( j = 0, \ldots, J \). We let \( c_{j,t}^i \) denote the consumption of an agent of age \( j \) in period \( t \) and country \( i \). In order to obtain a more realistic saving behavior for the old, we augment our baseline model with a bequest motive along the lines of Abel (2001). The lifetime utility of an agent born in period \( t \) in country \( i \) is

\[
U_t^i = \sum_{j=0}^{J} \beta^j u(c_{j,t+j}^i) + \phi \beta^j u(R_{t+j+1}^i b_{t+j}^i),
\]

where \( b_t^i \) denotes the amount of bequest left in period \( t \) by an agent born in period \( t - J \), and \( \phi \) captures the strength of the bequest motive. Agents of age \( j < J \) receive a fraction \( \vartheta_j \) of the bequests left in every period. Thus the amount of bequests received by an agent of age \( j \) in period \( t \), denoted by \( q_{j,t}^i \), is related to \( b_t^i \) as follows

\[
q_{j,t}^i = \vartheta_j L_{t-j}^i b_t^i,
\]

where \( L_t^i \) denotes the size of the generation born in period \( t \).

Production.—The production sector is analogous to the one in the qualitative model. Gross output in country \( i \) is

\[
Y_t^i = (K_t^i)^\alpha \left[ A_t^i \sum_{j=0}^{J} d_{j,t}^i L_{t-j}^i \right]^{1-\alpha} = A_t^i \bar{L}_t^i (k_t^i)^\alpha,
\]

where \( \bar{L}_t^i \equiv \sum_{j=0}^{J} e_{j,t}^i L_{t-j}^i \) denotes the total efficiency-weighted population, and \( k_t^i = K_t^i / (A_t^i \bar{L}_t^i) \) denotes the capital-effective-labor ratio. The efficiency weights \( \{ d_{j,t}^i \} \) capture the shape of the age-income profile in period \( t \) and country \( i \). Indeed, the competitive wage received by agent of age \( j \) in country \( i \) in period \( t \) is \( w_{j,t}^i = e_{j,t}^i (1 - \alpha) A_t^i (k_t^i)^\alpha \). The gross rate of return between \( t - 1 \) and \( t \) is \( R_t^i = 1 - \delta + \alpha (k_t^i)^{\alpha-1} \).

Credit Constraints.—Consider the intertemporal problem of a consumer born in period \( t \) and country \( i \). This agent faces a sequence of gross rates of return \( \{ R_{t+j+1}^i \} \), labor income \( \{ w_{j,t+j}^i \} \), and bequest transfers \( \{ q_{j,t+j}^i \} \). Let \( a_{j,t+j}^i \) denote his end-of-period net asset holdings at age \( j \). Flow budget constraints are

\[
c_{j,t+j} + a_{j,t+j}^i = R_{t+j}^i a_{j-1,t+j-1}^i + w_{j,t}^i + q_{j,t}, \quad 0 \leq j \leq J - 1,
\]

\[
c_{j,t+j} + b_{t+j}^i = R_{t+j}^i a_{j-1,t+j-1}^i + w_{j,t+j}^i,
\]
with \( a_{j,t-1}^i = 0 \). Define the discounted present value of current and future labor income

\[
H_{j,t}^i = w_{j,t}^i + \sum_{\tau=1}^{J-j} \frac{w_{j+t,\tau,t+\tau}^i}{\prod_{s=1}^{\tau} R_{t+s}^i}, \quad 0 \leq j \leq J - 1,
\]

and \( H_{J,t}^i = w_{J,t}^i \). The credit constraint faced by the agent at age \( j \leq J - 1 \) is

\[
a_{j,t+j}^i \geq -\theta_t \frac{H_{j+1,t+j+1}^i}{R_{t+j+1}^i}.
\]

**Equilibrium.**—We solve for the autarkic and integrated steady states of the model, as well as its transitory dynamics for a given evolution of productivity, demographics and efficiency parameters. In autarky, the model equilibrium is given by a path for the capital-effective-labor ratio \( k_t^i \) and bequests \( \{q_{j,t}^i, b_t^i\}_{j=0}^{J-1} \) such that:

(i) all agents maximize their intertemporal utility (equation (17)) with respect to their consumption decisions, subject to the sequence of budget constraints (equations (20)–(21)) and credit constraints (equation (23));

(ii) the consistency condition (equation (18)) between bequests received and bequests left is satisfied;

(iii) the market for capital clears in every period. Under financial integration, a similar definition of an equilibrium holds, with the market for capital clearing globally. When solving for equilibrium, the presence of bequests adds a layer of complexity for the reason that the paths of capital and bequests have to be determined together in a dynamic fixed point problem. A detailed description of the numerical solution method is provided in the online Appendix.

**B. Calibration**

Two economies are considered in the quantitative analysis, the US and China, \( i \in \{US, CH\} \). Each period lasts for 5 years and agents live for 11 periods, which map into the following age brackets: under 25, 25–29, 30–34, … , 65–69, and above 70. We consider an experiment where China grows faster than the US over four decades, from period \(-3\) to period 5 (corresponding to 1973–2013), and where the two economies integrate financially in period 0 (i.e., 1988) after 15 years of accelerated growth in China.37 Table 2 provides a complete summary of the model calibration. We now give a detailed description of our calibration methodology.

