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Abstract

A large portion of innovators do not patent their inventions. This is a relative puzzle since innovators are often perceived to be at the mercy of imitators in the absence of legal protection. In practice, innovators however invest actively in making their products technologically hard to reverse engineer. We consider the dynamics of imitation and investment in technological complexity, both by the innovator and by imitators. We show it can justify high level of profits beyond patents, can shed light on the regulation of reverse engineering and can explain delays in adoption of innovations.

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1 Introduction

Contrary to popular belief, a large share of innovators do not perceive patents as the best way to protect their innovations. Influential surveys of managers such as those by Levin et al. (1987) and Cohen et al. (2000) show that the legal protection conferred by patents is by far less preferred than other means of protection. This is confirmed by the results of the European CIS survey (Arundel (2001)).

These surveys provide very conclusive evidence that patents are not necessarily the best way to protect innovations. However, they are not very informative on the specific mechanism guaranteeing rents when patents are not used. The options other than patents respondents can choose from are typically secrecy and lead time, which are not descriptive of the precise strategies employed by firms. One exception is the CIS survey that includes complexity of product design as one of the choices. As documented in Arundel (2001), this is one of the most popular means of protection, far above patents, both for process and product innovations.[1]

In this paper we analyze formally the option available to innovators of investing in complexity to make their products harder to reverse engineer. Our main point of is to show that the *dynamics* of investment in such protective measures, by the innovator and by imitators, can explain some of the empirical evidence previously mentioned. We show that firms can collect very high profits without patenting even in environments where investing in technological complexity is costly and subsequent imitation is cheap and not time-consuming.

Concrete examples abound of how firms can strategically invest to hamper imitation efforts. Ichijo (2010) illustrates this for some consumer electronics products: "Sharp has put tremendous efforts into making imitation of its LCD TV sets time consuming and difficult. Various initiatives at Kameyama are aimed at increasing complexities (...) in order to make imitation difficult". The software industry is full of obfuscation strategies and tools designed to

[1] Arundel (2001) uses the 1993 CIS survey covering 7 European countries.

interfere with reading of the machine code or its decompilation. Not only software, but also hardware can be actively protected. For example, it is quite typical in the semiconductor industry to encase some of the important circuitry in epoxy blocks so that electronics are destroyed if someone tries to open them.[2] It is not unusual either to design the integrated circuits to have pieces that are seemingly unused but are required for the operation (for details and further examples, see Samuelson and Scotchmer (2002)). Finally, paying high wages to reduce researcher mobility is also an important way to hamper imitation.[3]

It may appear somewhat puzzling that innovators do not rely more heavily on patents since they are often seen as helplessly at the mercy of rampant imitation in the absence of legal protection. We argue in this paper that the common wisdom that free-riding by imitators is extremely harmful for an innovator misses two important aspects that our model with investments in protective measures does capture. First, free-riders find themselves in a similar situation to that of the innovator once they have imitated a protected innovation. Thus, the original innovator benefits from the incentive of imitators to keep imitation barriers high for those who have not yet imitated. Second, the innovator also benefits from the incentive that imitators have to free-ride on each other. If it is anticipated that the next imitator to enter will not actively pursue protective measures, all remaining imitators have incentives to delay their entry in the hope of benefiting from the imitation effort of the next one who happens to enter. These two aspects, coupled with the fact that an innovator moves first when pursuing protective measures, explain why rent dissipation may not be too severe.

All these ideas are formalized in an infinite-horizon model in which the original innovator faces a potentially large pool of ex ante identical imitators who are initially inactive. At every period, imitators who have not yet reverse engineered the innovator's technology decide whether to do so at some (pos-

[2] Another common way to reverse-engineer electronics and circuits is to use x-ray images and work out what components have been used. For this reason, firms try to hinder these imitation efforts by positioning parts in such a way that the x-ray recognition is hampered.

[3] See Henry and Ruiz-Aliseda (2012) for an application of our model to labor mobility.

sibly low) imitation cost c_i. If they do, they also decide whether or not to pay a given one-time protection cost c_p (investing in technological complexity). If all previous entrants have paid c_p, the cost of reverse engineering for the remaining imitators is c_i. If at least one of them did not, the innovation becomes freely available. Protection technologies are characterized by their cost, c_p, and the strength they confer, c_i.

We find in this context that the innovator can earn substantial rents, even in a very unfavorable environment, where for instance imitation is not time-consuming. Such high (post-innovation) rents may also be well above those attained by imitators. Surprisingly, the protection technologies that tend to yield a high payoff for the innovator are expensive and do not protect very well (relatively high c_p and small c_i). The intuition behind this result is as follows. The fact that the protection technology is relatively expensive means that the innovator uses it but, upon entry, imitators do not. Thus, as soon as the first imitator enters, the knowledge necessary to reproduce the technology enters the public domain and all remaining imitators enter for free. This creates a strong incentive for imitators to try to free-ride on other imitators' reverse engineering efforts and thus delay entry, which leaves potentially very high profits to the innovator. Our theory can explain the previously mentioned survey evidence and can shed light on why innovation was observed to flourish in sectors where legal protection did not exist and technologies to increase complexity were used, such as in the software industry.

