PREVENTING ENVIRONMENTAL DISASTERS:
MARKET BASED VS. COMMAND AND
CONTROL POLICIES

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Preventing Environmental Disasters:  
Market-Based vs. Command-and-Control Policies  

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Abstract

The paper compares the effects of market-based and command-and-control climate policies on the direction of technical change and the prevention of environmental disasters. Drawing on the model proposed in Acemoglu et al. (2012, American Economic Review), we show that market-based policies (carbon taxes and subsidies towards clean sectors) exhibit bounded window of opportunities: delays in their implementation make them completely ineffective both in redirecting technical change and in avoiding environmental catastrophes. On the contrary, we find that command-and-control interventions guarantee policy effectiveness irrespectively on the timing of their introduction. As command-and-control policies are always able to direct technical change toward “green” technologies and to prevent climate disasters, they constitute a valuable alternative to market-based interventions.

JEL: O33, O44, Q30, Q54, Q56, Q58

Keywords: Environmental Policy, Command and Control, Carbon Taxes, Disasters

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1 Introduction

In this work, we extend the seminal contribution of Acemoglu et al. (2012) to study the effectiveness of market-based (M-B) and command-and-control (C&C) policies in redirecting technical change towards “green” innovations, thus preventing environmental catastrophes related to climate change.

One of the major challenges faced by humankind today is the rising temperature caused by the increasing consumption of fossil fuels, and the appropriate policy responses. The debate is still unsettled as some researchers call for major and immediate actions (Stern, 2007), whereas others suggest limited and gradual policy interventions (see e.g. Nordhaus, 2007).

Acemoglu et al. (2012) contribute to such debate with a two-sector model of directed technical change, which allows to study how market-based environmental policies can affect the development of “dirty” and “green” technologies, thus impacting on climate change. When the clean and dirty inputs are “strong” substitute (more on that in Acemoglu et al., 2012, and in Section 3.3 below.), an optimal M-B environmental policy, grounded on a “carbon” tax and a green research subsidy, can redirect technical change towards the green sector, preventing environmental catastrophes. However, given path-dependency in the direction of technical change (Aghion et al., 2015), the window of opportunity of such policy actions is limited: if the technology gap between the dirty and green inputs becomes sufficiently high, M-B interventions are ineffective and environmental disasters will certainly occur. The policy window is shorter when the two inputs are “weak” substitutes. Moreover, in the latter case (as well as when inputs are complementary) M-B interventions cannot avoid an environmental catastrophe unless they stop the growth of the economy.

We expand the model of Acemoglu et al. (2012) to account for command-and-control policies and we study their impact on technical change and climate dynamics. The adoption of regulations not grounded on market incentives is quite common in environmental policy and it appears to be very effective. For instance, some international agreements (e.g. the Montreal Protocol on Substances that Deplete the Ozone Layer) fix an exogenous ceiling on specific polluting concentrations. Shapiro and Walker (2015) find that the increasing stringency of U.S. environmental regulation accounts for three quarters of the 60% decrease in pollution emissions (e.g. nitrogen oxides, particulate matter, sulfur dioxide, and volatile organic compounds) from U.S. manufacturing in the period from 1990 to 2008. More generally, both price and quantity forms of control can solve allocation problems and there is no a priori criterion to favor one instrument over the other (Weitzman, 1974). The choice of the most appropriate mode of control should be grounded only on its performance evaluated

\footnote{For an extension see also Acemoglu et al. (2015).}
according to “economic” criteria.  

We show that a command-and-control policy, which fixes the maximum amount of dirty inputs for each unit of clean ones, is always able to redirect technical change towards the green technology independently on the timing of its implementation, and to avoid environmental catastrophes. In that, C&C policies are a valuable alternative to M-B ones to reach a green transition whenever the window of opportunity for environmental interventions is bounded. Moreover, even if the dirty and clean inputs are weak substitutes or complementary, command-and-control interventions imposing a ceiling to the use of polluting inputs, avoid disasters without halting economic growth.

The rest of the paper is organized as follows. In Section 2 we present the model. Alternative policies interventions are compared in Section 3. Finally, we discuss the results in Section 4.

2 The model

The baseline structure of the model is akin to the one in Acemoglu et al. (2012). There is a continuum of households (composed of workers, entrepreneurs and scientists) with utility function:

$$\sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} u(C_t, S_t),$$  \hspace{1cm} (1)

where $\rho$ is the discount rate, $C$ is final consumption good, and $S \in [0, \bar{S}]$ captures the quality of the environment. Naturally, the instantaneous utility function is increasing in consumption and environmental quality.

On the supply side, a homogeneous final good is produced under perfect competition employing clean and dirty inputs $Y_c$ and $Y_d$:

$$Y_t = \left( Y^{\frac{\epsilon-1}{\epsilon}}_{ct} + Y^{\frac{\epsilon-1}{\epsilon}}_{dt} \right)^{\frac{\epsilon}{\epsilon-1}},$$  \hspace{1cm} (2)

where $\epsilon \in (0, +\infty)$ is the elasticity of substitution between the two inputs. Note that the two inputs are complements when $\epsilon < 1$ and substitutes if $\epsilon > 1$.