**Demographics.**—The age distribution for each country and its evolution over time are obtained from the World Population Prospects data, sampled every five years since 1970 (United Nations 2011).38 For each country, the demographic

---

37 The financial integration of China has been very progressive. A first accelerated phase of financial opening occurred in the late 1980s, followed by another one in the first half of the 1990s as Deng Xiaoping called for a faster pace of reforms (Southern Tour in 1992). See Bekaert, Harvey, and Lundblad (2007) for a detailed chronology. We use 1988 as the integration date since a sustained Chinese current account deficit is observed in the late 1980s. Simulations using 1993 as integration date produce very similar results, with a slightly larger deficit at opening.

38 Data availability limits us to set the demographic structure in 1968 (resp. 1973 up to 2008) to the one measured in the data for the year 1970 (resp. 1975 up to 2010).
growth rate before 1970 and the sequence of growth rates \( g^i_{L,t} \), post-1970 are chosen to best fit the observed age distributions from 1970 to 2010. Although the model does not have enough degrees of freedom to perfectly fit the data, our calibration produces a close match to the overall demographic structure, as shown in Table 3. Implied demographic growth rates are reported in Table 2. The main feature of the data is the large fall in population growth in China starting in 1990, largely a result of fertility controls (one-child policy), and the ensuing rapid aging of the population (see Table 3). Post 2008, the population growth rate is assumed to be 1 percent in both countries.\(^{39}\)

\(^{39}\) This corresponds to the average population growth rate in the US since 1970. We assume that the one-child policy in China will remain at least partially in place, implying slow population growth in line with the most recent years.
Age-Income Profiles.—The relative efficiency parameters \((e_{jt})\) are calibrated to the wage income profile (net of taxes) across age groups, as observed in the CEX for the US and in the UHS data for China. In the US, coefficients are remarkably stable over time. We therefore set US efficiency parameters \((e^t_{jt})\) equal to their 2008 values in every period. Panel A of Figure 8 displays these parameters, while panel B depicts wage income profiles in China, at the beginning and at the end of our sample period. Compared to the US, the Chinese profile reaches its peak earlier and falls more steeply in old age. This feature is particularly striking in the more recent

We only observe a slight flattening of the US age-income profile after age 55 in the recent period. Since this change is quantitatively small, our results are not affected if we take into account time variation in the US income profile. We set it constant in our benchmark calibration to eliminate one possible source of change in age-saving profiles and facilitate the interpretation of our results.
period, due to a marked increase in relative wages for the 30–49 age brackets. For periods $t = 1, \ldots, 4$, we set $e_{j,t}^{CH}$ to the values observed in the data for years 1993, 1998, 2003, and 2008, respectively. Relative efficiency parameters in earlier periods are set to their values in 1992, our first observation year for China. Going forward, we assume that relative efficiency parameters in China converge to the steady-state level of the US after seven decades.

**Credit Constraint Heterogeneity.**—In the absence of direct empirical counterparts to the country-specific credit constraint parameters $(\theta^{US}, \theta^{CH})$, we use various measures of household credit to calibrate the relative tightness of credit constraints across countries. In 1998, the total amount of mortgage debt represented only 1 percent of GDP in China, against 54 percent in the US, and despite some relatively rapid financial development in China over the subsequent decade, the difference remained vast in 2008 (11 percent against 87 percent). Looking more broadly at gross household debt-to-GDP, we observe for the year 2000 a ratio of 4 percent in China against about 67 percent in the US, a sixteen-fold difference. In 2008, the difference is reduced but still as large as about eight-fold (12 percent against 95 percent). In view of these numbers, we restrict the ratio of credit constraint parameters $\theta^{US}/\theta^{CH}$ to be equal to 16 in our benchmark calibration. Our results remain unaffected as long as $\theta^{CH}$ is an order of magnitude smaller than $\theta^{US}$.

**Initial Conditions and Productivity Growth.**—Initial relative productivity levels and subsequent productivity growth rates are set to match the output of China relative to the US over the period 1968–2008, and to allow the capital-effective-labor ratio in China to reach about 70 percent of that of the US in 1988, per Hall and Jones (1999). The resulting annual productivity growth rate for China is 4 percent between 1973–2003, slowing down to 3.75 percent between 2003–2008 and 3 percent between 2008–2013. We assume that US productivity grows at an annual rate of 1.5 percent throughout, and that China grows at the same rate after 2013. Such differences in productivity growth across countries may seem small compared to observed real GDP growth differentials between the US and China (5 percent on average over 1978–2008), but a significant part of Chinese growth in our experiment is driven by increases in labor (increasing share of middle-aged workers, who are the most productive) and capital inputs, and by the increased relative efficiency of workers in the 25–49 age groups.

**Other Calibrated Parameters.**—We use $\alpha = 0.28$ for the share of labor in value added, corresponding to the average share of labor income in the US over the period 1988–2008. The depreciation rate is set to 10 percent on an annual basis. Bequest transfers are assumed to be shared equally across the four age groups between

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41 Sources: Warnock and Warnock (2008), and Chinese National Bureau of Statistics.

