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Abstract

We examine how firm heterogeneity influences aggregate welfare through endogenous firm selection. We consider a homogeneous firm model that is a special case of a heterogeneous firm model with a degenerate productivity distribution. Keeping all structural parameters besides the productivity distribution the same, we show that the two models have different aggregate welfare implications, with larger welfare gains from reductions in trade costs in the heterogeneous firm model. Calibrating parameters to key U.S. aggregate and firm statistics, we find these differences in aggregate welfare to be quantitatively important (up to a few percentage points of GDP). Under the assumption of a Pareto productivity distribution, the two models can be calibrated to the same observed trade share, trade elasticity with respect to variable trade costs, and hence welfare gains from trade (as shown by Arkolakis, Costinot and Rodriguez-Clare, 2012); but this requires assuming different elasticities of substitution between varieties and different fixed and variable trade costs across the two models.

KEYWORDS: firm heterogeneity, welfare gains from trade
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1 Introduction

Over the last decade, new theories of heterogeneous firms in differentiated product markets have been developed to account for features of disaggregated trade data. Taking stock, Arkolakis, Costinot and Rodriguez-Clare (2012) ask whether these new insights for micro data have altered our understanding of the aggregate welfare gains from trade. They show that there exists a class of heterogeneous and homogeneous firm models in which a country's domestic trade share is a sufficient statistic for the aggregate welfare gains from trade. Thus, if the different models generate the same domestic trade share, then they also deliver the same welfare gains from that trade. Based on this result, they summarize the contribution of new theories of heterogeneous firms to our understanding of the aggregate welfare implications of trade as "So far, not much."

In this paper, we compare a heterogeneous firm model to a homogeneous firm model that is a special case with a degenerate productivity distribution. All other structural parameters are assumed to be the same in the two models. The heterogeneous firm model features an additional adjustment margin that is absent from the homogeneous firm model, namely the endogenous changes in aggregate productivity that result from the entry and exit decisions of heterogeneous firms. As a result, the two models have different aggregate welfare implications. Calibrating to an initial autarky equilibrium or to an initial open economy equilibrium, we show that welfare is the same in the two models for the calibrated value of trade costs but is strictly higher in the heterogeneous firm model than in the homogeneous firm model for all other values of trade costs.

The intuition for our results involves revealed preference arguments of the kind commonly used in international trade. We start from initial equilibria in the heterogeneous and homogeneous firm models that feature identical aggregate statistics (including, crucially, welfare). In the homogeneous firm model, aggregate productivity is exogenous, and hence remains unchanged following changes in trade costs. In contrast, in the heterogeneous firm model, aggregate productivity responds to changes in trade costs, associated with the endogenous adjustments in the productivity cutoffs for the domestic and export market. However, the open economy equilibrium in the heterogeneous firm model is efficient: a welfare-maximizing social planner faced with the same production and entry technologies would choose the same allocation as the market equilibrium. Since the social planner chooses a new allocation following the change in trade costs that has different aggregate productivity from the initial allocation, this new allocation must yield at least as high (and in general higher) welfare than another feasible allocation in which aggregate productivity is held constant. But this constant productivity allocation is identical to the new equilibrium in the homogeneous firm model following the change in trade costs. Therefore the new equilibrium in the heterogeneous firm model must yield at least as high (and in general higher) welfare than the new equilibrium in the homogeneous firm model.

The additional adjustment margin of heterogeneous firms entry and exit decisions expands the production set and enables higher welfare to be achieved in the heterogeneous firm model than in the homogeneous firm model following the change in trade costs. In the homogeneous firm model,

either all firms export or no firm exports. In contrast, in the heterogeneous firm model, there is the possibility of reallocating resources from low productivity firms that only serve the domestic market to higher productivity firms that export. Therefore the level of welfare that the social planner can achieve in a model with this extra adjustment margin must be at least as high (and in general higher) than the level of welfare that can be achieved in a model without it.

After developing these results, we discuss their relationship with Arkolakis, Costinot and Rodriguez-Clare (2012)’s result that a country’s domestic trade share is a sufficient statistic for the welfare gains from trade in the special case of a Pareto productivity distribution. The two sets of results are consistent with one another because they reflect fundamentally different approaches to comparing models. Our approach, which we refer to as the “micro” approach, is to compare models that differ in their productivity distribution but retain the same values for all other structural parameters including trade costs. In contrast, the approach of Arkolakis, Costinot and Rodriguez-Clare (2012), which we refer to as the “macro” approach, is to compare models that are calibrated to have the same reduced-form elasticity of trade with respect to trade costs and the same endogenous domestic trade share.

The macro approach has some advantages: it ensures that both models match key features of international trade data and compares the two models at empirically-observed moments. However, calibrating the two models to the same reduced-form trade elasticity involves assuming different structural demand parameters (different elasticities of substitution between varieties). Furthermore, calibrating the two models to the same endogenous domestic trade share involves assuming different trade costs (different values of both fixed and variable trade costs). Therefore, the macro approach changes the degree of firm heterogeneity, the ability of consumers to substitute between varieties and the value of trade costs between the two models. In contrast, our micro approach only changes the degree of firm heterogeneity between models, which enables us to isolate the effect of the degree of firm heterogeneity on aggregate welfare.

The macro approach’s assumption that there is a single empirical moment summarizing the elasticity of trade with respect to trade costs is also quite restrictive. Even under the assumption of a Pareto productivity distribution, the heterogeneous firm model has different elasticities of trade with respect to trade costs depending on whether these trade costs are variable or fixed. More broadly for general continuous productivity distributions (including Pareto distributions that are truncated from above), the elasticity of trade with respect to either variable or fixed trade costs is an endogenous variable. In this general case, we show how this elasticity varies with the magnitude of trade costs and the relative importance of fixed versus variable trade costs. Our micro approach can be applied to this more general case where the trade elasticity responds to the change in trade costs.

Finally, the fact that calibrating to the same domestic trade share involves assuming different trade costs in the two models has important implications for the interpretation of the macro approach. In some cases, it may be precisely the consequences of a given level of trade costs for trade and welfare that is of ultimate interest. The micro approach highlights that the answer to this question depends on the degree of firm heterogeneity.

The remainder of the paper is organized as follows. In Sections 2 and 3, we begin by contrasting the closed and open economy equilibria of models with and without firm heterogeneity. Given the same structural parameters and aggregate productivity, Section 2 shows that the heterogeneous and homogeneous firm models have the same closed economy welfare. Section 3 shows that the welfare gains from trade are higher in the heterogeneous firm model than in the homogeneous firm model. Section 4 provides further economic intuition for our results by showing that the market equilibrium in the heterogeneous firm model is efficient and developing our revealed preference argument. Section 5 examines the relationship between welfare and a country's trade share with itself. Section 6 shows that our results apply for a comparison of two open economy equilibria with different values of trade costs. Section 7 considers the special case of a Pareto productivity distribution. Section 8 shows that the differences in the aggregate implications of heterogeneous and homogeneous firm models are quantitatively relevant. Section 9 concludes.

2 Closed Economy

We compare the canonical heterogeneous and homogeneous firm models of Melitz (2003) and Krugman (1980).¹ The homogeneous firm model is a special case of the heterogeneous firm model with a degenerate productivity distribution.

2.1 Heterogeneous Firm Model

The specification of preferences, production and entry is the same as Melitz (2003).² There is a continuum of firms that are heterogeneous in terms of their productivity $\varphi \in (0, \infty)$, which is drawn from a common distribution $g(\varphi)$ after incurring a sunk entry cost of f_e units of labor. Labor is the sole factor of production. Production involves a fixed production cost and a constant marginal cost that depends on firm productivity, so that $l(\varphi) = f_d + q(\varphi)/\varphi$ units of labor are required to supply $q(\varphi)$ units of output. Consumers have constant elasticity of substitution (CES) preferences defined over the differentiated varieties supplied by firms. The equilibrium revenue for a firm with productivity φ is then:

$$r(\varphi) = RP^{\sigma-1}p(\varphi)^{1-\sigma},$$

where R is aggregate revenue; P is the aggregate CES price index; and $p(\varphi)$ is the price chosen by a firm with productivity φ . Profit maximization implies that equilibrium prices are a constant mark-up over marginal cost:

$$p(\varphi) = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}.$$

Hence profits are a constant fraction of revenue minus the fixed production cost:

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - wf_d.$$

¹A web-based technical appendix contains the derivations of all expressions in the paper.

²Following most of the subsequent international trade literature, including Arkolakis, Costinot and Rodriguez-Clare (2012), we consider a static version of Melitz (2003) in which there is zero probability of firm death.

Fixed production costs imply a productivity cutoff below which firms exit (φ_d^A) defined by the following zero-profit condition:

$$r_d(\varphi_d^A) = R \left(\frac{\sigma - 1}{\sigma} P \varphi_d^A \right)^{\sigma-1} w^{1-\sigma} = \sigma w f_d, \quad (1)$$

where the superscript A denotes autarky.

The equilibrium value of this zero-profit productivity is uniquely determined by the free entry condition that requires that the probability of successful entry times average profits conditional on successful entry is equal the sunk entry cost: $[1 - G(\varphi_d^A)] \bar{\pi} = w f_e$. Using the expression for profits above, this free entry condition can be expressed as:

$$f_d J(\varphi_d^A) = f_e, \quad (2)$$

$$J(\varphi_d^A) = \int_{\varphi_d^A}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^A} \right)^{\sigma-1} - 1 \right] dG(\varphi) = [1 - G(\varphi_d^A)] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A} \right)^{\sigma-1} - 1 \right], \quad (3)$$

where $J(\varphi_d^A)$ is a monotonically decreasing function and $\tilde{\varphi}_d^A$ is a weighted average of firm productivities:

$$\tilde{\varphi}_d^A = \left[\int_{\varphi_d^A}^{\infty} \varphi^{\sigma-1} \frac{dG(\varphi)}{1 - G(\varphi_d^A)} \right]^{\frac{1}{\sigma-1}}. \quad (4)$$

The mass of entrants (M_e) equals the mass of producing firms (M) divided by the probability of successful entry ($1 - G(\varphi_d^A)$):

$$M_e = \frac{M}{1 - G(\varphi_d^A)} = \frac{R}{\sigma w [f_e + [1 - G(\varphi_d^A)] f_d]}, \quad (5)$$

where the second equation uses the relationship between the mass of firms, aggregate revenue and average revenue ($M = R/\bar{r}$), the relationship between average revenue and average profits ($\bar{r} = \sigma(\bar{\pi} + w f_d)$) and free entry ($[1 - G(\varphi_d^A)] \bar{\pi} = w f_e$).

Combining the mass of entrants (5) and the free entry condition (2), aggregate revenue equals total labor payments ($R = wL$), where we choose labor as the numeraire and hence $w = 1$. Using this equality of aggregate revenue and total labor payments in the zero-profit condition (1), welfare can be written solely in terms of the zero-profit productivity (φ_d^A) and parameters:

$$\mathbb{W}_{\text{Het}}^A = \frac{w}{P} = \left(\frac{L}{\sigma f_d} \right)^{\frac{1}{\sigma-1}} \frac{\sigma - 1}{\sigma} \varphi_d^A. \quad (6)$$

Therefore the zero-profit productivity (φ_d^A) is a sufficient statistic for welfare.

2.2 Homogeneous Firm Model

We construct a homogeneous firm model that replicates the same aggregate equilibrium as the heterogeneous firm equilibrium that we just described. Firms pay a sunk entry cost of f_e units of labor and draw a productivity of either zero or $\bar{\varphi}_d$ with exogenous probabilities \bar{G}_d and $(1 - \bar{G}_d)$ respectively.

Fixed production costs imply that only firms drawing a productivity of $\bar{\varphi}_d$ find it profitable to produce. Therefore producing firms are homogeneous and there is a degenerate productivity distribution conditional on production at $\bar{\varphi}_d$.

Note that the only difference between the homogeneous and heterogeneous firm models is the productivity distribution of entrants: the distribution $G(\cdot)$ is replaced by the degenerate distribution with parameters \bar{G}_d and $\bar{\varphi}_d$. All other parameters are the same. This homogeneous firm model is isomorphic to Krugman (1980), in which the representative firm's productivity is set equal to $\bar{\varphi}_d$ and the fixed production cost is scaled to incorporate the expected value of entry costs ($F_d = f_d + f_e / [1 - \bar{G}_d]$). These values for the representative firm's productivity and the fixed production cost are exogenous and held constant in Krugman (1980). To simplify the exposition, we adopt this Krugman (1980) interpretation. The representative firm's production technology is:

$$l = \frac{q}{\bar{\varphi}_d} + F_d. \quad (7)$$

Consumers again have constant elasticity of substitution (CES) preferences defined over the differentiated varieties supplied by firms. Profit maximization implies that equilibrium prices are a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma - 1} \frac{w}{\bar{\varphi}_d}.$$

Profit maximization and free entry imply that equilibrium output and employment for the representative variety are proportional to the fixed production cost:

$$q = \bar{\varphi}_d F_d (\sigma - 1), \quad l = \sigma F_d.$$

Using the common employment for each variety, the mass of firms can be determined from the labor market clearing condition:

$$M = \frac{L}{\sigma F_d}. \quad (8)$$

Using the equilibrium pricing rule and the mass of firms, the CES price index is:

$$P^{1-\sigma} = M \left(\frac{\sigma}{\sigma - 1} \frac{w}{\bar{\varphi}_d} \right)^{1-\sigma}, \quad (9)$$

where we again choose labor as the numeraire and hence $w = 1$.