Both $Y_c$ and $Y_d$ are produced using labor and a continuum of sector-specific machines according to the production functions:

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jt}^{\alpha} di$$  \hspace{1cm} (3)

2 Different studies have compared the effects of market-based and command-and-control policies. In particular, Weitzman (1974); Buchanan (1969); Li and Shi (2010) and Li and Sun (2015) emphasize the drawbacks of marked-based instruments and support the use of regulation. See also Hepburn (2006) for a review.

3 The utility function is twice differentiable and jointly concave in $C$ and $S$. Moreover, conditions (2) and (3) in Acemoglu et al. (2012) hold. Households’ utility considerable falls when environmental quality approaches zero. Conversely, if $S = \bar{S}$, further increases of environmental quality do not lead to utility improvements.
with \( j \in \{c,d\} \) and \( \alpha \in (0,1) \), \( A_{jit} \) is the productivity of machine \( i \) in sector \( j \) and \( x_{jit} \) is the quantity of such machine. The aggregate productivity of the two sectors is defined as:

\[
A_{jt} \equiv \int_0^1 A_{jit} di.
\] (4)

Total labor supply is normalized to 1 and the market clearing condition for labor requires \( L_{ct} + L_{dt} \leq 1 \). Machines in both clean and dirty sectors are produced by monopolistic competitive firms. The cost of producing a single machine is constant across time and sectors and corresponds to \( \psi = \alpha^2 \).

In both industry, an innovation occurs if a scientist successfully discovers a new design. At the beginning of each period scientists try to develop a new clean or dirty technology. If she is successful, which happens with probability \( \eta_j \in (0,1) \), she obtains a one-year patent for its machine \( i \) and becomes a monopolistic supplier.\(^4\) Innovations increase the productivity of a machine by a factor \( 1 + \gamma \), with \( \gamma > 0 \). Normalizing the number of scientists to 1 the market clearing condition for scientists becomes: \( s_{ct} + s_{dt} \leq 1 \), where \( s_{jt} \) indicates the share of scientists conducting research in sector \( j = \{c,d\} \) at time \( t \). As scientists are randomly allocated to machines in the sector they choose, the average productivity of sector \( j \) evolves according to

\[
A_{jt} = (1 + \gamma \eta_j s_{jt}) A_{jt-1}.
\] (5)

The variation of environmental quality \( S_t \) depends on environmental degradation linked to the production of dirty inputs, as well as on environmental regeneration due to the intrinsic dynamics of the Earth’s physical and biological system:

\[
S_{t+1} = -\xi Y_{dt} + (1 + \delta)S_t,
\] (6)

with \( S_{t+1} \) bounded between 0 and \( \overline{S} \). The environmental degradation term catches the negative effects of \( \text{CO}_2 \) emissions,\(^5\) while the environmental regeneration term captures the absorption of \( \text{CO}_2 \) by the oceans and the biosphere (Oeschger et al., 1975; Goudriaan and Ketner, 1984; Nordhaus, 1992). Note that if \( S_t = 0 \) an environmental disaster occurs.

3 Climate policies and the direction of technical change

In this section we first recall the laissez-faire equilibrium (Section 3.1), where no environmental policies are in place. We then study the impact of different environmental policies aiming at redirecting technical change towards the green sector in order to reduce the total amount of dirty inputs used

\(^4\)In sectors where the innovation process is unsuccessful, a one-year patent is randomly assigned to one of the producers using the old technology.

\(^5\)One could reasonably define \( \text{CO}_2 \) emissions as directly proportional to the use of dirty inputs: \( E_{mt} \propto Y_{dt} \).
in the economy and avoid environmental disasters. More specifically, we compare the success of market-based policies (cf. Section 3.2), based on carbon tax and subsidies to the clean sector, vis-à-vis command-and-control interventions (cf. Section 3.3), which fix ceilings for the production of dirty inputs.

3.1 The laissez-faire equilibrium

As in Acemoglu et al. (2012), an equilibrium is represented by a sequence of wages \( w_t \), prices for inputs \( p_{jt} \) and machines \( p_{jit} \), demands for inputs \( Y_{jt} \) and machines \( x_{jit} \), labor \( L_{jt} \), quality of environment \( S_t \) and research allocations of scientists \( s_{jit} \) such that: firms maximize their profits, scientists maximize their expected profits, labor and input markets clear, and environmental quality evolves according to (6). We recall that the laissez-faire equilibrium occurs when no environmental policies are in place.

In line with Acemoglu et al. (2012), let us assume that the productivity of the green sector is sufficiently lower than the one of the dirty industry:

**Assumption 1.** \( \frac{A_{ct}}{A_{dt}} < \min \left( \left(1 + \gamma \eta_c \right)^{-\frac{\omega+1}{\varphi}} \left( \frac{\eta_c}{\eta_d} \right)^{\frac{1}{\varphi}}, \left(1 + \gamma \eta_d \right)^{-\frac{\omega+1}{\varphi}} \left( \frac{\eta_d}{\eta_c} \right)^{\frac{1}{\varphi}} \right) \),

with \( \phi \equiv (1 - \alpha)(1 - \epsilon) \). Assumption 1 will hold throughout the rest of the paper. If \( \epsilon > 1 \), innovation occurs only in the dirty sector and the long run growth rate of dirty inputs is \( \gamma \eta_d \). If \( \epsilon < 1 \), innovation happens first in the clean sector, then it will occur also in the fossil fuel one and the long run growth rate of dirty inputs will be \( (\eta_d \eta_c)/(\eta_d + \eta_c) < \eta_d \). If assumption 1 holds and \( \epsilon > 1 \), the laissez-faire allocation will always produce an environmental disaster, i.e. \( S_t = 0 \) for some \( t \) (Acemoglu et al., 2012).