42 Sources: McKinsey Global Institute and Federal Reserve. Large differences in terms of financial development are found across other indicators as shown in Appendix B, Table B1. See also IMF (2006) documenting the relative growth in household credit in the US and China.

43 In particular, our results are virtually unchanged if we set $\theta^{US}/\theta^{CH}$ equal to 8, corresponding to the ratio of gross household debt-to-GDP in the US versus China at the end of our sample period.

44 We use OECD Quarterly National Accounts data, correcting for mixed income as in Gollin (2002).
The remaining parameters are the elasticity of intertemporal substitution $\sigma$, the discount factor $\beta$, the bequest motive parameter $\phi$, and the credit constraint parameter $\theta^{US}$. These are calibrated to savings data in the integration period, while targeting the ratio of bequest-to-GDP in the US in that period. Specifically, let $s_0^i$ and $s_{j,0}^i$ denote the model-implied aggregate saving rate and the saving rate of agents of age $j$ in country $i$ in the integration period (using ten-year age brackets), and let $s_0^{i,d}$ and $s_{j,0}^{i,d}$ denote their counterparts in the data. $b_0^{US,d} = 2.65\%$ is the targeted US bequest-to-GDP ratio in the data and $b_0^{US}$ the model counterpart. We search over a large grid the vector of parameter values $\psi \equiv [\sigma, \beta, \phi, \theta^{US}]$ that minimizes the distance

$$\sum_i |s_0^i(\psi) - s_0^{i,d}| + \sum_i \sum_j \omega_j(s_{j,0}^i(\psi) - s_{j,0}^{i,d})^2$$

subject to

$$b_0^{US}(\psi) = b_0^{US,d},$$

where the weights $\omega_j$ on different age groups in country $i$ satisfy $\sum_j \omega_j = 1$ and reflect their shares in the total effective population. The optimal parameter values are described in Table 2. We obtain a value of 0.32 for the elasticity of intertemporal substitution $\sigma$, in the lower range of empirical estimates. We later perform sensitivity analysis for higher values of $\sigma$. Our estimate for $\beta$ is also in the lower range of conventional values. This is because our calibration aims at matching household savings in 1988, rather than aggregate savings, leading to a lower value for $\beta$.

C. Results

We now present the results for our benchmark calibration. When evaluating the performance of the model, one should keep in mind that free parameters are calibrated to match savings data at the date of integration, and not using any post-1990 data.

On the aggregate level, the qualitative implications of the three-period model are preserved. For comparison purposes, Figure 9 displays the empirical counterpart to the model for household savings and current account over the period 1978–2008, and the five-year US interest rate over the post-integration period (1988–2008), normalized to its value at time of opening. The model predicts a significant...

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45 In PSID data, Hendricks (2001) documents that a third of bequests goes to children, another third to other beneficiaries (e.g., grandchildren), and the remaining third to death expenses, taxes, and charitable donations. The way bequests are distributed is immaterial for our results.

46 When comparing micro data and model outcomes, we use age-saving profiles with ten-year age brackets, as micro data aggregated by finer age groups were unavailable for the US at the beginning of the period.

47 We set $b_0^{US,d} = 2.65\%$, as documented by Gale and Scholz (1994) and De Nardi (2004) from the Survey of Consumer Finance for the year 1986. Hendricks (2001) finds a similar number using PSID data.

48 Putting equal weight on different age groups does not affect our results. We adopt absolute deviations for the macro variables instead of squared differences, as otherwise the optimization would only weigh micro outcomes since macro discrepancies are on average much larger than macro ones.

49 See Appendix B for a description of the data. We focus on the US nominal interest rate in a period of stable inflation (post-1988). We do not show investment rates since our calibration is targeted toward household savings and thus cannot match the level of aggregate savings and investment.
increase of the aggregate household saving rate in China (+5.7 percentage points between 1988–2008) and a fall in the US (by about one percent over the same period), explaining about 30 percent of the savings divergence observed in the data. The model falls especially short of explaining the overall increase in savings in China. At the time of opening, China runs a small current account deficit, due to a growth-driven investment boom, before turning into a persistent surplus: the current account improves by about 7 percent between 1988 and 2008. In the data, we observe a similar pattern: China was running small current account deficits in the early 1990s—as did other Asian countries, to an even greater extent—before moving into a surplus in the late 1990s. For the US, the model implies a surplus of about 1 percent of GDP at opening, followed by a persistent deficit reaching 3 percent in 2008. This pattern of current account imbalances arises as the standard neoclassical forces (capital flowing toward the capital-scarce and fast growing economy) initially dominate when China is relatively small; but as its relative size in the world economy rapidly increases, the world interest rate significantly drops (as in the data but to a lower extent), and the asymmetric saving responses across countries induce a reversal in current account positions.