Using the price index (9), the mass of firms (8), and our choice of numeraire, welfare can be written in terms of productivity and other parameters:

$$\mathbb{W}_{\text{Hom}}^A = \frac{w}{P} = \left(\frac{L}{\sigma F_d} \right)^{\frac{1}{\sigma-1}} \frac{\sigma - 1}{\sigma} \bar{\varphi}_d. \quad (10)$$

2.3 Aggregate Equilibrium Equivalence

We now pick the parameters \bar{G}_d and $\bar{\varphi}_d$ of the degenerate productivity distribution with homogeneous firms such that the autarky equilibrium is isomorphic to the heterogeneous firm equilibrium, in the following sense:

Proposition 1 *Consider a homogeneous firm model that is a special case of the heterogeneous firm model with an exogenous probability of successful entry $[1 - \bar{G}_d] = [1 - G(\varphi_d^A)]$ and an exogenous degenerate distribution of productivity conditional on successful entry $\bar{\varphi}_d = \tilde{\varphi}_d^A$. Given the same value for all remaining parameters $\{f_d, f_e, L, \sigma\}$, all aggregate variables (welfare, wage, price index, mass of firms, and aggregate revenue) are the same in the closed economy equilibria of the two models.*

Proof. Combining $F_d = f_d + f_e / [1 - G(\varphi_d^A)]$ with the free entry condition (2) in the heterogeneous firm model, we obtain:

$$\frac{F_d}{f_d} = \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A} \right)^{\sigma-1}. \quad (11)$$

Substituting this result into closed economy welfare in the homogeneous firm model (10), we obtain the same welfare in the two models:

$$\mathbb{W}_{\text{Hom}}^A = \frac{w}{P} = \left(\frac{L}{\sigma f_d} \right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} \varphi_d^A = \mathbb{W}_{\text{Het}}^A.$$

Equal wages follow from our choice of numeraire ($w = 1$). Equal welfare and equal wages in turn imply equal price indices. Equal masses of firms follow from (8), (5) and (11). Equal aggregate revenue follows from $R = wL = L$ in both models. ■

3 Open Economy

We consider the canonical case of trade between two symmetric countries, as examined in Krugman (1980) and Melitz (2003). We assume that the heterogeneous and homogeneous firm models feature the same trade costs, so that there is a fixed exporting cost of f_x units of labor and an iceberg variable trade cost, where $\tau > 1$ units of a variety must be shipped from one country in order for one unit to arrive in the other country. We compare the effect of moving from the closed economy to the open economy on welfare in the two models, keeping all structural parameters other than the productivity distribution the same in the two models.

3.1 Heterogeneous Firm Model

Equilibrium firm revenues in the domestic and export markets are:

$$r_d(\varphi) = RP^{\sigma-1} p_d(\varphi)^{1-\sigma}, \quad r_x(\varphi) = \tau^{1-\sigma} r_d(\varphi),$$

where the subscript d indicates the domestic market and the subscript x indicates the export market.

Profit maximization implies that equilibrium prices are again a constant mark-up over marginal costs, with export prices a constant multiple of domestic prices due to the variable costs of trade:

$$p_d(\varphi) = \frac{\sigma}{\sigma-1} \frac{w}{\varphi}, \quad p_x(\varphi) = \tau p_d(\varphi), \quad (12)$$

This equilibrium pricing rule implies that profits in each market are a constant proportion of revenues minus the fixed costs:

$$\pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - wf_d, \quad \pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - wf_x.$$

Here, we assume that fixed exporting costs are incurred in the source country and we apportion the fixed production cost to the domestic market. The productivity cutoffs for serving the domestic market (φ_d^T) and export market (φ_x^T) are defined by the following zero-profit conditions:

$$r_d(\varphi_d^T) = R \left(\frac{\sigma - 1}{\sigma} P \varphi_d^T \right)^{\sigma-1} w^{1-\sigma} = \sigma w f_d. \quad (13)$$

$$r_x(\varphi_x^T) = R \left(\frac{\sigma - 1}{\sigma} P \varphi_x^T \right)^{\sigma-1} (\tau w)^{1-\sigma} = \sigma w f_x, \quad (14)$$

where the superscript T indicates the open economy equilibrium. Together these two zero-profit conditions imply that the export cutoff is a constant multiple of the domestic cutoff that depends on the fixed and variable costs of trade:

$$\varphi_x^T = \tau \left(\frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}} \varphi_d^T. \quad (15)$$

For sufficiently high fixed and variable trade costs ($\tau (f_x/f_d)^{\frac{1}{\sigma-1}} > 1$), only the most productive firms export, consistent with an extensive empirical literature (see for example the review in Bernard, Jensen, Redding and Schott 2007).

The free entry condition again equates the expected value of entry to the sunk entry cost, $[1 - G(\varphi_d^T)] \bar{\pi} = wf_e$, and can be re-written as follows:

$$f_d J(\varphi_d^T) + f_x J(\varphi_x^T) = f_e, \quad (16)$$

where $J(\cdot)$ is defined in (3) and we can define weighted average productivity in the export market ($\tilde{\varphi}_x^T$) in an analogous way to weighted average productivity in the domestic market ($\tilde{\varphi}_d^T$) in (4).

Using the relationship between the productivity cutoffs (15), and noting that $J(\cdot)$ is a decreasing function, the free entry condition (16) determines a unique equilibrium value of the domestic cutoff (φ_d^T), which in turn determines the export cutoff (φ_x^T). Furthermore, since $J(\cdot)$ is a decreasing function, the domestic cutoff in the open economy is strictly greater than the domestic cutoff in the closed economy ($\varphi_d^T > \varphi_d^A$) for positive values of fixed exporting costs.

As in the closed economy, the mass of entrants (M_e) equals the mass of firms (M) divided by the probability of successful entry ($1 - G(\varphi_d^T)$):

$$M_e = \frac{M}{1 - G(\varphi_d^T)} = \frac{R}{\sigma w [f_e + [1 - G(\varphi_d^T)] f_d + [1 - G(\varphi_x^T)] f_x]}, \quad (17)$$

and aggregate revenue equals total labor payments ($R = wL$).

Using the equilibrium pricing rule and the mass of firms, the CES price index in the open economy can be written as:

$$P^{1-\sigma} = M \left[(\tilde{\varphi}_d^T)^{\sigma-1} + \chi \tau^{1-\sigma} (\tilde{\varphi}_x^T)^{\sigma-1} \right] \left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} w^{1-\sigma}, \quad (18)$$

where $\chi = [1 - G(\varphi_x^T)] / [1 - G(\varphi_d^T)]$ is the proportion of exporting firms. We choose labor in one country as the numeraire. With symmetric countries, this implies a common unit wage in each country ($w = 1$).

Rearranging the price index (18), and using the mass of firms (17) and our choice of numeraire, welfare can be expressed in terms of productivity and parameters:

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \left(\frac{L \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1}}{\sigma \left[\frac{f_e}{1-G(\varphi_d^T)} + f_d + \chi f_x \right]} \right)^{\frac{1}{\sigma-1}} \left[(\tilde{\varphi}_d^T)^{\sigma-1} + \chi \tau^{1-\sigma} (\tilde{\varphi}_x^T)^{\sigma-1} \right]^{\frac{1}{\sigma-1}}. \quad (19)$$

In an open economy equilibrium with selection into export markets ($\varphi_x^T > \varphi_d^T$), the zero-profit condition for the domestic market (13) implies that open economy welfare can be written equivalently in terms of the domestic productivity cutoff and parameters:

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \left(\frac{L}{\sigma f_d} \right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} \varphi_d^T. \quad (20)$$

Comparing (6) and (20), and noting that the domestic cutoff is higher in the open economy than in the closed economy ($\varphi_d^T > \varphi_d^A$), there are necessarily welfare gains from trade.

In contrast, in an open economy equilibrium in which all firms export, the domestic and export productivity cutoffs are equal to one another ($\varphi_x^T = \varphi_d^T$), and are determined by the requirement that the sum of variable profits in the domestic and export markets is equal to the sum of fixed production and exporting costs. Using this zero-profit condition, open economy welfare also can be written in terms of the domestic productivity cutoff and parameters:

$$\mathbb{W}_{\text{Het}}^T = \frac{w}{P} = \left(\frac{(1 + \tau^{1-\sigma}) L}{\sigma (f_d + f_x)} \right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} \varphi_d^T. \quad (21)$$

Comparing (6) and (21), and noting that $f_x/f_d \leq \tau^{1-\sigma}$ in an open economy equilibrium in which all firms export, there are again necessarily welfare gains from trade.

3.2 Homogeneous Firm Model

In the homogeneous firm model, the probability of successful entry and productivity conditional on successful entry are exogenous and remain unchanged and equal to $[1 - \bar{G}_d]$ and $\bar{\varphi}_d$ respectively. For sufficiently high fixed and variable trade costs ($\tau^{\sigma-1} f_x / F_d > 1$), the representative firm does not find it profitable to export. In this case, welfare in the open economy equilibrium is necessarily higher in the heterogeneous firm model than in the homogeneous firm model, because the two models have the same

closed economy welfare, there are welfare gains from trade, and trade only occurs in the heterogeneous firm model. In contrast, for sufficiently low fixed and variable trade costs ($\tau^{\sigma-1}f_x/F_d < 1$), the representative firm finds it profitable to export. In this case, there is positive trade in both models, and we now compare their relative welfare in such an open economy equilibrium.

Profit maximization again implies that equilibrium prices are a constant mark-up over marginal costs, with export prices a constant multiple of domestic prices due to the variable costs of trade:

$$p_d = \frac{\sigma}{\sigma-1} \frac{w}{\bar{\varphi}_d}, \quad p_x = \tau p_d. \quad (22)$$

Together profit maximization and free entry imply that equilibrium output and employment for the representative variety are proportional to fixed costs:

$$q = \bar{\varphi}_d (F_d + f_x) (\sigma - 1),$$

$$l = \sigma (F_d + f_x).$$

Therefore both output and employment rise for the representative firm following the opening of trade to cover the additional fixed costs of exporting.

From the labor market clearing condition, this rise in employment for the representative firm implies a fall in the mass of domestically-produced varieties:

$$M = \frac{L}{\sigma (F_d + f_x)}. \quad (23)$$

Using the equilibrium pricing rule and the mass of firms, the CES price index in the open economy is:

$$P^{1-\sigma} = [1 + \tau^{1-\sigma}] M \left(\frac{\sigma}{\sigma-1} \frac{w}{\bar{\varphi}_d} \right)^{1-\sigma}, \quad (24)$$

where we again choose labor as the numeraire and hence $w = 1$.