Note that the direction of technical change is determined by the incentives scientists face when they decide to conduct their research in the clean or dirty sector. More specifically, the relative benefit from undertaking research in sector \( c \) rather than in sector \( d \) is expressed by the ratio (Acemoglu et al., 2012):

\[
\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c}{\eta_d} \left( \frac{p_{ct}}{p_{dt}} \right)^{\frac{1}{1-\alpha}} \frac{L_{ct} A_{ct-1}}{L_{dt} A_{dt-1}}.
\]

Equation (7) reveals that the relative profitability of research in the two sectors can be decomposed in three components which capture productivity differentials \( (A_{ct-1}/A_{dt-1}) \), relative prices \( ((p_{ct}/p_{dt})^{\frac{1}{1-\alpha}}) \), and market size \( (L_{ct}/L_{dt}) \). Finally, the equilibrium demand of the two inputs is determined by:

\[
Y_c = (A_c^\varphi + A_d^\varphi)^{-\frac{\alpha+\varphi}{\varphi}} A_c^{\alpha+\varphi} A_d,
\]

\[
Y_d = (A_c^\varphi + A_d^\varphi)^{-\frac{\alpha+\varphi}{\varphi}} A_d^{\alpha+\varphi} A_c,
\]
whose evolution over time depends on the sectoral allocation of scientists (cf. Equation 7), and on the stochastic process characterizing the dynamics of machines’ productivity.

### 3.2 Market-based environmental policies

In the model, a market-based (M-B) environmental policy is composed of a carbon tax and a subsidy towards clean research proportional to firms’ profits. When the two inputs $Y_c$ and $Y_d$ are substitutes ($\epsilon > 1$) and the economy is initially stacked in the bad laissez-faire equilibrium, Acemoglu et al. (2012) shows that the social planner can redirect technical change towards the green technology introducing a carbon-tax $t_d$ on the production of dirty inputs and a public subsidy $q_{ct}$ supporting the research in the clean sector. Moreover, if the two inputs are strong substitutes ($\epsilon > 1/(1-\alpha)$), M-B policy is temporary.

However, the direction of technical change is not only dependent on policy variables ($t_{dt}$ and $q_{ct}$). In fact, the past history of innovations, which in turns determines the relative productivity of the clean and dirty sectors and the profitability of performing research therein, plays a fundamental role. As a consequence, given the importance of path dependency (Aghion et al., 2015), even if carbon taxes and subsidies might affect the direction of technical change, they cannot always push the economy away from a bad, carbon-intensive equilibrium.

**Proposition 1.** Assume $\epsilon > 1$ and that Assumption 1 holds. Let $(q_{ct}, t_{dt})$ be a policy scheme composed by a finite subsidy $q_{ct}$ for the clean sector and a carbon tax $t_{dt}$ on the production of dirty inputs lower than their unitary price, $t_{dt} < p_{dt}$. Then, there exists a finite $\bar{A}_d > 0$ s.t. $\forall A_{dt} > \bar{A}_d$

(i) $(q_{ct}, t_{dt})$ is ineffective in re-directing technical change towards the clean sector,

(ii) the unique equilibrium allocation of scientists is $s_{dt} = 1$ and $s_{ct} = 0$ for any $t > 0$.

**Proof.** See Appendix A.1.

Proposition 1 shows that market-based environmental policies fail whenever the productivity differentials between the dirty and clean sectors is sufficiently large. The intuition behind such a result is that once the relative productivity advantage of dirty technology is sufficiently big, it will always compensate the cost of the carbon tax and the benefit of the subsidy. This happens because the policy scheme affects the relative profitability of clean technology only via the market size effect.

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6 Throughout the model we assume that subsidies have to be finite and the carbon tax, expressed as a percentage of the price of dirty inputs, must be lower than one. In particular $t_{dt} \leq \theta < p_{dt}$ and $q_{ct} \leq \theta < +\infty$. Notice that if such assumption does not hold, the model collapses to a degenerate case of a one-sector economy where the issue of directed technical change becomes completely meaningless.
Accordingly, potential entrepreneurs will always maximize expected profits by investing in the carbon-intensive sector, thereby undermining the effectiveness of the carbon tax. Furthermore, the next proposition states that the probability of passing the productivity threshold $\bar{A}_d$, above which market-based policies are ineffective, approaches 1 as time goes by

**Proposition 2.** Assume that $\epsilon > 1$ and that Assumption 1 holds. Then

$$\lim_{t \to +\infty} P(A_{dt} > \bar{A}_d \mid t \in \mathcal{N}) = 1$$

*Proof.* See Appendix A.2

The main consequence of Proposition 2 is that the timing of the introduction of market-based environmental policies is crucial. If the economy is stacked in the bad equilibrium ($s_d = 1$), then there exists a bounded window opportunity for policy scheme $(q_{ct}, t_{ct})$ to be effective. More precisely, the window opportunity for a market based policy introduced at time $T$ lasts $(\log(\frac{\bar{A}_d}{A_{dT}}))/(\log(1 + \eta_d \gamma))$ periods.