Turning to micro-level predictions, Figure 10 juxtaposes the model-implied age-saving profiles in 1988 and 2008 with those estimated from the data. For the US, the model matches the increasing spread in the saving rates of the young (under 25) and middle-aged (45–54) observed in the data over two decades. Yet it
overpredicts the fall in the young’s saving rate over this period and the saving rate of older workers (55–64) in 2008. For China, the model provides a reasonably good fit to the relatively flat age-saving profile observed at the beginning of the period. Over the subsequent two decades, the model-implied saving rates for individuals between 35–64 rise substantially (by 5 to 10 percentage points), although by less than in the data. The model falls especially short of explaining the increase in the savings of the 25–34. For individuals under 25, due to tight credit constraints in every period, the model predicts a roughly constant saving rate, instead of a slight increase in the data—in sharp contrast with the US. As a result, the model captures the increasing discrepancy between the saving rates of the very young and the middle-aged over time. At the other end of the age-saving profile however, the model is unable to explain the large increase in the saving rate of the elderly.50

In sum, our model can explain, with one mechanism, a significant portion of the rise in saving rates for most age groups in China, and the simultaneous increase in borrowing of the young in the US. However, it falls short of explaining the overall increase in China, pointing toward mechanisms more specific to the Chinese economy.

50The sharp rise in the elderly’s saving rate is most likely driven by factors outside of the model, such as a rising life expectancy and increasing out-of-pocket medical expenditures over the last ten years in China. De Nardi, French, and Jones (2010) argue these factors are key to understanding the saving rates of the old in the US.
D. Alternative Calibrations and Sensitivity Analysis

To provide further intuition on the channels driving the dynamics of savings across countries and age groups, we now examine the output of the model under alternative calibrations. In particular, we investigate the role played by the value of the elasticity of intertemporal substitution, the degree of asymmetry of credit constraints and the shape of income profiles. We also assess the quantitative contribution of changes in the Chinese income profile and of fast aging in China. For the sake of brevity, we display graphically only variables that exhibit significant changes relative to our benchmark calibration.

Elasticity of Intertemporal Substitution.—We first investigate the sensitivity of our results to the value of the elasticity of intertemporal substitution \( \sigma \). Empirical estimates and calibrated values typically range between 0 and 1, but are usually slightly higher than our benchmark value (see discussion in Guvenen 2006). Figure 11 shows the evolution of aggregate variables for higher values of \( \sigma \), keeping other parameters to their benchmark values. Higher values of \( \sigma \) generate a stronger divergence of saving rates across countries, amplifying the fall in US savings as the income effect gets weaker. In contrast, prices (interest rates) respond less over time and the fall of the interest rate is muted.

Asymmetry of Credit Constraints.—We next investigate the quantitative role of credit constraint heterogeneity—by reducing the asymmetry of the \( \theta \)’s across countries. First, the ratio \( \theta^{US}/\theta^{CH} \) is lowered to 8. This smaller difference in financial development across countries is more in line with the difference in the depth of household debt markets observed toward the very end of the sample period. The dynamics of the model (not reported here for the sake of brevity) are almost identical to those of the benchmark model. This indicates that our findings are robust to increasing financial development in China over the period as long as the difference with the US remains large throughout.

For comparison purposes, we also set the Chinese credit constraint parameter to the US level—shutting down any asymmetry in financial development \( (\theta^{US}/\theta^{CH} = 1) \). This experiment yields markedly different results (Figure 12). The
aggregate saving rate falls substantially in China upon integration, while increasing in the US. China experiences a very large current account deficit (−9.0 percent) at the time of opening, which persists over two decades. On the micro level, age-saving profiles for China are also markedly different from our benchmark calibration. In particular, the 1988 profile exhibits a much more pronounced inverted-U shape, given the massive borrowing of the young Chinese households in anticipation of faster growth. Simulations of the model in the absence of any credit constraints generate similar, counterfactual results, thus affirming the importance of both the presence of credit constraints and their being tighter in China for our findings.

Flat Age-Income Profiles.—We next demonstrate the importance of the shape of the income profile. The experiment sets relative efficiency parameters to unity at all ages in both countries, while keeping all other parameters at their benchmark values. As in the previous experiment, fast-growing China sees a large fall in aggregate savings and a massive current account deficit (Figure 13). There are two aspects of the shape of the calibrated age-income profiles that matter for our results. The downward-sloping part of the profile gives stronger saving motives to the middle-aged, and at the same time limits the wealth effect of growth. The upward-sloping part of the profile is even more crucial as credit constraints only matter to the extent that younger individuals have a desire to borrow. With flat age-income profiles, credit constraints are not binding in any country in the steady state (despite aggregate productivity growth), thus bringing the model dynamics very close to a frictionless neoclassical representative agent model. More generally, this experiment illustrates the dynamics induced by a growth shock in a model where most agents have a desire to save in the steady state.51

Contribution of Changes in Income Profile in China.—As noted earlier, the age-income profile in China changed significantly over time. Toward the end of the sample period, the income profile reaches a higher peak at a younger age and falls

51 However, such a model could still produce an increase in savings in a fast-growing country if the saving motive happens to be stronger upon fast growth, as pointed out in Carroll and Jeanne (2009).
more steeply in old age (see Figure 8). By providing further incentives to save for retirement and by reducing wealth effects for middle-aged consumers, this evolution contributes to the rise in Chinese savings. We assess the quantitative importance of this channel by keeping the relative efficiency parameters in China \((e_{jt}^{CH} = 1\) for all \(j\)) and all other parameters remain at their benchmark values displayed in Table 2. Figure 14 depicts the evolution of macro variables of interest, along with the 2008 Chinese age-savings profile. Compared to our benchmark, aggregate savings and current
account surplus in China rise less over the period 1988–2008, due to a smaller increase in the middle-aged’s saving rates over the period. As a consequence, the fall in interest rate is reduced.