Rearranging the price index (24), and using the mass of firms (23) and our choice of numeraire, welfare can be again expressed in terms of productivity and parameters:

$$\mathbb{W}_{\text{Hom}}^T = \frac{w}{P} = \left(\frac{[1 + \tau^{1-\sigma}] L}{\sigma (F_d + f_x)} \right)^{\frac{1}{\sigma-1}} \frac{\sigma-1}{\sigma} \bar{\varphi}_d. \quad (25)$$

3.3 Relative Welfare

In the homogeneous firm model, aggregate productivity is exogenous and hence constant by assumption. In contrast, in the heterogeneous firm model, aggregate productivity is determined by the endogenous entry and exit decisions of heterogeneous firms. This provides a new adjustment margin through which the economy can respond to the opening of trade. The presence of this new adjustment margin implies that the relative change in welfare following the opening of trade is strictly larger in the heterogeneous firm model than in the homogeneous firm model. Since the homogeneous firm model is a special case of the heterogeneous firm model, this comparison of the welfare gains from trade across

the two models is equivalent to a comparative static within the heterogeneous firm model on the productivity distribution (from a non-degenerate to a degenerate distribution). This comparative static interpretation requires that we hold all other parameters equal when comparing the two models (same $f_d, f_e, f_x, \tau, L, \sigma$). We maintain this assumption throughout the paper whenever we compare the heterogeneous and homogeneous firm models. We pick the parameters of the degenerate productivity distribution under homogeneous firms (\bar{G}_d and $\bar{\varphi}_d$) such that the autarky equilibrium is isomorphic to the heterogeneous firm one (as outlined in Proposition 1): $\bar{G}_d = G(\varphi_d^A)$ and $\bar{\varphi}_d = \tilde{\varphi}_d^A$

Proposition 2 *The proportional welfare gains from trade are strictly larger in the heterogeneous firm model than in the homogeneous firm model ($\mathbb{W}_{Het}^T/\mathbb{W}_{Het}^A > \mathbb{W}_{Hom}^T/\mathbb{W}_{Hom}^A$), except in the special case with no fixed exporting cost. In this special case, the proportional welfare gains from trade are the same in the two models.*

Proof. See the Appendix. ■

In the special case with no fixed exporting cost, the domestic productivity cutoff does not respond to the opening of trade in the heterogeneous firm model. As a result, the additional adjustment margin provided by heterogeneous firms entry and exit decisions is inoperable, and the welfare gains from trade are the same in the two models. But this special case is uninteresting, because firm productivity dispersion plays no role in the heterogeneous firm model (the exit threshold and average productivity are the same in the closed and open economies). Furthermore, this special case stands at odds with an extensive body of empirical evidence that only some firms export, exporters are larger and more productive than non-exporters, and there are substantial fixed exporting costs.³

Since the proportional welfare gains from trade are strictly lower in the homogeneous firm model than in the heterogeneous firm model for positive fixed exporting costs, and since open economy welfare in the homogeneous firm model is monotonically decreasing in trade costs, we also obtain the following result.

Proposition 3 *To achieve the same proportional welfare gains from trade requires strictly lower trade costs (either lower f_x and/or lower τ) in the homogeneous firm model than in the heterogeneous firm model, except in the special case with no fixed exporting cost.*

Proof. The proposition follows immediately from $\mathbb{W}_{Het}^T/\mathbb{W}_{Het}^A > \mathbb{W}_{Hom}^T/\mathbb{W}_{Hom}^A$ in Proposition 2 and from $\frac{d\mathbb{W}_{Hom}^T}{df_x} < 0$ and $\frac{d\mathbb{W}_{Hom}^T}{d\tau} < 0$ in (25). ■

Although we chose the productivity of the representative firm ($\tilde{\varphi}_d^A$) to ensure the same closed economy welfare in both models, the ratio of open to closed economy welfare in the homogeneous firm model $\mathbb{W}_{Hom}^T/\mathbb{W}_{Hom}^A$ is independent of the representative firm's productivity (from (10) and (25)). It

³For reviews of the extensive empirical literatures on firm export market participation, see Bernard, Jensen, Redding and Schott (2007) and Melitz and Redding (2012). For evidence of substantial fixed exporting costs, see Roberts and Tybout (1997) and Das, Roberts and Tybout (2007).

follows that both of the above propositions hold for any value of the representative firm's productivity. Both propositions also hold for general continuous productivity distributions.

4 Revealed Preference

The theoretical results throughout the paper are proved using the free entry condition in the market equilibrium of the heterogeneous firm model. But to provide further economic intuition for these results, we consider the problem of a social planner choosing the productivity cutoffs and the mass of entrants to maximize welfare in the heterogeneous firm model. We begin with the planner's problem in the closed economy, in which welfare in the homogeneous and heterogeneous firm models is the same. We next consider the planner's problem in the open economy. The planner is assumed to maximize world welfare. With symmetric countries, this is equivalent to maximizing the welfare of the representative consumer in each country.⁴ We show that the planner's choices in the closed and open economies coincide with the market allocations, and hence the market allocations in the heterogeneous firm model are efficient.⁵ We also show that the social planner in general chooses to adjust the productivity cutoffs following the opening of trade, even though it is feasible to leave them unchanged and replicate the open economy equilibrium of the homogeneous firm model. Therefore, by revealed preference, open economy welfare must be at least as high in the heterogeneous firm model as in the homogeneous firm model, and we show that it is in general higher.

4.1 Closed Economy

In the closed economy of the heterogeneous firm model, the real consumption index and aggregate labor constraint for the social planner can be written as:

$$Q = \{ [1 - G(\varphi_d^A)] M_e \}^{\sigma/(\sigma-1)} \tilde{q}_d^A,$$

$$L = [1 - G(\varphi_d^A)] M_e \left[\frac{\tilde{q}_d^A}{\tilde{\varphi}_d^A} + f_d + \frac{f_e}{1 - G(\varphi_d^A)} \right],$$

where $\tilde{q}_d^A = \tilde{q}_d^A(\tilde{\varphi}_d^A)$ is the output of a firm with a productivity equal to domestic weighted average productivity $\tilde{\varphi}_d^A$; and we assume that the social planner faces the same productivity distribution $G(\varphi)$ and entry cost f_e per firm as in the market allocation.

Using the aggregate labor constraint in the real consumption index, the social planner chooses the domestic cutoff φ_d^A (and hence the productivity range of producing firms) and the output of a firm with weighted average productivity \tilde{q}_d^A (and hence the mass of entrants) to solve the following

⁴To highlight the efficiency properties of the market equilibrium, we assume a world planner, which abstracts from the incentives of national planners to manipulate the terms of trade between countries.

⁵For an analysis of how the efficiency of the monopolistically competitive equilibrium depends on the extent to which the elasticity of substitution between varieties is constant or variable, see Dixit and Stiglitz (1977) for homogeneous firm models and Dhingra and Morrow (2012) for heterogeneous firm models.

unconstrained maximization problem:⁶

$$\max_{\varphi_d^A, \tilde{q}_d^A} \left\{ L^{\sigma/(\sigma-1)} \left[\frac{\tilde{q}_d^A}{\tilde{\varphi}_d^A} + f_d + \frac{f_e}{1 - G(\varphi_d^A)} \right]^{-\sigma/(\sigma-1)} \tilde{q}_d^A \right\}. \quad (26)$$

The trade-off faced by the social planner is as follows. On the one hand, a lower productivity cutoff reduces expected entry costs conditional on successful entry, which releases more labor for production. On the other hand, the lower productivity cutoff involves production by lower productivity firms, which reduces expected output conditional on successful entry. From the first-order conditions for \tilde{q}_d^A and φ_d^A , we obtain:

$$[1 - G(\varphi_d^A)] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^A} \right)^{\sigma-1} - 1 \right] f_d = f_e, \quad (27)$$

which corresponds to the free entry condition in the market economy (2).

Therefore, the social planner chooses the same productivity cutoff φ_d^A (and the same values of all other endogenous variables) as in the market equilibrium of the heterogeneous firm model. This confirms that the market equilibrium is efficient. In the heterogeneous firm model, changes in fixed costs and other parameters induce the social planner to change both the productivity cutoff φ_d^A and the output of a firm with weighted average productivity \tilde{q}_d^A (and hence the mass of entrants M_e). In contrast, in the homogeneous firm model, productivity is constant by assumption, which implies that changes in fixed costs and other parameters only induce changes in the mass of producing firms and output per firm.

4.2 Open Economy

In the open economy of the heterogeneous firm model, the real consumption index and the aggregate labor constraint for the social planner can be written as:

$$\begin{aligned} Q &= M_t^{\sigma/(\sigma-1)} \tilde{q}_t^T, \\ L &= M_t \left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} + \frac{1}{1 + \chi} \left(f_d + \chi f_x + \frac{f_e}{1 - G(\varphi_d^T)} \right) \right], \end{aligned}$$

where M_t is the mass of firms serving each market and $\tilde{q}_t^T \equiv q_d(\tilde{\varphi}_t^T)$ is the domestic output of a firm with a productivity equal to a weighted average across all non-exporting and exporting firms $\tilde{\varphi}_t^T$.⁷ The mass of firms serving each market and weighted average productivity are defined as follows:

$$M_t = [1 + \chi] M = [1 + \chi] [1 - G(\varphi_d^T)] M_e, \quad (28)$$

$$\tilde{\varphi}_t^T = \left\{ \frac{1}{1 + \chi} \left[(\tilde{\varphi}_d^T)^{\sigma-1} + \chi \tau^{1-\sigma} (\tilde{\varphi}_x^T)^{\sigma-1} \right] \right\}^{1/(\sigma-1)}. \quad (29)$$

Using the aggregate labor constraint in the real consumption index, the social planner chooses the domestic cutoff φ_d^T (and hence the productivity range for producing firms), the export cutoff φ_x^T

⁶In the web appendix, we also derive the same results from an equivalent representation of the planner's problem as a Lagrangian.

⁷Recall that $\chi = [1 - G(\varphi_x^T)] / [1 - G(\varphi_d^T)]$ is the proportion of exporting firms.

(and hence the proportion of exporting firms) and the output of a firm with the weighted average productivity (and hence the mass of firms) as the solution to the following unconstrained maximization problem:

$$\max_{\varphi_d^T, \varphi_x^T, \tilde{q}_t^T} \left\{ L^{\sigma/(\sigma-1)} \left[\frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} + \frac{1}{1+\chi} \left(f_d + \chi f_x + \frac{f_e}{1-G(\varphi_d^T)} \right) \right]^{-\sigma/(\sigma-1)} \tilde{q}_t^T \right\}. \quad (30)$$

The trade-offs faced by the social planner are as follows. A lower domestic cutoff again reduces expected entry costs conditional on successful entry and thereby releases more labor for production. But this lower cutoff involves production lower productivity firms, which reduces expected output conditional on successful entry. A lower export cutoff for a given domestic cutoff increases the proportion of exporting firms. This uses more labor to cover fixed exporting costs and reduces expected output conditional on exporting, but increases the fraction of foreign varieties available to domestic consumers. The first-order condition for \tilde{q}_t^T is:

$$[1 - G(\varphi_d^T)] \left[\frac{(1+\chi)\tilde{q}_t^T}{(\sigma-1)\tilde{\varphi}_t^T} - f_d - \chi f_x \right] = f_e. \quad (31)$$

It equates the expected profits from entry to the sunk entry cost, since $\frac{(1+\chi)\tilde{q}_t^T}{(\sigma-1)\tilde{\varphi}_t^T}$ is expected variable profits conditional on successful entry. The first-order condition for φ_d^T yields:

$$\left(\frac{\varphi_d^T}{\tilde{\varphi}_t^T} \right)^{\sigma-1} \frac{1}{\sigma-1} \frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} = f_d. \quad (32)$$

It equates the domestic variable profits of the least productive firm with the fixed production cost. The first-order condition for φ_x^T yields:

$$\left(\frac{\varphi_x^T}{\tilde{\varphi}_t^T} \right)^{\sigma-1} \frac{\tau^{1-\sigma}}{\sigma-1} \frac{\tilde{q}_t^T}{\tilde{\varphi}_t^T} = f_x. \quad (33)$$

It equates the export variable profits of the least productive exporter with the fixed exporting costs. Combining these three first-order conditions, we obtain the same relationship between the domestic and export cutoffs (15) and free entry condition (16) as in the open economy market equilibrium. As a result, the social planner chooses the same productivity cutoffs φ_d^T and φ_x^T (and the same value of all other endogenous variables) as in the open economy market equilibrium of the heterogeneous firm model. Hence the open economy market equilibrium of the heterogeneous firm model is efficient.

4.3 Open Versus Closed Economy

Recall that the heterogeneous and homogeneous firm models generate the same values of all aggregate variables (including welfare) in the closed economy. Furthermore, it is technologically feasible for the social planner to choose the same domestic productivity cutoff in the open economy as in the closed economy ($\varphi_d^T = \varphi_d^A$) and to let all firms export ($\varphi_x^T = \varphi_d^T$). In this hypothetical allocation, the heterogeneous and homogeneous firm models would also generate the same values of all aggregate variables (including welfare) in the open economy.

However, comparing the closed and open economy free entry conditions (2) and (16), the social planner in general chooses a different domestic productivity cutoff in the open economy than in the closed economy ($\varphi_d^T \neq \varphi_d^A$) and in general restricts exporting to a proper subset of more productive firms ($\varphi_d^T < \varphi_x^T < \infty$). Since the social planner chooses different productivity cutoffs in the open economy when it is technological feasible to choose the same productivity cutoffs as in the closed economy, revealed preference implies that these different productivity cutoffs must yield welfare levels at least as high as a hypothetical allocation with the same productivity cutoffs. Furthermore, the social planner's objective (30) is globally concave in $\{\varphi_d^T, \varphi_x^T, \tilde{q}_t^T\}$, which implies that the different productivity cutoffs must yield strictly higher welfare than the hypothetical allocation with the same productivity cutoffs. Additionally, the social planner's choice corresponds to the open economy equilibrium of the heterogeneous firm model, while the hypothetical allocation corresponds to the open economy equilibrium of the homogeneous firm model. Therefore it follows that open economy welfare is strictly higher in the heterogeneous firm model than in the homogeneous firm model whenever the productivity cutoffs differ in the open and closed economies of the heterogeneous firm model.⁸

Comparing the social planner's objectives in the closed and open economies (26) and (30), she faces different combinations of fixed versus variable costs in the closed versus open economy. The social planner's optimal response to these different cost combinations is to adjust the productivity range for both producing and exporting firms (as well as average output per firm). Thus the greater welfare gains in the heterogeneous firm model reflect the presence of an additional adjustment margin (the firm productivity ranges) relative to the homogeneous firm model. In the heterogeneous firm model, the social planner can allocate low-productivity firms to serve only the domestic market and reallocate resources to higher-productivity exporting firms. Therefore the welfare that the social planner can achieve in a model with this additional adjustment margin must be at least as high (and in general higher) than in a model without it.