**Remark 1.** Given the evolution of the quality of the environment (Equation 6), a too much delayed policy intervention inevitably leads to an environmental disaster. Indeed, as the economy is stacked in a bad equilibrium ($s_d = 1$), $Y_{dt}$ increases at a rate of $\gamma \eta_d$, M-B policies become ineffective and $S_t$ reaches zero in finite time, thereby producing a disaster.

**Remark 2.** The higher the elasticity of substitution between the two inputs ($\epsilon$), the shorter the time window in which M-B policies are effective.

The intuition underlying the last remark is straightforward. If the elasticity of substitution of clean and dirty inputs increases, producers will have higher incentives to switch towards the cheaper dirty ones, creating additional demand for these inputs. Accordingly, the relative profitability of dirty inputs increases, reducing the threshold $\bar{A}_d$ above which policy interventions are ineffective.

Carbon taxes are a meaningful tool for emission control policy. At the same time, Proposition 1 and 2 show that, when markets are competitive and dirty and clean inputs are substitutes, their effectiveness cannot be guaranteed a priori. The productivity gap between carbon-intensive and green technologies becomes crucial for the success of M-B environmental policies. In addition such a gap widens over time due to the sub-martingale nature of machines’ productivity (cf. proof of Proposition 2), implying that delayed interventions are likely to be ineffective.

Table 1 provides numerical experiments supporting these claims. It considers different scenarios defined by the expected productivity growth, initial relative backwardness of clean technologies
and the size of the subsidy. In many cases, the window of opportunity for market-based policy lasts very few periods and, the carbon tax needed to redirect technical change is extremely high. For example, when the productivity of clean technologies is initially half of the dirty ones and the expected productivity growth is high, the carbon tax required to move the economy towards the “green” equilibrium is above 35% and it has to be introduced by 30 periods. Would the backwardness of clean technologies be higher, the minimum tax would correspond to 61% of dirty sector’s revenues and the window of opportunity reduces to around 20 periods.

Table 1: Minimum carbon taxes to redirect technical change and corresponding window of opportunities (in parenthesis) under different policy schemes.

<table>
<thead>
<tr>
<th>Backwardness of clean technologies</th>
<th>Expected productivity growth</th>
<th>Subsidy (proportion of clean sector’s profits)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>20%</td>
</tr>
<tr>
<td>low (70% of dirty)</td>
<td>19.0% (118)</td>
<td>20.4% (117)</td>
</tr>
<tr>
<td>medium (50% of dirty)</td>
<td>33.8% (106)</td>
<td>34.4% (105)</td>
</tr>
<tr>
<td>high (20% of dirty)</td>
<td>61.8% (76)</td>
<td>62.1% (75)</td>
</tr>
<tr>
<td>low (70% of dirty)</td>
<td>21.6% (36)</td>
<td>22.3% (36)</td>
</tr>
<tr>
<td>medium (50% of dirty)</td>
<td>35.9% (33)</td>
<td>36.5% (32)</td>
</tr>
<tr>
<td>high (20% of dirty)</td>
<td>63.0% (23)</td>
<td>63.3% (22)</td>
</tr>
<tr>
<td>low (70% of dirty)</td>
<td>30.5% (33)</td>
<td>31.1% (33)</td>
</tr>
<tr>
<td>medium (50% of dirty)</td>
<td>43.2% (30)</td>
<td>43.7% (30)</td>
</tr>
<tr>
<td>high (20% of dirty)</td>
<td>67.2% (20)</td>
<td>67.5% (20)</td>
</tr>
</tbody>
</table>

Note: Low, medium and high expected productivity growth correspond to an average growth rate of machines’ productivity of, 1%, 3% and 8% respectively. With asymmetric expected productivity growth, they corresponds to 8% for dirty technologies and 3% for clean ones.

A further insight on the potential shortness of M-B’s effectiveness period is provided by figure 1. Numerical simulations of the model under different technological regimes (that correspond to the technological opportunities in the two sectors) allow to illustrate the dynamics of productivities, which in turns drive the magnitude of the minimum carbon tax that would move economy toward a “green” development path. Results shows that in many circumstances, minimum taxes rapidly reach the unit and become ineffective as a policy instrument.

Given such dismal results, are there alternative policy interventions that always redirect technical change towards the green sector, thus preventing climate catastrophes? In the next Section, we will show that such objectives can be achieved by appropriate command-and-control policies.
Figure 1: Window opportunities, minimum carbon tax and productivities under different technological regimes

Note: figures on the left column (a,c,e) show the Monte Carlo average dynamics of productivities and minimum carbon tax to redirect technical change obtained across 100 independent runs; figures on the right column (b,d,f) show the relationship between average carbon taxes and the ratio between clean and dirty sector productivity, each point represents a period. Each row of figures is characterized by different technological regimes: low (a,b) corresponds to an average productivity growth of 1%, medium (c,d) corresponds to an average productivity growth of 3%, high (e,f) corresponds to an average productivity growth of 8%. All simulations are obtained setting $\epsilon = 10$, $\alpha = 1/3$ and initial the initial productivity ratio $A_{c0}/A_{d0} = 0.8$.