**Contribution of Fast Aging in Asia.**—Since the early 1970s, China has experienced an accelerated demographic transition due to fertility-restriction policies. The benchmark experiment takes this demographic evolution into account. To investigate its quantitative role, we consider an alternative experiment in which demographic growth in China remains at 3 percent per year until 2008 (a scenario where the fertility rate stays identical to its 1968 value) before converging to its steady-state value of 1 percent. All other parameter values remain unchanged from the benchmark case. Results for the evolution of macro variables, displayed in Figure 15, are not very different from the benchmark simulation. Aggregate savings in China increase at a slower pace in the two decades following integration, while the interest rate decreases by less over that period. Indeed in the absence of a demographic transition, the share of middle-aged savers does not increase. As a result of this composition effect, the extent of the rise in savings in China (and the fall in the world interest rate) is smaller compared to our benchmark simulation. Higher demographic growth also limits the drop in interest rate by raising the marginal productivity of capital. These effects tend to dominate in the short run—but in the long run, as China reaches an even greater weight in the world economy, the world interest rate falls further, causing a larger divergence in savings.

**IV. Conclusion**

This paper develops a life-cycle theory of savings in large open economies with heterogeneous levels of household credit constraints. We show that faster growth in (more constrained) emerging markets can lead to a divergence in household saving rates across developed and emerging economies, as well as a persistent decline in the world interest rate. The theory provides, with a single mechanism, micro-foundations

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The change in demography has little impact on age-savings profiles.
to the global saving glut (Bernanke 2005) and a potential answer to the “allocation puzzle” (Gourinchas and Jeanne 2013). The age-saving profiles estimated from US and China’s survey data are broadly consistent with the life-cycle hypothesis, and at the same time lend empirical support to our theoretical predictions on the contrasted evolution of saving profiles between these two economies. A quantitative version of the model calibrated to macro and micro data for the US and China can explain about a third of the divergence in their aggregate household saving rates and a substantial share of the evolution of saving rates across age-groups in both countries. Our model, however, falls short of explaining the full extent of the “Chinese saving puzzle” (Modigliani and Cao 2004).

In examining micro-level evidence for China, we point out the biases that may arise from employing household-level data to estimate age-specific saving behavior. Our endeavors to correct for these biases allow us to establish new empirical facts. The novel evidence we provide, along with remaining discrepancies between data and theory, can potentially form the basis for future research. In particular, the saving behavior of the old in China warrants further study. Plausible explanations for their puzzling behavior, not considered in this model, include the evolution of pension systems and health insurance or changes in life expectancy. Finally, our theory can be easily applied to a larger cross-section of countries, thus providing an additional dimension for assessing its performance in accounting for savings and current account patterns across countries.

APPENDIX A: Proofs

PROOF OF THEOREM 1:

Consider a country $i$ with credit constraint parameter $\theta^i$. Note that for $\delta = 1$, we have $R^i_t = \alpha (k^i_t)^{1-\alpha}$. The law of motion for $k^i_t$ is implicitly given by

$$k^i_{t+1} + \beta^{-\sigma} \alpha^{1-\sigma} (k^i_{t+1})^{\alpha(1-\sigma)+\sigma}$$

$$= \frac{(1 - \theta^i)(1 - \alpha)}{(1 + g_A^i t + 1)(1 + g_L^i t)\{1 + e^i_{t+1}(1 + g_L^i t + 1) + \theta^i t 1 - \alpha \alpha \}} (k^i_t)^{\alpha}.$$ 

If a steady-state level of capital $k^i$ exists, it therefore satisfies

$$k^i + \beta^{-\sigma} \alpha^{1-\sigma} (k^i)^{\alpha(1-\sigma)+\sigma}$$

$$= \frac{(1 - \theta^i)(1 - \alpha)}{(1 + g_A^i)(1 + g_L^i)\{1 + e(1 + g_L^i) + \theta^i t 1 - \alpha \alpha \}} (k^i)^{\alpha}.$$ 

Substituting the steady-state gross rate of return $R^i = \alpha (k^i)^{1-\alpha}$, we can write

$$1 + \beta^{-\sigma} (R^i)^{1-\sigma} = C(\theta^i) R^i,$$
with \( C(\theta) = \frac{(1 - \alpha)(1 - \theta)}{(1 + g_A)(1 + g_L)\{\alpha[1 + e(1 + g_L)] + \theta(1 - \alpha)\}} \). Note in particular that \( \frac{\partial C}{\partial \theta} < 0 \). If \( \sigma = 1 \), the steady-state exists, is unique, and satisfies

\[
R^i = \frac{1 + \beta}{\beta C(\theta)}
\]

\[
= (1 + g_A)(1 + g_L)\frac{1 + \beta \alpha[1 + e(1 + g_L)] + \theta^i(1 - \alpha)}{(1 - \alpha)(1 - \theta^i)}.
\]