While we have focused on opening the closed economy to trade, an analogous analysis can be undertaken for the reverse comparative static of closing the open economy. Choosing the degenerate productivity distribution in the homogeneous firm model such that the two models to have the same welfare in an initial open economy equilibrium, the welfare costs of closing the open economy are smaller in the heterogeneous firm model than in the homogeneous firm. Again the welfare that the social planner can achieve in the heterogeneous firm model with its additional adjustment margin must be at least as high (and in general higher) than in the homogeneous model. As a result, whether we consider increases or reductions in trade costs, welfare in the two models is the same for the calibrated value of trade costs, but is strictly higher in the heterogeneous firm model than in the homogeneous firm model for all other values of trade costs.

⁸While to make our argument as clearly as possible we focus on symmetric countries, this revealed preference argument applies more generally for asymmetric countries.

5 Trade Shares and Welfare

We now examine the relationship between the heterogeneous and homogeneous firm models' predictions for welfare and their predictions for domestic trade shares. In an open economy equilibrium of the heterogeneous firm model, the domestic trade share is:

$$\lambda_{\text{Het}} = \frac{p_d (\tilde{\varphi}_d^T)^{1-\sigma}}{p_d (\tilde{\varphi}_d^T)^{1-\sigma} + \chi \tau^{1-\sigma} p_d (\tilde{\varphi}_x^T)^{1-\sigma}} = \frac{1}{1 + \tau^{1-\sigma} \Lambda}, \quad (34)$$

$$\text{where } \Lambda = \frac{\int_{\varphi_x^T}^{\infty} \varphi^{\sigma-1} dG(\varphi)}{\int_{\varphi_d^T}^{\infty} \varphi^{\sigma-1} dG(\varphi)}.$$

In contrast, in an open economy equilibrium of the homogeneous firm model in which the representative firm exports, the domestic trade share is:

$$\lambda_{\text{Hom}} = \frac{p_d (\tilde{\varphi}_d^A)^{1-\sigma}}{(1 + \tau^{1-\sigma}) p_d (\tilde{\varphi}_d^A)^{1-\sigma}} = \frac{1}{1 + \tau^{1-\sigma}}. \quad (35)$$

Comparing the domestic trade shares in the heterogeneous and homogeneous firm models ((34) and (35) respectively) for the same trade costs $\{f_x, \tau\}$, we obtain the following result:

Proposition 4 *Whenever the representative firm exports in the homogeneous firm model and there is selection into export markets in the heterogeneous firm model ($0 < \tau (f_x/F_d)^{1/(\sigma-1)} < 1 < \tau (f_x/f_d)^{1/(\sigma-1)}$), the domestic trade share λ is strictly higher in the heterogeneous firm model than in the homogeneous firm model. If all firms export in both models ($0 \leq \tau (f_x/F_d)^{1/(\sigma-1)} < \tau (f_x/f_d)^{1/(\sigma-1)} \leq 1$), the domestic trade shares are the same.*

Proof. For $0 < \tau (f_x/F_d)^{1/(\sigma-1)} < 1 < \tau (f_x/f_d)^{1/(\sigma-1)}$, we have $\varphi_x^T > \varphi_d^T$, which implies $0 < \Lambda < 1$ and hence $\lambda_{\text{Het}} > \lambda_{\text{Hom}}$ in the domestic trade shares (34) and (35). For $0 \leq \tau (f_x/F_d)^{1/(\sigma-1)} < \tau (f_x/f_d)^{1/(\sigma-1)} \leq 1$, we have $\varphi_x^T = \varphi_d^T$, $\Lambda = 1$ and $\lambda_{\text{Het}} = \lambda_{\text{Hom}}$. ■

Proposition 4 implies a non-monotonic relationship between relative domestic trade shares in the two models and trade costs. We illustrate this non-monotonic relationship for the special case of a Pareto distribution in our simulations of the model in Section 8 below (see in particular Panel D of Figures 1 and 2). For sufficiently high trade costs, there is trade in the heterogeneous firm model, but the representative firm does not find it profitable to export in the homogeneous firm model. As a result, the domestic trade share is *lower* in the heterogeneous firm model than in the homogeneous firm model. As trade costs fall, the representative firm starts to export in homogeneous firm model, at which point the domestic trade share in the homogeneous firm model falls discretely from one to a value below that in the heterogeneous firm model. For all values of trade costs with selection into export markets in the heterogeneous firm model and positive trade in the homogeneous firm model, the domestic trade share is *higher* in the heterogeneous firm model than in the homogeneous firm model. As trade costs fall further, $\varphi_d^T \rightarrow \varphi_x^T$ and the domestic trade share in the heterogeneous firm

model converges towards the domestic trade share in the homogeneous firm model. Once trade costs reach the point at which all firms export in the heterogeneous firm model, the domestic trade share is the same in the two models.

Proposition 4 highlights the complex relationship between domestic trade shares and welfare in the heterogeneous and homogeneous firm models. The domestic trade share is an endogenous variable and in general differs between the heterogeneous and homogeneous firm models given the same structural parameters and trade costs. Furthermore, the domestic trade share can be strictly higher in the heterogeneous firm model than in the homogeneous firm model even though welfare is strictly higher in the heterogeneous firm model (Propositions 2 and 4).

Heterogeneous firms' endogenous entry and exit decisions also have implications for the elasticity of trade flows with respect to trade costs. In the homogeneous firm model, the elasticity of the domestic trade share with respect to variable trade costs depends solely on the elasticity of substitution between varieties and the domestic trade share:

$$\frac{d\lambda_{\text{Hom}}}{d\tau} \frac{\tau}{\lambda_{\text{Hom}}} = (\sigma - 1)(1 - \lambda),$$

and the elasticity of the domestic trade share with respect to fixed exporting costs is equal to zero:

$$\frac{d\lambda_{\text{Hom}}}{d\tau} \frac{f_x}{\lambda_{\text{Hom}}} = 0.$$

In contrast, in the heterogeneous firm model, the elasticities of the domestic trade share with respect to variable trade costs and fixed exporting costs are endogenous variables that depend on the entire productivity distribution:

$$\begin{aligned} \frac{d\lambda_{\text{Het}}}{d\tau} \frac{\tau}{\lambda} &= \left[(\sigma - 1) - \frac{d\Lambda}{d\tau} \frac{\tau}{\lambda_{\text{Het}}} \right] (1 - \lambda), \\ \frac{d\lambda_{\text{Het}}}{d\tau} \frac{f_x}{\lambda_{\text{Het}}} &= - \frac{d\Lambda}{d\tau} \frac{f_x}{\lambda_{\text{Het}}} (1 - \lambda), \\ \frac{d\Lambda}{d\tau} \frac{\tau}{\lambda_{\text{Het}}} &= - \left[\frac{(\varphi_x^T)^{\sigma-1} g(\varphi_x)}{\int_{\varphi_x^T}^{\infty} \varphi^{\sigma-1} dG(\varphi)} - \frac{(\varphi_d^T)^{\sigma-1} g(\varphi_d)}{\int_{\varphi_d^T}^{\infty} \varphi^{\sigma-1} dG(\varphi)} \right] \tau, \\ \frac{d\Lambda}{d\tau} \frac{f_x}{\lambda_{\text{Het}}} &= - \left[\frac{(\varphi_x^T)^{\sigma-1} g(\varphi_x)}{\int_{\varphi_x^T}^{\infty} \varphi^{\sigma-1} dG(\varphi)} - \frac{(\varphi_d^T)^{\sigma-1} g(\varphi_d)}{\int_{\varphi_d^T}^{\infty} \varphi^{\sigma-1} dG(\varphi)} \right] f_x, \end{aligned}$$

where $\varphi_x^T > \varphi_d^T$ and $\int_{\varphi_x^T}^{\infty} \varphi^{\sigma-1} dG(\varphi) < \int_{\varphi_d^T}^{\infty} \varphi^{\sigma-1} dG(\varphi)$.

Therefore the elasticity of the domestic trade share with respect to trade costs varies with the functional form of the productivity distribution, the level of trade costs, and the relative importance of fixed versus variable trade costs. It follows that there is no single elasticity of trade flows with respect to trade costs in the heterogeneous firm model with a general continuous productivity distribution. Rather empirical measures of this elasticity capture an endogenous variable that depends on the full structure of the model.

6 Trade Liberalization in the Open Economy

We now examine the aggregate welfare implications of the two models starting from an open economy equilibrium. In Proposition 4, we showed that whenever the heterogeneous firm model features export market selection, the homogeneous firm counterpart model cannot be calibrated to generate the same initial domestic trade share given the same trade costs $\{f_x, \tau\}$.⁹

Thus, we cannot construct an open economy homogeneous firm model that features (a) the same structural parameters as our heterogeneous firm model with export market selection and (b) the same initial domestic trade share and welfare. We therefore consider an extension of the homogeneous firm model to allow for two types of firms: exporting and non-exporting firms. In this extension, firms again pay a sunk entry cost of f_e units of labor before observing their productivity. An entering firm draws a productivity level $\bar{\varphi}_x$ with probability $[1 - \bar{G}_x]$, and draws a productivity level $\bar{\varphi}_{dx}$ with probability \bar{G}_{dx} . The remaining firms draw a productivity level of zero (which occurs with probability $[\bar{G}_x - \bar{G}_{dx}]$).

We pick the parameters of the “extended” homogeneous firm model $(\bar{\varphi}_x, \bar{\varphi}_{dx}, \bar{G}_x, \bar{G}_{dx})$ such that the open economy equilibrium features the same aggregate variables as in the initial open economy equilibrium with heterogeneous firms (same welfare, wage, price index, mass of firms, aggregate revenue, and domestic trade share). This requires equating these parameters with their corresponding averages under firm heterogeneity:

$$\begin{aligned} \bar{G}_x &= G(\varphi_x^T), & \bar{G}_{dx} &= G(\varphi_x^T) - G(\varphi_d^T), \\ \bar{\varphi}_x &= \tilde{\varphi}_x^T, & \bar{\varphi}_{dx} &= \left\{ \frac{1}{G(\varphi_x^T) - G(\varphi_d^T)} \int_{\varphi_d^T}^{\varphi_x^T} \varphi^{\sigma-1} dG(\varphi) \right\}^{\frac{1}{\sigma-1}}, \end{aligned}$$

since this last term represents the average productivity of non-exporters in the heterogeneous firm model. As with all our previous comparative statics, we keep the values of all the other structural parameters $(f_d, f_e, f_x, \tau, L, \sigma)$ constant across the two models. These assumptions ensure that, in our extended-homogeneous-firm setup, the firms with productivity $\bar{\varphi}_x$ will find it profitable to produce for both the domestic and export markets, while the firms with productivity $\bar{\varphi}_{dx}$ will only find it profitable to produce for the domestic market (leading to the property that the aggregate statistics line-up across the two models in the initial open economy equilibrium).

These two models will nevertheless feature a key difference in response to a reduction in trade costs from this common initial equilibrium: In the heterogeneous firm model, the endogenous selection responses to trade costs lead to changes in the average productivity of exporting and non-exporting firms (also inducing a response in the proportion of exporting firms). In contrast, those average productivity levels for exporters and non-exporters remain fixed in our extended-homogeneous-firm

⁹The calibration of the heterogeneous and homogeneous firm models to achieve the same domestic trade share and welfare under a Pareto productivity distribution (as in Arkolakis, Costinot and Rodriguez Clare 2012) involves assuming different fixed and variable trade costs in the two models, as discussed further in the next section.

model.¹⁰ Once again, this endogenous response of firm selection and average productivity leads to larger welfare gains from a given reduction in trade costs under firm heterogeneity:

Proposition 5 *Starting from an initial open economy equilibrium with the same welfare and domestic trade share, a common reduction in variable and/or fixed trade costs generates greater welfare gains in the heterogeneous firm model than in our extended-homogeneous model. Achieving the same welfare gains in both models requires a larger reduction in trade costs in the extended homogeneous firm model than in the heterogeneous firm model.*

Proof. See the Appendix. ■

Note that the extended homogeneous firm model is equivalent to a version of the heterogeneous firm model in which the domestic and export productivity cutoffs are held constant at their values in the initial equilibrium. Therefore the comparison of the heterogeneous and extended homogeneous firm models in Proposition 5 is equivalent to the following thought experiment. Start from an initial open economy equilibrium of the heterogeneous firm model with symmetric countries and selection into export markets. Now reduce trade costs and consider two cases. In the first case, let the domestic and export cutoffs adjust endogenously. In the second case, hold the thresholds constant at their initial values before the reduction in trade costs. As shown in the proof of the proposition, welfare is higher in the first case in which the domestic and export cutoffs adjust endogenously.