3.3 Command-and-control policies

A command-and-control (C&C) policy refers to an environmental intervention that relies on regulation (permission, prohibition, standard setting and enforcement) as opposed to financial incentives,
that is, economic instruments of cost internalization (UN, 1997). In particular, we consider a policy that establishes the maximum amount $\kappa$ of dirty inputs that can be used for each unit of clean ones:

$$\frac{Y_{ct}}{Y_{dt}} > \kappa, \quad \forall t > T,$$

where $\kappa \in \mathbb{R}^+$ represents the command-and-control policy chosen by the government and introduced at time $T$. Given Assumption 1, in the laissez-faire equilibrium innovations start in the more productive dirty sector. Moreover, if $\epsilon > 1$ and given Equations (8) and (9), C&C policies (cf. Equation 10) will be binding in equilibrium.

The dirty ($Y_{dt}$) and clean ($Y_{ct}$) inputs are employed competitively by final good producers, which maximizes their profits according to:

$$\max_{Y_{ct}, Y_{dt}} \left\{ p_t Y_t - p_{ct} Y_{ct} - p_{dt} Y_{dt} \right\} \quad s.t. \quad Y_t = \left( \frac{Y_{ct}}{Y_{ct}} + \frac{Y_{dt}}{Y_{dt}} \right)^{\frac{\epsilon}{1-\epsilon}}.$$ (11)

Under the C&C policy, first-order conditions give:

$$\frac{Y_{ct}}{Y_{dt}} = \left( \frac{p_{ct}}{p_{dt}} \right)^{-\epsilon} = \kappa.$$ (12)

In both the laissez-faire equilibrium and market-based policies, the relative demand of inputs is determined competitively (the relative price of clean inputs compared to dirty ones is decreasing in its relative supply). In contrast, under command-and-control policies, the government indicates a relative upper bound on the use of dirty inputs ($\kappa$), which implicitly constrains the gap between the prices of the two sectors. Furthermore, relying on (12), it is possible to express the relative employment in the clean sector as

$$\frac{L_{ct}}{L_{dt}} = \frac{Y_{ct}}{Y_{dt}} \left( \frac{p_{ct}}{p_{dt}} \right)^{-\frac{\alpha}{1-\alpha}} \left( \frac{A_{ct}}{A_{dt}} \right)^{-1} = \kappa^{(1+\frac{\alpha}{1-\alpha})} \left( \frac{A_{ct}}{A_{dt}} \right)^{-1}.$$ (13)

Equation (13) implies that whenever the relative demand of dirty inputs is constrained by the C&C policy, any productivity gain in the carbon intensive sector is labor destroying. Indeed, since firms cannot expand production, any increases in machine productivity will increase profits by reducing the number of employees needed to serve a constant demand. The profitability ratio of conducting research in the two sectors than becomes:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \eta_{ct} \left( \frac{p_{ct}}{p_{dt}} \right)^{\frac{1}{1-\alpha}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}}^{-1} = \eta_{ct} \kappa^{\frac{1}{1-\alpha}} \frac{A_{ct}}{A_{dt}}^{-1} = \eta_{ct} \kappa^{\frac{1}{1-\alpha}} \left( \frac{1 + \eta_{ct}s_{ct}}{1 + \eta_{dt}s_{dt}} \right)^{-1},$$ (14)

where the second equality follows combining (7) with (12) and (13), and the third one is obtained via (5). From the last equation it follows that under a command-and-control policy $\kappa$ the expected

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3The effects of such policy on the direction of technical change are equivalent to the ones of an absolute upper bound for the use of dirty inputs. We refer to the relative threshold per unit of clean inputs to simplify computations.
profitability of the two sectors in equilibrium does not depend on the productivity of currently available machines as in the laissez-faire and market-based policy cases, but only on the relative likelihood of obtaining a successful innovation and $\kappa$. We can now state the following proposition.

**Proposition 3.** Consider a C&C policy $\kappa$. If $\epsilon > 1$, then there exists a finite $\bar{\kappa} > 0$ such that any $\kappa > \bar{\kappa}$ always redirects technical change towards the green sector.

**Proof.** See Appendix A.3

The main intuition behind Proposition 3 is that any C&C policy, which sufficiently limits the relative share of dirty inputs, creates an additional demand for clean ones, increasing the profitability of the green sector. If such a policy is implemented, technical change moves towards a development path where innovations occur only in the clean sector. Moreover, once the new equilibrium is achieved, the economy behaves as in the “good” laissez faire scenario, where output $Y_t$ and use of clean inputs $Y_{ct}$ grows at the long-run rate $\gamma\eta_c$ (see equations 8, 9 and 4 and recall that $A_{ct}$ grows at a rate of $\gamma\eta_c$ while $A_{dt}$ remains constant).

**Remark 3.** A temporary C&C policy intervention $\kappa$ is sufficient to redirect technical change permanently.

Indeed, as the economy moves toward a “good” equilibrium, $Y_{ct}$ grows faster than $Y_{dt}$, as the clean sector is more profitable than the dirty one, thus increasing the ratio $Y_{ct}/Y_{dt}$. The command-and-control policy sustains the improvement of machines’ productivity in the clean sector, thereby increasing the ratio $A_{ct}/A_{dt}$. When such ratio becomes sufficiently high, the C&C policy is not needed anymore as research is spontaneously performed only in the clean sector.

Command-and-control policies can always redirect technical change towards the clean sector, but are they able to prevent of natural disasters ($S_t = 0$)? In other words, what happens to the environment when in equilibrium innovation occurs only in the green sector?