For \( \sigma < 1 \), \( R^i \) is such that \( v_\theta(R^i) = 0 \), where \( v_\theta(R) \equiv 1 + \beta^{-\sigma} R^{1-\sigma} - C(\theta) R \) for \( R > 0 \). We now show that \( v_\theta(R) = 0 \) has a unique solution. Differentiating \( v_\theta \) with respect to \( R \), we get

\[
\frac{\partial v_\theta}{\partial R} = \beta^{-\sigma}(1 - \sigma) R^{-\sigma} - C(\theta),
\]

which implies the following equivalence:

\[
\frac{\partial v_\theta}{\partial R} \geq 0 \iff R \leq \frac{1}{\beta} (1 - \sigma) \frac{1}{2} C(\theta)^{-\frac{1}{2}}.
\]

Hence \( v_\theta \) is increasing for \( R \in \left[0, \frac{1}{\beta} (1 - \sigma) \frac{1}{2} C(\theta)^{-\frac{1}{2}}\right] \) and decreasing for \( R \geq \frac{1}{\beta} (1 - \sigma) \frac{1}{2} C(\theta)^{-\frac{1}{2}} \). We also have \( \lim_0 v_\theta(R) = 1 > 0 \) and \( \lim_{\infty} v_\theta(R) = -\infty \). Since \( v_\theta \) is a continuous function, it follows that \( v_\theta(R) = 0 \) has a unique solution, \( R(\theta) \). This is our first result. We also note in passing that our characterization of \( v_\theta \) implies

\[
(A1) \quad R < R(\theta) \iff v_\theta(R) > 0.
\]

We now show that countries with a higher \( \theta \) have a higher rate of return in autarky steady state. Consider \( \theta^i < \theta^j \) and let \( R^i = R(\theta^i) \) (resp. \( R^j = R(\theta^j) \)) denote the well-defined solution to \( v_\theta(R^i) = 0 \) (resp. \( v_\theta(R^j) = 0 \)). For any \( R > 0 \), we can write

\[
v_\theta(R) - v_\theta(R) = (C(\theta^i) - C(\theta^j)) R > 0,
\]

where the first equality follows from the definition of \( v_\theta \), and the inequality follows from \( \frac{\partial C}{\partial \theta} < 0 \). In particular, for \( R = R^i \), we have \( v_\theta(R^i) - v_\theta(R^i) = v_\theta(R^i) > 0 \), which by remark (A1) above, is equivalent to \( R^i < R^j \). We therefore have shown that \( \theta^i < \theta^j \) if and only if \( R^i < R^j \). This establishes our second result, \( \frac{\partial R^i}{\partial \theta^j} > 0 \), and the fact that \( \frac{d k^i}{d \theta^i} < 0 \) follows immediately. It is worthwhile to note that the theorem also holds for \( \sigma > 1 \). Our proof naturally extends to that case. \( \blacksquare \)
PROOF OF PROPOSITION 1:
For $\delta = 1$ and any $\sigma \leq 1$, one can easily show that the steady-state world interest rate $R$ satisfies

\begin{equation}
F(R) = \sum_i \frac{\lambda^i(1 - \theta^i)}{\sum_j \lambda^j(1 - \theta^j)} F(R^i),
\end{equation}

where $F(x) \equiv x/(1 + \beta^{-\sigma}x^{1-\sigma})$ and $R^i$ denotes the autarky steady-state interest rate in country $i$. The bounds on $R$ in (14) follow from $F'(\cdot) > 0$. Note that the proposition also holds for $\sigma > 1$. ■

PROOF OF PROPOSITION 2:
The result follows immediately from equation (A2). ■

PROOF OF PROPOSITION 3:
The result follows immediately from Proposition 2 along with the observation that $\frac{\partial(S/Y)}{\partial \theta} < 0$ and $\frac{\partial^2(S/Y)}{\partial \theta \partial R} > 0$. ■

PROOF OF PROPOSITION 4:
We first derive the expressions for savings by age groups given in Section IE. The level of saving of the young in country $i$ and period $t$ is

$$S^i_{y,t} = L^i_{y,t}(w^i_{y,t} - c^i_{y,t}) = L^i_{y,t}a^i_{y,t+1} = -L^i_{y,t}R^i_{t+1}w^i_{m,t+1},$$

where the last equality follows from (9). Using (3) and normalizing by $Y^i_t = \left[A^i_t(e^i_tL^i_{y,t} + L^i_{m,t})\right]k^\alpha_t$, we get

$$\frac{S^i_{y,t}}{Y^i_t} = -(1 + g^i_{A,t+1})\frac{1 + g^i_{L,t}}{1 + e^i_t(1 + g^i_{L,t}) R^i_{t+1}} \frac{\theta^i}{(1 - \alpha)} \left(\frac{k^i_{t+1}}{k^i_t}\right)^\alpha$$

$$= -(1 + g^i_{A,t+1})\frac{1 + g^i_{L,t}}{1 + e^i_t(1 + g^i_{L,t})} \frac{(1 - \alpha)\theta^i}{k^\alpha_t R^i_{t+1}} \left(\frac{\alpha}{R^i_{t+1} - 1 + \delta}\right)^{1-\alpha},$$

where the second equality obtains by expressing $k^i_{t+1}$ as a function of $R^i_{t+1}$.