Atkeson and Burstein (2010) consider this thought experiment and show that endogenous changes in the domestic and export cutoffs have only second-order effects on welfare. Proposition 5 is consistent with this result. As shown in the previous section, the initial equilibrium of the heterogeneous firm model is efficient. Therefore the envelope theorem implies that the changes in the productivity cutoffs in the heterogeneous firm model have only second-order effects on welfare (30). But for substantial reductions in trade costs, these changes in the productivity cutoffs can have substantial effects on welfare, because the second-order terms can be large. In Section 8, we examine the quantitative magnitude of these effects using a calibration of the heterogeneous firm model to key U.S. aggregate and firm statistics.

While Proposition 5 focuses on reductions in trade costs, an analogous analysis can be undertaken for the reverse comparative static of increases in trade costs, and the same arguments apply as already discussed above. Proposition 5 makes clear that the new aggregate welfare implications of the heterogeneous firm model do not arise from firm heterogeneity *per se*, since exogenous differences between exporters and non-exporters exist in the extended homogeneous firm model. Instead, they stem from *endogenous* firm heterogeneity: the endogenous selection of firms into the domestic and export markets. This introduces a new adjustment margin through which the economy can respond to trade liberalization; a margin that is absent in both the homogeneous and extended-homogeneous firm models.

¹⁰Unless the reduction in trade costs is so large that the non-exporting firms in the initial equilibrium find it profitable to start exporting in the new equilibrium. In this case, the productivity levels for the two types of firms remain constant, but the proportion of exporting firms changes.

7 Pareto Productivity Distribution

While our results in the previous sections are developed for general continuous productivity distributions, we now consider the special case of a Pareto productivity distribution. Under this functional form assumption, our heterogeneous firm model falls within the class considered by Arkolakis, Costinot and Rodriguez-Clare (2012), where a country's domestic trade share is a sufficient statistic for the welfare gains of trade. Assuming that productivity has the following Pareto distribution:

$$g(\varphi) = k(\varphi_{\min})^k \varphi^{-(k+1)}, \quad k > \sigma - 1, \quad \varphi_{\min} > 0,$$

we can solve in closed form for the domestic and export productivity cutoffs. Using these solutions, welfare in the heterogeneous firm model (20) can be expressed solely in terms of a country's domestic trade share and parameters:

$$\mathbb{W}_{\text{Het}} = \frac{w}{P} = \lambda_{\text{Het}}^{-1/k} L^{1/(\sigma-1)} \left[\frac{\varphi_{\min}^k f_d^{1-\frac{k}{\sigma-1}}}{f_e \left(\frac{\sigma}{\sigma-1}\right)^k \sigma^{\frac{k}{\sigma-1}} k - (\sigma-1)} \frac{\sigma-1}{k - (\sigma-1)} \right]^{\frac{1}{k}}. \quad (36)$$

Therefore, under this assumption of a Pareto productivity distribution, knowing a country's domestic trade share (λ) and the shape parameter of the productivity distribution (k) is sufficient to determine the welfare gains from trade in the heterogeneous firm model:

$$\frac{\mathbb{W}_{\text{Het}}^T}{\mathbb{W}_{\text{Het}}^A} = \left[\frac{1}{\lambda_{\text{Het}}^T} \right]^{\frac{1}{k}}. \quad (37)$$

With a Pareto productivity distribution, the elasticity of the domestic trade share with respect to variable trade costs is:

$$\frac{d\lambda_{\text{Het}}}{d\tau} \frac{\tau}{\lambda_{\text{Het}}} = \begin{cases} k(1 - \lambda_{\text{Het}}) & \tau (f_x/f_d)^{1/(\sigma-1)} \geq 1 \\ (\sigma-1)(1 - \lambda_{\text{Het}}) & \tau (f_x/f_d)^{1/(\sigma-1)} < 1 \end{cases}, \quad (38)$$

In contrast, in the homogeneous firm model, the domestic trade share (35) and welfare in the closed and open economies ((10) and (25) respectively) imply that the welfare gains from trade can be expressed as:

$$\frac{\mathbb{W}_{\text{Hom}}^T}{\mathbb{W}_{\text{Hom}}^A} = \left[\frac{F_d}{\lambda_{\text{Hom}}^T (F_d + f_x)} \right]^{\frac{1}{\sigma-1}}. \quad (39)$$

In this homogeneous firm model, knowing a country's domestic trade share (λ) and the elasticity of substitution (σ) is sufficient to determine the welfare gains from trade only in the absence of fixed export costs. Otherwise, the ratio of those fixed export costs to the remaining fixed costs is also needed to compute those welfare gains. In the homogeneous firm model, the elasticity of the domestic trade share with respect to variable trade costs remains as specified in Section 5.

As in Arkolakis, Costinot and Rodriguez-Clare (2012), the heterogeneous and homogeneous firm models can be calibrated to have the same domestic trade share (λ^T), the same elasticity of trade flows with respect to variable trade costs ($k_{\text{Het}} = \sigma_{\text{Hom}} - 1$), and the same welfare gains from trade (if there

are no fixed exporting costs in the homogeneous firm model). Arkolakis, Costinot and Rodriguez-Clare (2012) conclude that this result implies “not much” insight from heterogeneous firm models for the aggregate implications of trade. However, Propositions 1-5 were derived for general continuous productivity distributions and hence hold in the special case of a Pareto productivity distribution. Therefore, considering a homogeneous firm model that is a special case of the heterogeneous firm model with degenerate productivity distribution and that has the same values of all other parameters including fixed and variable trade costs, the two models have different aggregate welfare implications.

These two divergent results reflect fundamentally different approaches to comparing models, which we referred to in the introduction as the macro and micro approaches. In the macro approach of Arkolakis, Costinot and Rodriguez-Clare (2012), the homogeneous and heterogeneous firm models are calibrated to match the endogenous domestic trade share and a reduced-form trade elasticity. This approach requires different assumptions regarding the structural demand and trade cost parameters in the two models: the elasticity of substitution between goods must differ ($\sigma_{\text{Het}} < \sigma_{\text{Hom}}$ since $\sigma_{\text{Het}} - 1 < k_{\text{Het}} = \sigma_{\text{Hom}} - 1$); variable trade costs must differ ($\tau_{\text{Het}} \neq \tau_{\text{Hom}}$ from Proposition 4); and the homogeneous firm model must assume no fixed exporting cost whereas the heterogeneous firm model requires positive fixed exporting costs to generate selection into export markets ($f_{x,\text{Hom}} = 0 < f_{x,\text{Het}}$). Therefore, the macro approach simultaneously changes the degree of firm heterogeneity, the ability of consumers to substitute between varieties and the value of trade costs between the two models. In contrast, our micro approach only changes the degree of firm heterogeneity between models. This enables us to isolate the effect of the degree of firm heterogeneity on aggregate welfare.

Since the homogeneous firm model is a special case of the heterogeneous firm model, our comparison of the two models is equivalent to a discrete comparative static of moving from a non-degenerate to a degenerate productivity distribution within the heterogeneous firm model. If productivity in the heterogeneous firm model is assumed to be Pareto distributed, the degree of firm heterogeneity is summarized by a single parameter: the shape parameter k , with lower values of k corresponding to greater firm heterogeneity. Therefore, in this special case, we can complement the above discrete comparative static with a continuous comparative static in the degree of firm heterogeneity.

Proposition 6 *Assuming a Pareto productivity distribution with shape parameter $k > \sigma - 1$ and positive fixed exporting costs, the greater the dispersion of firm productivity (smaller k), (a) the larger the proportional welfare gains from opening the closed economy to trade in the heterogeneous firm model (larger $\mathbb{W}_{\text{Het}}^T/\mathbb{W}_{\text{Het}}^A$), (b) the larger (smaller) the proportional welfare gains (costs) from a reduction (increase) in variable trade costs.*

Proof. See the appendix. ■

In Proposition 6, we change the degree of firm heterogeneity (k) holding all other parameters of the model constant. Therefore, in part (a), we compare closed and open economy equilibria in two heterogeneous firm models that have the same values of all parameters except for the degree of firm

heterogeneity. Similarly, in part (b), we compare two open economy equilibria with different values of trade costs in two heterogeneous firm models that have the same values of all parameters except for the degree of firm heterogeneity. In both cases, we determine the proportional changes in welfare between the two equilibria as a function of the parameters of the model. Again the same arguments apply irrespective of whether we consider reductions or increases in trade costs.

As mentioned in the introduction, the macro approach's assumption that there is a single empirical moment summarizing the elasticity of trade with respect to trade costs is also quite restrictive. Even under the assumption of a Pareto productivity distribution, the elasticity of trade with respect to trade costs is a reduced-form parameter in the heterogeneous firm model that depends on several structural parameters. For variable trade costs, this elasticity is $k > \sigma - 1$ for parameter values for which there is selection into export markets ($\tau^{\sigma-1} f_x / f_d > 1$) and $\sigma - 1$ for parameter values for which all firms export ($\tau^{\sigma-1} f_x / f_d \leq 1$). Assuming $k = \sigma_{\text{Hom}} - 1 > \sigma_{\text{Het}} - 1$ ensures the same elasticity of trade with respect to variable trade costs in the two models for parameter values for which there is selection into export markets in the heterogeneous firm model ($\tau^{\sigma-1} f_x / f_d > 1$), but implies a different trade elasticity in the two models ($\sigma_{\text{Hom}} - 1 \neq \sigma_{\text{Het}} - 1$) for parameter values for which all firms export in the heterogeneous firm model ($\tau^{\sigma-1} f_x / f_d \leq 1$).

Additionally, even with a Pareto productivity distribution, the heterogeneous firm model features different elasticities of trade with respect to trade costs depending on whether these trade costs are fixed or variable. The trade elasticity for variable trade costs is k when only some firms export (or $\sigma - 1$ when all firms export), whereas the trade elasticity for fixed exporting costs is $(k - (\sigma - 1)) / (\sigma - 1)$. Therefore empirically-measured elasticities of trade flows with respect to observed trade barriers (e.g. tariffs, transport costs, distance) can differ depending on whether these empirical barriers affect fixed versus variable trade costs.

More broadly for general continuous productivity distributions, we showed that the elasticity of trade with respect to either variable or fixed trade costs is an endogenous variable. In this general case, the trade elasticity varies with the magnitude of trade costs, the relative importance of fixed versus variable trade costs, and the functional form of the productivity distribution. Our micro approach can be applied to this more general case where the trade elasticity responds to the change in trade costs.

Even small departures from the Pareto distribution can generate substantial variation in this key endogenous trade elasticity. One empirically relevant example is the introduction of an upper bound on firm productivity draws, while maintaining their Pareto shape. Helpman, Melitz and Rubinstein (2008) show that allowing for such a bound in the gravity estimation of aggregate bilateral trade flows leads to substantial variation in the elasticity of trade with respect to distance. This estimation nests the case considered by Arkolakis, Costinot and Rodriguez-Clare (2012) of an unbounded Pareto distribution or the case of upper bounds on the productivity distribution that are sufficiently high that there is little predicted variation in that trade elasticity. In the resulting estimates, the trade elasticity varies to the order of 30-60 percent for developed and developing countries.

Finally, we return to the point that calibrating the heterogeneous and homogeneous firm models to the same domestic trade share requires an assumption that the level of trade costs (both fixed and variable) is different across the two models. This requirement holds even under the special case of an unbounded Pareto distribution of productivity and, as mentioned in the introduction, has important implications for the interpretation of the macro approach when evaluating the question of the trade and welfare effects of a given level of trade costs. For some policy issues, it may be precisely the answer to this question that is of ultimate interest, and the micro approach highlights that the answer depends on the degree of firm heterogeneity.

8 Quantitative Relevance

In this section, we use the assumption of a Pareto productivity distribution to show that the differences in the aggregate implications of heterogeneous and homogeneous firms are of quantitative relevance. We choose standard values for the model's parameters based on central estimates from the existing empirical literature and moments of the U.S. data.