Given the results in Acemoglu et al. (2012) and using equations (8) and (9), one can conclude that when all scientists are allocated to the clean sector ($s_{ct} = 1$), the production of dirty inputs ($Y_{dt}$) grows at a rate $(1 + \gamma\eta_c)^{\alpha+\gamma} - 1 > 0$, if the the two inputs are weak substitutes (i.e. $1 < \epsilon < 1/(1 - \alpha)$). In contrast, when inputs are strong substitutes ($\epsilon > 1/(1 - \alpha)$), $Y_{dt}$ behaves in the long run as $A_{ct}^{\alpha+\varphi}$, which in turns decreases over time.

The above results imply that appropriate climate policies are able to prevent environmental catastrophes only when dirty and clean inputs are strong substitutes. More precisely, C&C policies are always successful in avoiding $S_t$ to reach zero, whereas M-B solutions are effective only if they are implemented at the right time. If inputs are weak substitutes instead, final good production
requires increasing amount of dirty inputs which cannot be replaced by clean ones, even though the productivity of dirty machines keeps constant. As a result, an environmental disaster looks inevitable.

Given such a gloomy perspective, is there an environmental policy that work even if clean and dirty inputs are weak substitutes? The next proposition provides a positive answer:

**Proposition 4.** Let \( \hat{\kappa} \) be a C&C policy imposing a fixed ceiling on the use of dirty inputs \( Y_{dt} \) such that \( Y_{dt} \leq \hat{\kappa} \). Then, there exists a finite \( \hat{\kappa}' > 0 \) such that \( \forall \hat{\kappa} \in (0, \hat{\kappa}') \) the policy always redirects technical change towards the green sector and prevents an environmental disaster.

**Proof.** See Appendix A.4

The claim follows straightforwardly from proposition 3 and equation (6).\(^8\) A fixed-ceiling C&C policy is always effective, even when the dirty and clean inputs are complementary, whereas in Acemoglu et al. (2012) disasters can be avoided only by switching off economic growth. Table 2 summarizes the results of the different types of policy explored in the paper.

<table>
<thead>
<tr>
<th></th>
<th>redirection of technical change</th>
<th>prevention of environmental disaster</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>weak substitutes</td>
<td>strong substitutes</td>
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<tr>
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<td>yes</td>
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<tr>
<td>Absolute limit</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

### 4 Conclusions

In this work, we have extended the model developed by Acemoglu et al. (2012) to compare the impact of environmental market-based (M-B) policies with command-and-control (C&C) ones. M-B

\(^8\)The maximum \( Y_{dt} \) allowed by the regulation policy at time \( t \) must be below \( S_{t-1}/\xi \) in order to prevent the realization of an environmental disaster.
policies are grounded on a carbon tax and a subsidy to the green sector, whereas C&C interventions fix limits to the production of dirty inputs (and greenhouse gas emissions).

Table 2 provides a summary of our results. We find that market-based policies are not always successful to redirect technical change from the dirty to the green sector. Given the cumulativeness of technical change (Aghion et al., 2015), time is fundamental: there is a limited window of opportunity to trigger a green transition, after that the productivity gap between the dirty and green sector becomes so high that M-B policies are ineffective. The time for an effective intervention gets shorter if the two inputs are “weak” substitutes. In latter case, market-based policies are never able to prevent an environmental catastrophes. On the contrary, if the dirty and green inputs are “strong” substitutes, timely M-B interventions can successfully avoid the occurrence of disasters.

Command-and-control policies can always redirect technical change toward the green sector. In that, they are more effective than market-based interventions. Such result occurs because M-B policies work only via the market size channel (larger input sector stimulates innovation), whereas C&C interventions also affect the relative price and are not limited by the productivity gap between the dirty and green technologies (cf. Equation 7). This explains why the evolution of the technologies do not affect the success of command-and-control policies, which are also always effective in preventing environmental disasters when inputs are strong substitutes. If the dirty and green inputs are weak substitutes, the only environmental policy that allows to avoid a climate catastrophe is a C&C intervention fixing an absolute limit on the use of polluting inputs.

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References


A Mathematical Appendix

A.1 Proposition 1

We divide the proof in two parts. First, we show that the policy scheme \((q_{ct}, t_{dt})\) with \(q_{ct} \leq \vartheta < +\infty\) and \(t_{dt} \leq \theta < p_{dt}\) is not able to redirect technical change, then we characterize the unique equilibrium.

In the model, the more the dirty sector is profitable, the more researchers devote effort to innovate therein, the more dirty machines become productive and the relative share of dirty inputs increases. Under the policy scheme \((q_{ct}, t_{dt})\), the profitability of the two sectors is determined by three elements, namely the productivity ratio, the price of dirty inputs and the carbon tax:

\[
\frac{\Pi_{ct}}{\Pi_{dt}} = (1 + q_{ct}) \frac{\eta_c}{\eta_d} \left( \frac{p_{dt} - t_{dt}}{p_d} \right)^{-\epsilon} \left( \frac{A_{ct}}{A_{dt}} \right)^{-\varphi-1} \left( \frac{A_{ct-1}}{A_{dt-1}} \right)^{t_{dt}}.
\]

where the second line follows from (5) and where \(\varphi = (1 - \epsilon)(1 - \alpha) < 0\) and \(s_{ct} = 1 - s_{dt}\). Let

\[
f(s) = (1 + q_{ct}) \frac{\eta_c}{\eta_d} \left( \frac{p_{dt} - t_{dt}}{p_d} \right)^{-\epsilon} \left( 1 + \gamma \eta_{c} s_{ct} \right)^{-\varphi-1} \left( \frac{A_{ct-1}}{A_{dt-1}} \right)^{t_{dt}},
\]

where \(s = s_{ct} = 1 - s_{dt}\). If \(f(0) < 1\), then \(s = 0\) is an equilibrium where all scientists devote their effort toward the dirty sector.