The level of saving of the middle-aged in country $i$ and period $t$ is

$$S^i_{m,t} = L^i_{m,t}[w^i_{m,t} + (R^i_t - 1)a^i_{y,t} - c^i_{m,t}]$$

$$= L^i_{m,t}\left[ (w^i_{m,t} + R^i_t a^i_{y,t} - c^i_{m,t}) - a^i_{y,t} \right]$$

$$= L^i_{m,t}(a^i_{m,t+1} - a^i_{y,t})$$

$$= L^i_{m,t}\left[ \frac{1 - \theta^i}{1 + \beta^{-\sigma}R^i_{t+1} - \alpha} + \frac{\theta^i}{R^i_t} \right]w^i_{m,t},$$
where the third equality follows from (6), and the last equality follows from (9) and (10). Using (3) and normalizing by GDP, we get

\[
\frac{S_{m,t}^i}{Y_t^i} = \frac{1}{1 + e_t^i(1 + g_{L,t}^i)} \left[ \frac{1 - \theta_t^i}{1 + \beta^{-\sigma}R_{t+1}^{1-\sigma}} + \frac{\theta_t^i}{R_t} \right] (1 - \alpha).
\]

Finally, the level of saving of the old in country \(i\) and period \(t\) is given by

\[
S_{o,t}^i = r_{K,t} K_t^i + (R_t - 1) \left[ L_{m,t-1} a_{m,t}^i - K_t^i \right] - L_{o,t} c_{o,t}^i.
\]

The first two terms in the expression correspond to the rental rate earned on capital and to interests received on other savings. Using the relationship between \(r_{K,t}\) and \(R_t\), and substituting for \(c_{o,t}^i\) from (7), we can write

\[
S_{o,t}^i = (R_t - 1 + \delta) K_t^i + (R_t - 1) L_{o,t} a_{m,t}^i - (R_t - 1) K_t^i - L_{o,t} R_t a_{m,t}^i
\]

\[
= -L_{o,t} a_{m,t}^i + \delta K_t^i
\]

\[
= -L_{o,t} \frac{1 - \theta_t^i}{1 + \beta^{-\sigma}R_t^{1-\sigma}} w_{m,t-1}^i + \delta K_t^i,
\]

where the last equality follows from (10). The last term is dropped when net savings are considered. Normalizing by GDP, we then get

\[
\frac{S_{o,t}^i}{Y_t^i} = -\frac{1}{1 + g_{A,t}^i} \frac{1}{1 + g_{L,t-1}^i + e_t^i(1 + g_{L,t}^i)} \left[ \frac{1}{1 + \beta^{-\sigma}R_t^{1-\sigma}} (1 - \theta_t^i)(1 - \alpha) \left( \frac{k_{t-1}}{k_t} \right)^\alpha \right].
\]

Looking at savings by the young, we observe that

\[
\frac{\partial}{\partial R_{t+1}} \left( \frac{S_y,t}{Y_t^i} \right) > 0, \quad \frac{\partial^2}{\partial \theta_t^i \partial R_{t+1}} \left( \frac{S_y,t}{Y_t^i} \right) > 0.
\]

Looking at savings by the middle-aged, we observe that when \(\sigma < 1\)

\[
\frac{\partial}{\partial R_{t+1}} \left( \frac{S_{m,t}}{Y_t^i} \right) < 0, \quad \frac{\partial^2}{\partial \theta_t^i \partial R_{t+1}} \left( \frac{S_{m,t}}{Y_t^i} \right) > 0.
\]

These four inequalities prove Proposition 4. ■
Appendix B: Data

A. Aggregate Data (For the Figures Shown in the Introduction)

**Developed Countries:** Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Israel, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

**Asian Countries:** Bangladesh, Cambodia, China, Fiji, Hong Kong SAR, China, India, Indonesia, Kiribati, Korea, Lao P.D.R., Malaysia, Maldives, Nepal, Pakistan, Papua New Guinea, Philippines, Singapore, Solomon Islands, Sri Lanka, Thailand, Tonga, Vanuatu, Vietnam.

**Data on Savings, Private Savings, and Current Account (percent of GDP):** Data for Emerging Asia and Developed Countries are from World Development Indicators (World Bank), Penn World Tables and Asian Development Bank (ADB). Private savings are computed as the difference between Aggregate saving and Primary Government Surplus. Data for Primary Government Surplus in Asian countries are only available starting in 1988 for a large sample of Asian countries.

**Data on Household Saving Rates:** Data for Developed Countries are from the OECD. For the US, the OECD series is used in Figure 1 for consistency, otherwise we use NIPA personal saving rate. Data for India are from the Central Bank of India. Data for China are from CEIC Data based on Urban Household Survey (UHS).