We set the elasticity of substitution between varieties $\sigma = 4$, which is consistent with the estimates using plant-level U.S. manufacturing data in Bernard, Eaton, Jensen and Kortum (2003). Since productivity is Pareto distributed and firm revenue is a power function of firm productivity, firm revenue also has a Pareto distribution: $G(r) = 1 - \left(\frac{r_d}{r}\right)^{\frac{k}{\sigma-1}}$, where r_d is the revenue of the least productive firm. Existing empirical estimates suggest that the firm size distribution is well approximated by a Pareto distribution with a shape parameter close to one (see for example Axtell 2001). Therefore, we set the Pareto shape parameter for firm productivity $k = 4.25$, which ensures a Pareto shape parameter for firm revenue close to one and that log firm revenue has a finite mean ($\frac{k}{\sigma-1} = 1.42 > 1$). A choice for the Pareto scale parameter is equivalent to a choice of units in which to measure productivity, and hence we normalize $\varphi_{\min} = 1$.

We consider trade between two symmetric countries, and choose labor in one country as the numeraire ($w = 1$), which implies that the wage in both countries is equal to one. With a Pareto productivity distribution, scaling L and $\{f_e, f_d, f_x\}$ up or down by the same proportion leaves the productivity cutoffs $\{\varphi_d^T, \varphi_x^T\}$ and the mass of entrants unchanged (M_e), and merely scales average firm size (\bar{r}) up or down by the same proportion. Therefore we set L equal to the U.S. labor force and normalize f_d to one. We choose values for $\{\tau, f_e, f_x\}$ to match moments of the U.S. data. We calibrate τ to match the average fraction of exports in firm sales in U.S. manufacturing ($\frac{\tau^{1-\sigma}}{1+\tau^{1-\sigma}} = 0.14$, as reported in Bernard, Jensen, Redding and Schott 2007), which implies $\tau = 1.83$ (which is in line with the estimate of 1.7 in Anderson and van Wincoop 2004). Given our choice for the parameters $\{\sigma, k, \varphi_{\min}, f_d, \tau\}$, we choose $\{f_e, f_x\}$ to ensure that the model is consistent with the annual exit rate for U.S. firms with more than 500 employees (0.0055, as reported in Atkeson and Burstein 2010) and the average fraction of U.S. manufacturing firms that export (0.18, as reported in Bernard, Jensen, Redding and Schott 2007). We match both these moments with $f_e = 0.0145$ and $f_x = 0.545$.

Given these parameter values, we solve for the closed economy equilibrium of the heterogeneous

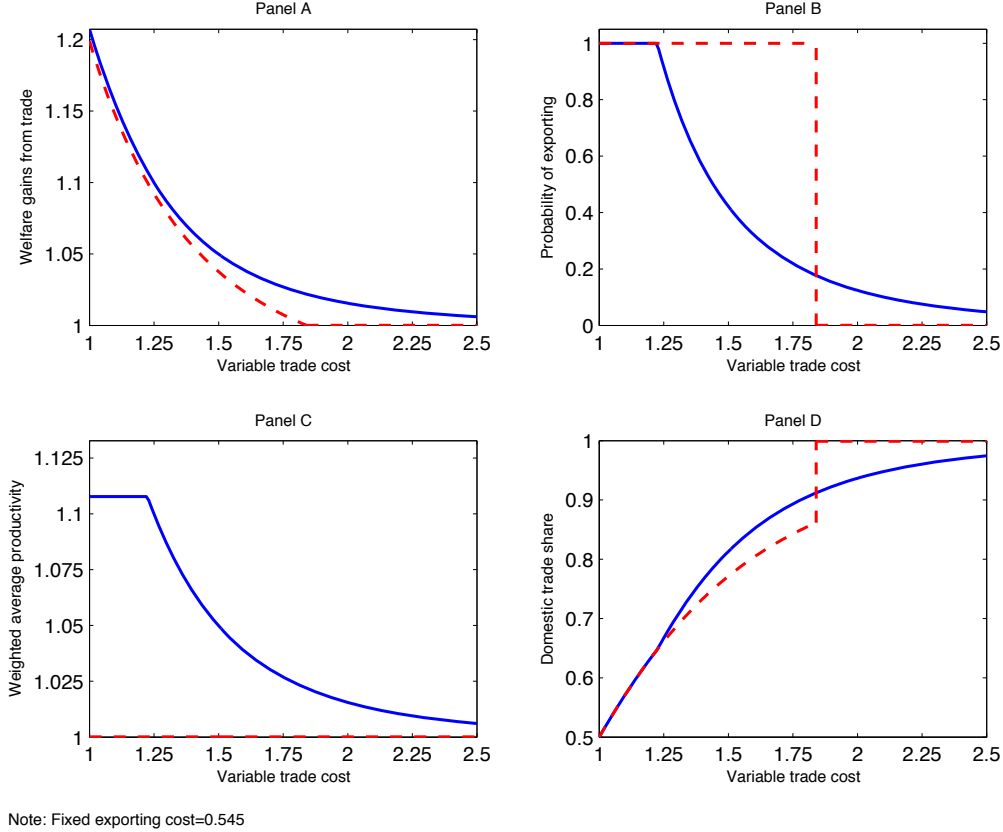


Figure 1: Reductions in variable trade costs

firm model, including the probability of successful firm entry $[1 - G(\varphi_d^A)]$ and weighted average productivity $(\tilde{\varphi}_d^A)$. In the homogeneous firm model, we set the probability of successful firm entry and productivity conditional on successful entry equal to these values: $[1 - \bar{G}_d] = [1 - G(\varphi_d^A)]$ and $\bar{\varphi}_d = \tilde{\varphi}_d^A$. All parameters besides the productivity distribution $\{f_d, f_e, f_x, \tau, L, \sigma\}$ are assumed to be the same in the two models, which implies $F_d = f_d + f_e / [1 - G(\varphi_d^A)]$. Given these same parameters, welfare and all other aggregate variables are identical in the closed economy equilibria of the two models.

In Figures 1 and 2, we show the impact of reducing trade costs from their infinite values under autarky to a range of finite values in the open economy equilibrium. Figure 1 sets fixed exporting costs equal to their calibrated value of $(f_x = 0.545)$ and considers reductions in variable trade costs from infinity to $\tau \in [1, 2.5]$ (including the calibrated value of 1.83). Panel A displays the welfare gains from opening the closed economy to trade $(\mathbb{W}^T/\mathbb{W}^A)$; Panel B displays the probability of exporting (χ) ; Panel C displays domestic weighted average productivity relative to its value under autarky $(\tilde{\varphi}_d^T/\tilde{\varphi}_d^A)$; Panel D displays the domestic trade share (λ) . The solid blue line shows values in the heterogeneous firm model, while the red dashed line shows values in the homogeneous firm model.

As shown in Panel A, the welfare gains from trade in the heterogeneous firm model are strictly greater than those in the homogeneous firm model for all finite values of variable trade costs. Across

the range of variable trade costs shown in the figure, the welfare gains from trade range from 1.00 to 1.21, which is broadly in line with existing empirical estimates from quantitative trade models.

For sufficiently high variable trade costs, the representative firm does not find it profitable to export in the homogeneous firm model. At this value for variable trade costs, the probability of exporting in the homogeneous firm model falls from one to zero (Panel B); the domestic trade share in the homogeneous firm model rises to one (Panel D); and welfare in the homogeneous firm model equals its autarkic value, even though there remain substantial welfare gains from trade in the heterogeneous firm model (Panel A).

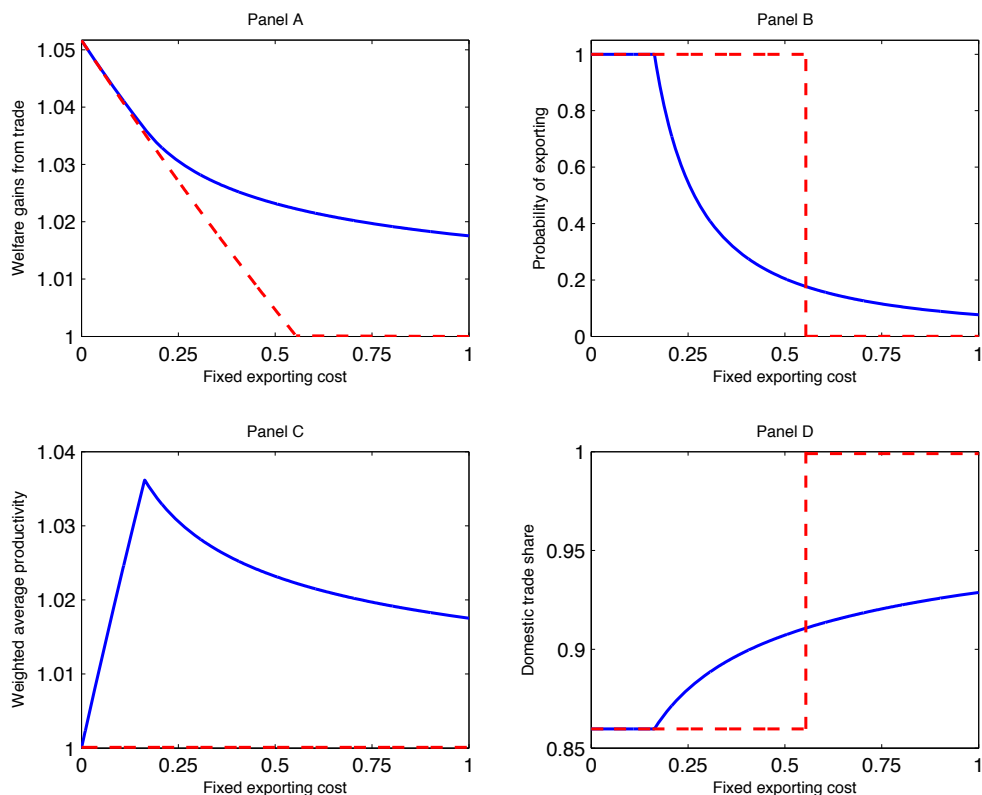
However, even for variable trade costs for which there is trade in both models, the heterogeneous firm model generates substantially greater welfare gains from trade. For example, for $\tau = 1.60$, the difference in welfare gains from trade between the two models (two percentage points) is as large as the overall welfare gains from trade in the homogeneous firm model (two percentage points). These differences in the welfare gains from trade between the two models are driven by the endogenous responses of the domestic and export productivity cutoffs to changes in trade costs in the heterogeneous firm model. In Panel C, productivity is constant by assumption in the homogeneous firm model. In contrast, weighted average productivity in the domestic market in the heterogeneous firm model rises by around 10 percent relative to its autarkic value as variable trade costs fall to one.

For sufficiently low values of variable trade costs, all firms export in the heterogeneous firm model. For this range of parameter values, the probability of exporting is one in both models (Panel B); the domestic trade share is the same in the two models (Panel D); and once all firms export further reductions in variable trade costs leave weighted average productivity unchanged (Panel C). Even for this range of trade costs (including $\tau = 1$), welfare in the heterogeneous firm model is strictly higher than in the homogeneous firm model, because of positive fixed exporting costs, which imply that the domestic productivity cutoff is different in the open and closed economy of the heterogeneous firm model.

Figure 2 sets variable trade costs equal to their calibrated value of $\tau = 1.83$ and considers reductions in fixed exporting costs from infinity to $f_x \in [0, 1]$ (which includes our calibrated value of 0.545). Each panel of the figure is constructed in the same way as for Figure 1. Again the solid blue line shows values in the heterogeneous firm model, while the dashed red line shows values in the homogeneous firm model.

As shown in Panel A, the welfare gains from trade in the heterogeneous firm model are again strictly greater than those in the homogeneous firm model, except in the limiting case in which fixed exporting costs are equal to zero. In general, the welfare gains from trade for these reductions in fixed exporting costs are smaller in Figure 2 than for the reductions in variable trade costs in Figure 1, which reflects the relatively high calibrated value of variable trade costs.

For sufficiently high fixed exporting costs, the representative firm ceases to export in the homogeneous firm. At this value for fixed exporting costs, the probability of exporting in the homogeneous firm model falls from one to zero (Panel B); the domestic trade share in the homogeneous firm model



Note: Variable trade costs=1.83

Figure 2: Reductions in fixed exporting costs

rises to one (Panel D); and welfare in the homogeneous firm model equals its autarkic value, even though there remain substantial welfare gains from trade in the heterogeneous firm model (Panel A).