By assumption the economy is initially stacked in the bad equilibrium where productivity-improving innovations take place only for dirty machines. A carbon tax on the production of dirty inputs \(t_{dt}\), and a subsidy \(q_{ct}\), introduced at time \(T\), are able to redirect technical change if they guarantee \(f(0) > 1\), which corresponds to:

\[
(1 + q_{cT}) \frac{\eta_c}{\eta_d} \left( \frac{p_{dT} - t_{dT}}{p_d} \right)^{-\epsilon} (1 + \gamma \eta_{c} s_{ct})^{t_{dT}} \left( \frac{A_{cT-1}}{A_{dT-1}} \right)^{t_{dT}} > 1.
\]

We analyze the case where the government provides the maximum possible subsidy, \(q_{cT} = \vartheta\). If the tax is not able to redirect technical change in such scenario, then it would not be effective for all \(q_{cT} < \vartheta\) as well (results do not change if the maximum available tax is fixed and one studies how subsidy affects technical change). The tax is effective in redirecting technical change if the following condition is satisfied:

\[
t_{dT} > p_{dT} - \left[ (1 + \vartheta) \frac{\eta_c}{\eta_d} (1 + \gamma \vartheta) \frac{A_{cT-1}}{A_{dT-1}} \right]^{\frac{1}{\varphi}} p_{dT}.
\]
Given the productivity of machines in the dirty sector \( r \), define \( g(r) \):

\[
g(r) := p_{dT} - \left[ (1 + \vartheta) \frac{\eta_c}{\eta_d} (1 + \eta_d \gamma)^{\varphi + 1} \left( \frac{A_{dT-1}}{r} \right)\right]^{\frac{1}{\varphi}} p_{dT}.
\]

\( g(r) \) is a continuous function in \((0; +\infty)\) and satisfies:

\[
\lim_{r \to +\infty} g(r) = p_{dT}.
\]

Without loss of generality, let \( \theta = p_{dT} - \delta \) with \( \delta \in \mathbb{R}^+ \). Then, using the definition of limit, for all \( \delta > 0 \), it exists a \( \bar{A}_d \ll \infty \), such that for all, \( r > \bar{A}_d \), one obtains

\[
p_{dT} - g(r) < \delta,
\]

which in turn implies \( g(r) > p_{dT} - \delta \). Finally, there exists a finite \( r \) and a sufficiently low \( \delta \) such that, in order to redirect technical change, it is required

\[
t_{dT} > g(r) > \theta,
\]

which is impossible because it contradicts our assumptions.

Now let us show that the equilibrium where all researchers are employed in the dirty sector \( s = 0 \) is also the unique equilibrium when \( A_{dT-1} \) is sufficiently large. Two cases must be distinguished.

First, if \( 1 + \varphi > 0 \), then \( f(s) \) is strictly decreasing in \( s \) and \( f(0) < 1 \) guarantees that \( s = 0 \) is the unique equilibrium. The previous condition can be rewritten as follows:

\[
f(0) = (1 + \vartheta) \frac{\eta_c}{\eta_d} \left( \frac{p_{dT} - t_{dT}}{p_{dT}} \right)^{-\varepsilon} (1 + \eta_d \gamma)^{\varphi + 1} \left( \frac{A_{dT-1}}{A_{dT-1}} \right)^{-\varphi} < 1,
\]

which implies

\[
\left( \frac{A_{dT-1}}{A_{dT-1}} \right)^{-\varphi} < \frac{\eta_d}{\eta_c (1 + \vartheta)} \left( \frac{p_{dT} - t_{dT}}{p_{dT}} \right)^{\varepsilon} \left( \frac{1}{1 + \eta_d \gamma} \right)^{\varphi + 1} = \Psi,
\]

where \( \Psi > 0 \). As \( \varepsilon > 1 \), the left hand side of (18) is a continuous function monotonically decreasing in \( A_{dT-1} \) which tends to 0 as the productivity of machines in the dirty sector becomes larger and larger. This proves that for a sufficiently large \( A_{dT-1} \), the unique equilibrium allocation of scientists satisfies \( s = 0 \).

Now consider the second case where \( 1 + \varphi < 0 \). As \( f(s) \) is strictly increasing in \( s \), the unique equilibrium is \( s = 0 \) only if \( f(0) < f(1) < 1 \), where the first inequality is obviously satisfied. Consider the second inequality:

\[
f(1) = (1 + \vartheta) \frac{\eta_c}{\eta_d} \left( \frac{p_{dT} - t_{dT}}{p_{dT}} \right)^{-\varepsilon} (1 + \eta_c \gamma)^{-\varphi - 1} \left( \frac{A_{dT-1}}{A_{dT-1}} \right)^{-\varphi} < 1.
\]
Accounting for the time the tax is introduced and after some algebra it becomes

\[
\left( \frac{A_{cT-1}}{A_{dT-1}} \right)^{-\varphi} < \frac{\eta_d}{\eta_c(1 + \varphi)} \left( \frac{p_{dT} - l_{dT}}{p_{dT}} \right)^\epsilon (1 + \eta_c \varphi + 1) = \Psi',
\]

with \( \Psi' > 0 \). Analogously to the previous case, it is easy to see that for a sufficiently high \( A_{dT-1} \) the equilibrium \( s = 0 \) is unique. Finally, if \( 1 + \varphi = 1 \) then \( f(s) \equiv f \) is constant and \( f < 1 \), surely verified for some large \( A_{dT-1} \), is sufficient to obtain the unique equilibrium \( s = 0 \).