B. Data for the United States

Definitions

- *Household disposable income:* sum of individual income net of taxes (in USD).
- *Household expenditure:* household consumption expenditures (in USD).
- *Household saving:* difference between household disposable income and consumption expenditure (in USD).
- *Household saving rate:* household saving divided by disposable income.

**Consumer Expenditures Survey Data (CEX):** Annual data over the period 1986–2008 for consumption expenditures and income. Disaggregated by age groups (6 age groups): under 25, 25–34, 35–44, 45–54, 55–64, and above 65. Disaggregated by sectors of expenditures. The sectors covered in the CEX data are: Food and alcoholic beverages, Shelter, Utilities and public services, Household expenses, Clothing and apparel, Vehicle purchases, Gas and motor oil, Other vehicle expenses, Public transportation, Health, Entertainment, Education, Tobacco, Miscellaneous and cash contributions, Life/personal insurance.

**NIPA Data from US Bureau of Economic Analysis (BEA):** Consumption and income data for 1986–2008. Consumption expenditures data are disaggregated by sectors of expenditures. We match sectors in NIPA with the corresponding sectors in
CEX. Only two categories in NIPA consumption expenditures (accounting for about 1 percent of total expenditures) do not appear in CEX data (Net foreign travel and expenditures abroad by US residents, and Final consumption expenditures of non-profit institutions serving households). Aggregate consumption expenditures from CEX data do not match aggregate NIPA data, as a result of underreporting of consumption in CEX—a bias which has increased over time. Income displays a similar bias but without trend.

C. Data for China

Definitions

Household disposable income: sum of individual disposable income net of taxes within a household.

Household consumption expenditures: sum of consumption expenditures within household.

Household savings: difference between household disposable income and household consumption expenditure. Rates are computed by dividing by household disposable income.

Individual savings: difference between individual disposable income and individual consumption expenditure (estimated). Rates are computed by dividing by individual disposable income.

Urban Household Survey Data (UHS): Annual data for the year 1986 and over the period 1992–2009 for consumption expenditures, income and household characteristics (number of household members, age of household members, employment status of household members...), for a large sample of urban households in China. Starting from 1992, households are chosen randomly—based on several stratifications at the provincial, city, country, township, and neighborhood levels—and are expected to stay in the survey for 3 years. The 1986 survey covers 47,221 individuals in 12,185 households across 31 provinces. The 1992–2009 surveys cover 112 prefectures across 9 representative provinces (Beijing, Liaoning, Zhejiang, Anhui, Hubei, Guangdong, Sichuan, Shaanxi, and Gansu). The sample size has been extended over time from roughly 5,500 households in the 1992–2001 surveys to nearly 16,000 households in the 2002–2009 surveys. Disposable income is provided at the individual level for the years 1992–2009 and at the household level for all years. Data for consumption expenditures are given at the household level. When estimating individual consumption expenditures and savings, we restrict our attention to individuals above 25 and income earners aged between 19–24 (annual income above 100 yuans). All individuals below 18 and those under 25 who do not qualify as income earners (unless they are the household head’s spouse) are considered as children, whose consumption is imputed to other household members (typically their parents).

53 Information on individual income is available for 1986, but the data are very noisy and therefore not reliable.
54 In our final specification, old dependents (i.e., individuals above 65 who do not qualify as income earners and living with their offspring) are treated in the same way as children, but the treatment of old dependents has little...
Chinese Households Income Project Data (CHIP): CHIP survey data are available for the years 1995 and 2002. Income and consumption by age for these two years are consistent across the UHS and CHIP datasets.

D. Financial Development and Household Credit in the United States and China

Table B1, computed from various sources, exhibits large differences in financial development and household credit between the US and China. Data for the US are from Federal Reserve for Household Debt over GDP, Warnock and Warnock (2008) for mortgage debt over GDP. Data for China are from McKinsey Global Institute for Household Debt over GDP, Chinese National Bureau of Statistics for mortgage debt over GDP. Outstanding domestic private debt securities to GDP are from GFDD World Bank for both countries. Data on mortgage and credit card penetration in 2011 are from Global Financial Inclusion Database (World Bank) for both countries.

REFERENCES


Table B1—Indicators of Financial Development and Household Credit

<table>
<thead>
<tr>
<th>Variable</th>
<th>Year</th>
<th>US</th>
<th>China</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gross household debt (percent of GDP)</td>
<td>2000</td>
<td>66.7</td>
<td>4.0</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>95.2</td>
<td>12.0</td>
</tr>
<tr>
<td>Mortgage debt (percent of GDP)</td>
<td>1998</td>
<td>53.8</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>86.5</td>
<td>10.9</td>
</tr>
<tr>
<td>Outstanding private debt securities (percent of GDP)</td>
<td>1990</td>
<td>68.2</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>1998</td>
<td>80.5</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>2008</td>
<td>120.0</td>
<td>15.9</td>
</tr>
<tr>
<td>Percent of population above 15 with mortgage</td>
<td>2011</td>
<td>33.4</td>
<td>5.0</td>
</tr>
<tr>
<td>Percent of population above 15 with credit card</td>
<td>2011</td>
<td>61.9</td>
<td>8.2</td>
</tr>
</tbody>
</table>