However, even for fixed exporting costs for which there is trade in both models, the differences in welfare are again substantial. For example, for $f_x = 0.40$, the difference in welfare gains from trade between the two models (2 percentage points) is larger than the welfare gains from trade in the homogeneous firm model (1 percentage point). These differences in welfare gains between the two models are again explained by the endogenous responses of the domestic and export productivity cutoffs to changes in trade costs in the heterogeneous firm model. As fixed exporting costs fall, weighted average productivity in the domestic market in the heterogeneous firm model rises until the point at which all firms export (Panels B, C and D). From this point onwards, further reductions in fixed exporting costs reduce weighted average productivity in the domestic market until in the limiting case of zero fixed exporting costs weighted average productivity is the same as in the closed economy.

Taken together, our analysis of the calibrated model suggests that for empirically plausible parameter values the differences in the aggregate predictions of the heterogeneous and homogeneous firm models are of quantitative relevance.

9 Conclusions

We compare a heterogeneous firm model to a homogeneous firm model that is a special case with a degenerate productivity distribution. For a given level of trade costs, we show that welfare – as well as other key aggregate variables – is the same in the two models. But welfare is strictly higher in the heterogeneous firm model than in the homogeneous firm model for all other levels of trade costs. The source of the higher welfare in the heterogeneous firm model is the presence of an additional adjustment margin that is absent from the homogeneous firm model, namely the endogenous selection of heterogeneous firms into domestic and export markets. We demonstrate these results for general continuous productivity distributions and show that they hold both for movements between the closed and the open economy and for changes in trade costs in the open economy equilibrium.

While we focus on the case of CES preferences and monopolistic competition considered by Krugman (1980) and Melitz (2003), the point that the heterogeneous firm model features an additional adjustment margin relative to the homogeneous firm model is more general. In the case of variable elasticity of substitution preferences or departures from monopolistic competition, the market equilibrium need not be efficient. Nonetheless, endogenous firm selection has the potential to generate higher welfare in the heterogeneous firm model than in the homogeneous firm model, as long as adjustment along this margin is similar in both the market equilibrium and social optimum.

In the special case where firm productivity is distributed Pareto with no upper bound, the heterogeneous firm model falls within the class considered by Arkolakis, Costinot and Rodriguez-Clare (2012). While in this special case the heterogeneous and homogeneous firm models can be calibrated to achieve the same endogenous domestic trade share, reduced-form trade elasticity and welfare gains from trade, this requires assuming a different elasticity of substitution between varieties and different values of fixed and variable trade costs in the two models. Given the same structural parameters, the two models have different aggregate welfare implications, with larger welfare gains from reductions in trade costs in the heterogeneous firm model. These differences in aggregate welfare implications are quantitatively substantial for empirically plausible parameter values.

A Appendix

A.1 Proof of Proposition 2

Proof. We establish the proposition for the various possible types of open economy equilibria depending on parameter values. **(I)** First, we consider parameter values for which the representative firm does not find it profitable to export in the homogeneous firm model ($\tau (f_x/F_d)^{1/(\sigma-1)} > 1$). For these parameter values, the proposition follows immediately from the fact that the two models have the same closed economy welfare, there are welfare gains from trade, and trade only occurs in the heterogeneous firm model. **(II)** Second, we consider parameter values for which the representative firm exports in the homogeneous firm model and there is selection into export markets in the heterogeneous firm model ($0 < \tau (f_x/F_d)^{1/(\sigma-1)} < 1 < \tau (f_x/f_d)^{1/(\sigma-1)}$). From (19) and (25), open economy

welfare is higher in the heterogeneous firm model than in the homogeneous firm model if the following inequality is satisfied:

$$\frac{(\tilde{\varphi}_d^T)^{\sigma-1} + \chi\tau^{1-\sigma} (\tilde{\varphi}_x^T)^{\sigma-1}}{\frac{f_e}{1-G(\varphi_d^T)} + f_d + \chi f_x} > \frac{(1 + \tau^{1-\sigma}) (\tilde{\varphi}_d^A)^{\sigma-1}}{F_d + f_x}. \quad (40)$$

To show that this inequality must be satisfied, we use the open economy free entry condition in the heterogeneous firm model, which implies:

$$\begin{aligned} f_d \int_{\varphi_d^T}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) + f_x \int_{\varphi_x^T}^{\infty} \left[\left(\frac{\varphi}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) &= f_e, \\ f_d [1 - G(\varphi_d^T)] \left[\left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] + f_x [1 - G(\varphi_x^T)] \left[\left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] &= f_e, \\ f_d \left(\frac{\tilde{\varphi}_d^T}{\varphi_d^T} \right)^{\sigma-1} + f_x \frac{1 - G(\varphi_x^T)}{1 - G(\varphi_d^T)} \left(\frac{\tilde{\varphi}_x^T}{\varphi_x^T} \right)^{\sigma-1} &= \frac{f_e}{1 - G(\varphi_d^T)} + f_d + \chi f_x. \end{aligned}$$

Using $(\varphi_x^T)^{\sigma-1} = (\varphi_d^T)^{\sigma-1} \tau^{\sigma-1} f_x/f_d$, we obtain:

$$\frac{f_d}{(\varphi_d^T)^{\sigma-1}} \left[(\tilde{\varphi}_d^T)^{\sigma-1} + \chi\tau^{1-\sigma} (\tilde{\varphi}_x^T)^{\sigma-1} \right] = \frac{f_e}{1 - G(\varphi_d^T)} + f_d + \chi f_x. \quad (41)$$

Note that the open economy free entry condition in the heterogeneous firm model also implies:

$$f_d \int_{\varphi_d^A}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) + f_x \int_{\varphi_d^A}^{\infty} \left[\left(\frac{\varphi}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) < f_e, \quad (42)$$

since $\varphi_d^A < \varphi_d^T < \varphi_x^T$ and

$$\begin{aligned} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] &< 0, \quad \text{for } \varphi < \varphi_d^T, \\ \left[\left(\frac{\varphi}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] &< 0 \quad \text{for } \varphi < \varphi_x^T. \end{aligned}$$

Rewriting (42), we have:

$$\begin{aligned} f_d [1 - G(\varphi_d^A)] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] + f_x [1 - G(\varphi_d^A)] \left[\left(\frac{\tilde{\varphi}_d^A}{\varphi_x^T} \right)^{\sigma-1} - 1 \right] &< f_e, \\ f_d \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T} \right)^{\sigma-1} + f_x \left(\frac{\tilde{\varphi}_d^A}{\varphi_x^T} \right)^{\sigma-1} &< \frac{f_e}{1 - G(\varphi_d^A)} + f_d + f_x. \end{aligned}$$

Using $(\varphi_x^T)^{\sigma-1} = (\varphi_d^T)^{\sigma-1} \tau^{\sigma-1} f_x/f_d$, we obtain:

$$\frac{f_d}{(\varphi_d^T)^{\sigma-1}} (1 + \tau^{1-\sigma}) (\tilde{\varphi}_d^A)^{\sigma-1} < \frac{f_e}{1 - G(\varphi_d^A)} + f_d + f_x. \quad (43)$$

From (41) and (43), we have:

$$\frac{\frac{f_d}{(\varphi_d^T)^{\sigma-1}} \left[(\tilde{\varphi}_d^T)^{\sigma-1} + \chi\tau^{1-\sigma} (\tilde{\varphi}_x^T)^{\sigma-1} \right]}{\frac{f_e}{1-G(\varphi_d^T)} + f_d + \chi f_x} = 1, \quad (44)$$

$$\frac{\frac{f_d}{(\varphi_d^T)^{\sigma-1}} (1 + \tau^{1-\sigma}) (\tilde{\varphi}_d^A)^{\sigma-1}}{\frac{f_e}{1-G(\varphi_d^A)} + f_d + f_x} = \frac{\frac{f_d}{(\varphi_d^T)^{\sigma-1}} (1 + \tau^{1-\sigma}) (\tilde{\varphi}_d^A)^{\sigma-1}}{F_d + f_x} < 1,$$

which establishes that inequality (40) is satisfied. From (20) and (25), the condition for open economy welfare to be higher in the heterogeneous firm model than in the homogeneous firm model can be also written as:

$$\left(\frac{1}{f_d}\right)^{\frac{1}{\sigma-1}} \varphi_d^T > \left(\frac{1 + \tau^{1-\sigma}}{F_d + f_x}\right)^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A.$$

Using (40) and (44), this (equivalent) inequality is necessarily satisfied. Since closed economy welfare is the same in the two models, and open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model, it follows that the proportional welfare gains from trade are larger in the heterogeneous firm model ($\mathbb{W}_{\text{Het}}^T/\mathbb{W}_{\text{Het}}^A > \mathbb{W}_{\text{Hom}}^T/\mathbb{W}_{\text{Hom}}^A$). **(III)** Third, we consider parameter values for which the representative firm exports in the homogeneous firm model and all firms export in the heterogeneous firm model, but fixed exporting costs are still positive ($0 < \tau (f_x/F_d)^{1/(\sigma-1)} < \tau (f_x/f_d)^{1/(\sigma-1)} \leq 1$). This is simply a special case of **(II)** in which $\varphi_x^T = \varphi_d^T$, $\tilde{\varphi}_x^T = \tilde{\varphi}_d^T$ and $\frac{1-G(\varphi_x^T)}{1-G(\varphi_d^T)} = 1$. Therefore the same line of reasoning as in **(II)** can be used to show that the inequality (40) is satisfied and hence open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model. In this special case in which all firms export, the free entry condition in the open economy equilibrium of the heterogeneous firm model implies:

$$\begin{aligned} (f_d + f_x) \int_{\varphi_d^T}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1 \right] dG(\varphi) &= f_e, \\ (f_d + f_x) \int_{\varphi_d^A}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1 \right] dG(\varphi) &< f_e, \end{aligned} \quad (45)$$

since $\varphi_d^A < \varphi_d^T$ and

$$\left[\left(\frac{\varphi}{\varphi_d^T}\right)^{\sigma-1} - 1 \right] < 0, \quad \text{for } \varphi < \varphi_d^T.$$

Rewriting (45), we obtain:

$$(f_d + f_x) \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T}\right)^{\sigma-1} < \frac{f_e}{1-G(\varphi_d^A)} + f_d + f_x = F_d + f_x. \quad (46)$$

From (21) and (25), the condition for open economy welfare to be higher in the heterogeneous firm model than in the homogeneous firm model can be also written as:

$$\left(\frac{1}{f_d + f_x}\right)^{\frac{1}{\sigma-1}} \varphi_d^T > \left(\frac{1}{F_d + f_x}\right)^{\frac{1}{\sigma-1}} \tilde{\varphi}_d^A.$$

From (46), this inequality is necessarily satisfied. Since closed economy welfare is the same in the two models, and open economy welfare is higher in the heterogeneous firm model than in the homogeneous firm model, it follows that the proportional welfare gains from trade are larger in the heterogeneous firm model ($\mathbb{W}_{\text{Het}}^T/\mathbb{W}_{\text{Het}}^A > \mathbb{W}_{\text{Hom}}^T/\mathbb{W}_{\text{Hom}}^A$). **(IV)** Finally, when fixed exporting costs are zero, we have

$0 = \tau (f_x/F_d)^{1/(\sigma-1)} = \tau (f_x/f_d)^{1/(\sigma-1)}$. This is a special case of **(III)** in which $\varphi_x^T = \varphi_d^T = \varphi_d^A$, $\tilde{\varphi}_x^T = \tilde{\varphi}_d^T = \tilde{\varphi}_d^A$ and $\frac{1-G(\varphi_x^T)}{1-G(\varphi_d^T)} = 1$. In this special case of zero fixed exporting costs, the free entry condition in the open economy equilibrium of the heterogeneous firm model implies:

$$(f_d + f_x) \int_{\varphi_d^A}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^T} \right)^{\sigma-1} - 1 \right] dG(\varphi) = f_e,$$

$$(f_d + f_x) \left(\frac{\tilde{\varphi}_d^A}{\varphi_d^T} \right)^{\sigma-1} = \frac{f_e}{1-G(\varphi_d^A)} + f_d + f_x = F_d + f_x,$$

where we have used $\varphi_d^A = \varphi_d^T$. From (21) and (25), it follows immediately that open economy welfare is the same in the two models when fixed exporting costs are equal to zero. ■

A.2 Proof of Proposition 5

Proof. In the initial open economy equilibrium before the change in trade costs, (19) implies that welfare in both the heterogeneous firm model and in the extended homogeneous firm model can be written as:

$$\left(\mathbb{W}_{\text{Het}}^{T_1} \right)^{\sigma-1} = \frac{L \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \left[\left(\tilde{\varphi}_d^{T_1} \right)^{\sigma-1} + \chi_1 \tau_1^{1-\sigma} \left(\tilde{\varphi}_x^{T_1} \right)^{\sigma-1} \right]}{\sigma \left[\frac{f_e}{1-G(\varphi_d^{T_1})} + f_d + \chi_1 f_{x1} \right]}.$$