### A.2 Proposition 2

The proof relies upon the nature of the stochastic process characterizing the evolution of machines’ productivity, i.e.

\[
A_{jt} = \begin{cases} 
(1 + \gamma s_{jt}) A_{jt-1}, & \text{with probability } \eta_j \\
A_{jt-1}, & \text{with probability } (1 - \eta_j)
\end{cases}
\]

with \( j = \{c, d\} \) as usual. As \( E(A_{jt} | A_{j0}, ..., A_{jt-1}) \geq A_{jt-1} \) for all \( t \), \( A_{jt} \) is a submartingale. This implies that the expected productivity of dirty machines explodes as time goes by if no emission control policy is undertaken. Moreover, let \( \bar{A}_d \) be the lowest threshold such that any \( A_{dT} > \bar{A}_d \) prevents inequality (16) to be satisfied while let (18) holding. In principle, \( \bar{A}_d \) can take any real number. However, we assume, without loss of generality, that \( \bar{A}_d \) is an element of the sequence \( \{A_{d0} (1 + \gamma)^n\}_{n=0}^\infty \). Since the economy starts from the initial bad equilibrium where \( s_{d0} = 1 \), the probability of exceeding the threshold can be written as

\[
P(A_{dt} > \bar{A}_d, t \in N, t > n) = 1 - \sum_{i=0}^{n-1} \binom{t}{t-i} (1 - \eta_d)^i \eta_d^{t-i},
\]

\[
= 1 - \left[ q^t + t p q^{t-1} + \cdots + \frac{t(t-1) \cdots (t-n+2)}{(n+1)!} p^{n-1} q^{t-n-1} \right],
\]

where \( q = \eta_d \) and \( p = 1 - \eta_d \). Now let us study the probability limit of exceeding the threshold as time goes by. Since \( 0 < \eta_d < 1 \), one obtains that

\[
\lim_{t \to +\infty} P(A_{dt} > \bar{A}_d | t \in N, t > n) = 1.
\]

### A.3 Proposition 3

First, we notice that equation (14) can be expressed as

\[
g(s) = \frac{\eta_c}{\eta_d} \left( \frac{1 + \eta_c s}{1 + \eta_d(1 - s)} \right)^{-1},
\]

where \( s = s_{ct} = 1 - s_{dt} \). As the economy is stacked in the bad equilibrium, in order to redirect technical change, a command-and-control policy \( \kappa \) should satisfy \( g(0) > 1 \). This implies that \( s = 1 \).
is the unique equilibrium allocation of scientists. Imposing previous condition \( g(0) > 1 \) one obtains:

\[
\frac{\eta_c}{\eta_d} \kappa \left( 1 + \eta_c \right)^{-1} > 1.
\]

The latter condition can be easily expressed as

\[
\kappa > \bar{\kappa} = \left( \frac{\eta_d(1+\eta_c)}{\eta_c} \right)^\frac{\bar{\epsilon}}{1+\epsilon}.
\]

Notice that \( \bar{\kappa} \) is strictly positive and does not depend on the relative productivity of machines (at any instant of time). In addition as \( \kappa \in (0, \infty) \), there will always be a C&C policy \( \kappa > \bar{\kappa} \) that successfully redirects technical change.

### A.4 Proposition 4

Let us start noticing that Assumption 1 implies the command-and-control policy scheme to be binding, that is \( Y_{dt} = \hat{\kappa} \). Therefore, by solving the model in section 3.3, one obtains that (similarly to proposition 3’s proof) technical change is redirected towards the “green” equilibrium \( s_{ct} = 1 \) and \( s_{dt} = 0 \) if

\[
\frac{Y_{ct}}{\hat{\kappa}} > \left( \frac{\eta_d(1+\eta_c)}{\eta_c} \right)^\frac{\bar{\epsilon}}{1+\epsilon},
\]

which easily translates in

\[
\hat{\kappa} < Y_{ct} \frac{\eta_c}{\eta_d(1+\eta_c)}^\frac{\bar{\epsilon}}{1+\epsilon}.
\]

Since \( Y_{ct}, \eta_c, \eta_d, \) and \( \epsilon \) are strictly positive, there always exists an absolute C&C policy \( \hat{\kappa} > 0 \) able to redirect technical change.

An environmental disaster is avoided if \( S_t \) is permanently positive after policy intervention. Since \( Y_{dt} \) is bounded from above, it suffices that \( \hat{\kappa} < S_{t-1}/\xi \) to prevent an environmental catastrophe. Therefore, any C&C policy such that

\[
\hat{\kappa} < \hat{\kappa}', \\
\hat{\kappa}' = \min \left( S_{t-1}/\xi, Y_{ct} \frac{\eta_c}{\eta_d(1+\eta_c)}^\frac{\bar{\epsilon}}{1+\epsilon} \right)
\]

always guarantees the redirection of technical change towards the clean sector and the avoidance of environmental disaster.