In the new open economy equilibrium after the change in trade costs, (19) implies that welfare in the heterogeneous firm model is:

$$\left(\mathbb{W}_{\text{Het}}^{T_2} \right)^{\sigma-1} = \frac{L \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \left[\left(\tilde{\varphi}_d^{T_2} \right)^{\sigma-1} + \chi_2 \tau_2^{1-\sigma} \left(\tilde{\varphi}_x^{T_2} \right)^{\sigma-1} \right]}{\sigma \left[\frac{f_e}{1-G(\varphi_d^{T_2})} + f_d + \chi_2 f_{x2} \right]}.$$

In contrast, in the new open economy equilibrium after the change in trade costs, welfare in the extended homogeneous firm model is:

$$\left(\mathbb{W}_{\text{Hom}}^{T_2} \right)^{\sigma-1} = \frac{L \left(\frac{\sigma-1}{\sigma} \right)^{\sigma-1} \left[\left(\tilde{\varphi}_d^{T_1} \right)^{\sigma-1} + \chi_1 \tau_2^{1-\sigma} \left(\tilde{\varphi}_x^{T_1} \right)^{\sigma-1} \right]}{\sigma \left[\frac{f_e}{1-G(\varphi_d^{T_1})} + f_d + \chi_1 f_{x2} \right]}.$$

To show that welfare in the new open economy equilibrium is higher in the heterogeneous firm model than in the homogeneous firm model, we need to show that:

$$\frac{\left(\tilde{\varphi}_d^{T_2} \right)^{\sigma-1} + \chi_2 \tau_2^{1-\sigma} \left(\tilde{\varphi}_x^{T_2} \right)^{\sigma-1}}{\frac{f_e}{1-G(\varphi_d^{T_2})} + f_d + \chi_2 f_{x2}} > \frac{\left(\tilde{\varphi}_d^{T_1} \right)^{\sigma-1} + \chi_1 \tau_2^{1-\sigma} \left(\tilde{\varphi}_x^{T_1} \right)^{\sigma-1}}{\frac{f_e}{1-G(\varphi_d^{T_1})} + f_d + \chi_1 f_{x2}}. \quad (47)$$

To establish this inequality, we use the free entry condition in the new open economy equilibrium of the heterogeneous firm model, which implies:

$$f_d \int_{\varphi_d^{T_2}}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] dG(\varphi) + f_{x2} \int_{\varphi_x^{T_2}}^{\infty} \left[\left(\frac{\varphi}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] dG(\varphi) = f_e,$$

$$f_d \left[1 - G(\varphi_d^{T_2}) \right] \left[\left(\frac{\tilde{\varphi}_d^{T_2}}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] + f_{x2} \left[1 - G(\varphi_x^{T_2}) \right] \left[\left(\frac{\tilde{\varphi}_x^{T_2}}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] = f_e,$$

$$f_d \left(\frac{\tilde{\varphi}_d^{T_2}}{\varphi_d^{T_2}} \right)^{\sigma-1} + f_{x2} \frac{1 - G(\varphi_x^{T_2})}{1 - G(\varphi_d^{T_2})} \left(\frac{\tilde{\varphi}_x^{T_2}}{\varphi_x^{T_2}} \right)^{\sigma-1} = \frac{f_e}{1 - G(\varphi_d^{T_2})} + f_d + \frac{1 - G(\varphi_x^{T_2})}{1 - G(\varphi_d^{T_2})} f_{x2}.$$

Using $(\varphi_x^{T_2})^{\sigma-1} = (\varphi_d^{T_2})^{\sigma-1} \tau_2^{\sigma-1} f_{x2}/f_d$, we obtain:

$$\frac{f_d}{(\varphi_d^{T_2})^{\sigma-1}} \left[(\tilde{\varphi}_d^{T_2})^{\sigma-1} + \chi_2 \tau_2^{1-\sigma} (\tilde{\varphi}_x^{T_2})^{\sigma-1} \right] = \frac{f_e}{1 - G(\varphi_d^{T_2})} + f_d + \chi_2 f_{x2}. \quad (48)$$

Note that the free entry condition in the new open economy equilibrium of the heterogeneous firm model also implies:

$$f_d \int_{\varphi_d^{T_1}}^{\infty} \left[\left(\frac{\varphi}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] dG(\varphi) + f_{x2} \int_{\varphi_x^{T_1}}^{\infty} \left[\left(\frac{\varphi}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] dG(\varphi) < f_e, \quad (49)$$

since $\varphi_d^{T_1} < \varphi_d^{T_2}$ and $\varphi_x^{T_1} > \varphi_x^{T_2}$ and

$$\left[\left(\frac{\varphi}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] < 0, \quad \text{for } \varphi < \varphi_d^{T_2},$$

$$\left[\left(\frac{\varphi}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] > 0 \quad \text{for } \varphi_x^{T_2} < \varphi < \varphi_x^{T_1}.$$

Rewriting (49), we have:

$$f_d \left[1 - G(\varphi_d^{T_1}) \right] \left[\left(\frac{\tilde{\varphi}_d^{T_1}}{\varphi_d^{T_2}} \right)^{\sigma-1} - 1 \right] + f_{x2} \left[1 - G(\varphi_x^{T_1}) \right] \left[\left(\frac{\tilde{\varphi}_x^{T_1}}{\varphi_x^{T_2}} \right)^{\sigma-1} - 1 \right] < f_e,$$

$$f_d \left(\frac{\tilde{\varphi}_d^{T_1}}{\varphi_d^{T_2}} \right)^{\sigma-1} + f_{x2} \frac{1 - G(\varphi_x^{T_1})}{1 - G(\varphi_d^{T_1})} \left(\frac{\tilde{\varphi}_x^{T_1}}{\varphi_x^{T_2}} \right)^{\sigma-1} < \frac{f_e}{1 - G(\varphi_d^{T_1})} + f_d + \frac{1 - G(\varphi_x^{T_1})}{1 - G(\varphi_d^{T_1})} f_{x2}.$$

Using $(\varphi_x^{T_2})^{\sigma-1} = (\varphi_d^{T_2})^{\sigma-1} \tau_2^{\sigma-1} f_{x2}/f_d$, we obtain:

$$\frac{f_d}{(\varphi_d^{T_2})^{\sigma-1}} \left[(\tilde{\varphi}_d^{T_1})^{\sigma-1} + \chi_1 \tau_2^{1-\sigma} (\tilde{\varphi}_x^{T_1})^{\sigma-1} \right] < \frac{f_e}{1 - G(\varphi_d^{T_1})} + f_d + \chi_1 f_{x2}. \quad (50)$$

From (48) and (50), we have:

$$\frac{\frac{f_d}{(\varphi_d^{T_2})^{\sigma-1}} \left[(\tilde{\varphi}_d^{T_2})^{\sigma-1} + \chi_2 \tau_2^{1-\sigma} (\tilde{\varphi}_x^{T_2})^{\sigma-1} \right]}{\frac{f_e}{1 - G(\varphi_d^{T_2})} + f_d + \chi_2 f_{x2}} = 1,$$

$$\frac{\frac{f_d}{(\varphi_d^{T_2})^{\sigma-1}} \left[(\tilde{\varphi}_d^{T_1})^{\sigma-1} + \chi_1 \tau_2^{1-\sigma} (\tilde{\varphi}_x^{T_1})^{\sigma-1} \right]}{\frac{f_e}{1 - G(\varphi_d^{T_1})} + f_d + \chi_1 f_{x2}} < 1,$$

which establishes the inequality (47). ■

A.3 Proof of Proposition 6

Proof. (a) First, consider parameter values for which the representative firm exports in the homogeneous firm model and there is selection into export markets in the heterogeneous firm model ($0 < \tau (f_x/F_d)^{1/(\sigma-1)} < 1 < \tau (f_x/f_d)^{1/(\sigma-1)}$). From (6) and (20), we have:

$$\frac{\mathbb{W}_{\text{Het}}^T}{\mathbb{W}_{\text{Het}}^A} = \frac{\varphi_d^T}{\varphi_d^A}. \quad (51)$$

In the special case of a Pareto productivity distribution and for these parameter values for which there is selection into export markets in the heterogeneous firm model, we have:

$$\frac{\varphi_d^T}{\varphi_d^A} = \left[1 + \left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k \frac{f_x}{f_d} \right]^{1/k},$$

which can be written as:

$$\ln \left(\frac{\varphi_d^T}{\varphi_d^A} \right) = k^{-1} \ln \left[1 + \left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k \frac{f_x}{f_d} \right].$$

Note that

$$\frac{d \ln(\varphi_d^T/\varphi_d^A)}{dk} = -k^{-2} \ln \left[1 + \left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k \frac{f_x}{f_d} \right] - \frac{k^{-1} \ln(\tau (f_x/f_d)^{1/(\sigma-1)}) \left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k \frac{f_x}{f_d}}{\left[1 + \left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k \frac{f_x}{f_d} \right]} < 0, \quad (52)$$

where we have used $d(a^x)/dx = (\ln a) a^x$. Since a smaller value of k corresponds to greater productivity dispersion, it follows that greater productivity dispersion implies larger φ_d^T/φ_d^A . Second, consider parameter values for which the representative firm exports in the homogeneous firm model and all firms export in the heterogeneous firm model, but fixed exporting costs are still positive ($0 < \tau (f_x/F_d)^{1/(\sigma-1)} < \tau (f_x/f_d)^{1/(\sigma-1)} \leq 1$). From (6) and (21), we have:

$$\frac{\mathbb{W}_{\text{Het}}^T}{\mathbb{W}_{\text{Het}}^A} = \left(\frac{(1 + \tau^{1-\sigma}) f_d}{f_d + f_x} \right)^{\frac{1}{\sigma-1}} \frac{\varphi_d^T}{\varphi_d^A}.$$

In the special case of a Pareto productivity distribution and for these parameter values for which all firms export in the heterogeneous firm model, we have:

$$\frac{\varphi_d^T}{\varphi_d^A} = \left[1 + \frac{f_x}{f_d} \right]^{1/k},$$

which can be written as:

$$\ln \left(\frac{\varphi_d^T}{\varphi_d^A} \right) = k^{-1} \ln \left[1 + \frac{f_x}{f_d} \right].$$

Note that

$$\frac{d \ln(\varphi_d^T/\varphi_d^A)}{dk} = -k^{-2} \ln \left[1 + \frac{f_x}{f_d} \right] < 0. \quad (53)$$

Since a smaller value of k corresponds to greater productivity dispersion, it follows that greater productivity dispersion again implies larger φ_d^T/φ_d^A . Taking (52) and (53) together and using (51), it follows that greater dispersion of firm productivity (smaller k) implies larger proportional welfare gains from opening the closed economy to trade. **(b)** Consider parameter values for which there is selection into export markets in the open economy equilibrium of the heterogeneous firm model ($\tau (f_x/f_d)^{1/(\sigma-1)} > 1$). In the special case of a Pareto productivity distribution, we have:

$$\varphi_d^T = \left(\frac{\sigma - 1}{k - (\sigma - 1)} \right)^{1/k} \left[\frac{f_d + \left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k f_x}{f_e} \right]^{1/k} \varphi_{\min}.$$

We have:

$$\begin{aligned} \frac{d\varphi_d^T}{d\tau} \frac{\tau}{\varphi_d^T} d\tau &= - \frac{\left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k f_x}{f_d + \left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k f_x} d\tau \\ &= -\xi d\tau. \end{aligned}$$

Hence:

$$\frac{d \left(\frac{d\varphi_d^T}{d\tau} \frac{\tau}{\varphi_d^T} d\tau \right)}{dk} = \frac{\ln \left(\tau (f_x/f_d)^{1/(\sigma-1)} \right) \left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k f_x}{f_d + \left(\frac{1}{\tau (f_x/f_d)^{1/(\sigma-1)}} \right)^k f_x} (1 - \xi) d\tau,$$

which implies:

$$\begin{aligned} \frac{d \left(\frac{d\varphi_d^T}{d\tau} \frac{\tau}{\varphi_d^T} d\tau \right)}{dk} &< 0 \quad \text{for} \quad d\tau < 0, \\ \frac{d \left(\frac{d\varphi_d^T}{d\tau} \frac{\tau}{\varphi_d^T} d\tau \right)}{dk} &> 0 \quad \text{for} \quad d\tau > 0. \end{aligned}$$

Therefore greater dispersion of firm productivity (smaller k) implies a larger elasticity of the domestic productivity cutoff with respect to reductions in variable trade costs, which from (20) implies greater proportional welfare gains from reductions in variable trade costs. By the same reasoning, greater dispersion of firm productivity (smaller k) implies a smaller elasticity of the domestic productivity cutoff with respect to increases in variable trade costs, which from (20) implies smaller proportional welfare costs from increases in variable trade costs. ■

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