We focused our previous discussion on the most profitable protection technology, but we characterize in the paper the symmetric mixed-strategy equilibria for arbitrary technologies. This leads us to characterize a theoretically interesting pattern where typically a series of preemption games is followed with some probability by a waiting game. Imitators are involved in a series of preemption games taking place quasi-instantaneously at the outset of the game. All the imitators who happen to enter in this phase pay the protection cost, but fear mis-coordination and thus mix at each instant between waiting and imitating. They invest in complexity in the hope of securing some rents, anticipating that the initial phase of massive entry will be followed with some probability by a waiting game played by the imitators left to enter. Such a game involving delayed imitation arises because once a sufficient number of imitators have entered, the protection cost is too large relative to the post-entry payoff, and hence the next imitator to enter does so without investing in complexity. These imitators thus engage in a waiting game and delay entry in the hope that another imitator enters before them.

Our theory thus predicts an interesting pattern of entry by imitators: a certain number of firms quasi simultaneously enter, and the remaining imitators then wait to enter until one of them chooses to do so, followed by all the others. If the process of diffusion of new technologies is a process of imitation, this provides a strategic explanation for the well documented delay in diffusion (see survey by Hoppe (2002)). We show that this delay is increasing in c_i and c_p and show it is coherent with some preliminary statistical evidence. Our theory also has distinctive features for the pattern of entry that could allow to empirically distinguish it from other strategic explanations for delay.

We also show that our results can enrich the debate on when reverse engineering should be regulated. As stated in Samuelson and Scotchmer (2002), "reverse engineering has always been a lawful way to acquire a trade secret as long as acquisition of the known product is by a fair and honest means". The authors nevertheless emphasize that attempts have been made in certain industries to legally restrict reverse engineering (such as the 1984 Semiconductor Chip Protection Act (SCPA)). Samuelson and Scotchmer (2002) argue that lead time before reverse engineers can enter and costliness of reverse engineering provide natural protection, but when these barriers disappear, legislation might be needed. Our paper complements this analysis by introducing dynamic considerations and introducing another dimension that can evolve through time, the cost of technical protection c_p. An increase in c_p can increase profits of the innovator, even when imitation is not time consuming and is not very costly.

There is a growing literature discussed below on mechanisms that can generate profits for innovators in the absence of patents (new stream of papers following Boldrin and Levine (2002, 2008a, 2008b)). However, these papers do not address directly the empirical puzzle presented by the seminal surveys of Levin et al. (1987) and Cohen et al. (2000). We take a different approach and focus on one dimension apparent in these surveys and of large empirical relevance: the choice between patents and protection through technical complexity. This allows us to draw important implications not only for profits, but also on regulation of reverse engineering and on diffusion paths of innovations.

One strand of the literature examines the choice between patents and secrecy (see survey by Hall et al. (2012)). However, all of these papers consider secrecy as the default option when patents are not used without focusing on the precise strategy used by firms to generate profits when not patenting. Horstmann, MacDonald and Slivinski (1985) emphasize the signaling dimension of patents in an environment where innovators have private information on the value of imitation for potential imitators, whereas Gallini (1992) is interested in analyzing the optimal trade-off between patent length and breadth. Kultti et al. (2007) deal with the comparison between patenting and secrecy in a setting with multiple independent discoveries and where the idea becomes public under secrecy with a certain exogenous probability. Anton and Yao (2004), Maurer and Scotchmer (2002) and Henry and Ponce (2011) do consider precise strategies such as disclosure decisions or licensing, but these strategies cannot be tied directly to the evidence provided in the survey.

A second strand of the literature examines how innovators can disclose the value of their inventions without being expropriated, in an environment with weak property rights. This essential topic (dating back to Arrow (1962)) is however not as directly linked to the evidence presented in the surveys where the respondents are typically not entrepreneurs in small startups.[4] Anton and Yao (1994, 2002) analyze how a financially-constrained innovator with an innovative idea can earn substantial post-innovation rents even if her idea can be expropriated when revealed to either of two firms capable of commercializing it. In turn, Baccara and Razin (2007) analyze the incentives to disclose ideas when there is possibly more than one innovator with the same idea and

[4]The issue of selling the idea is less relevant for larger firms that are not financially constrained and can commercialize products on their own unlike entrepreneurs.

patenting is not feasible.

Our paper also contributes to the literatures on adoption of new technologies and on entry games with an infinite horizon of play, both of which are interrelated. Our game exhibits a theoretically interesting pattern of a series of preemption games followed by a waiting game. Our approach towards analyzing continuous-time preemption games builds upon the technology adoption model of Fudenberg and Tirole (1985), except that we have more than two players (possibly) mixing over more than two actions. The existence of equilibrium coordination failures directly relates our work to that of Dixit and Shapiro (1986), Vettas (2000) and Bertomeu (2009). Vettas (2000) is of particular relevance because he finds the remarkable result that the payoff expected by an incumbent first increases then decreases as more firms become active in the market. A similar nonmonotonicity result is derived in our setting, even though we allow flow profits to strictly decrease in the number of firms active in the market, unlike Vettas (2000). Our focus on continuous time allows us to dispense with his assumption, showing that his insights carry over to settings in which decisions can be made very often.[5]

The remainder of the paper is organized as follows. In Section 2 we introduce the model. In Section 3 we solve for the equilibrium entry and protection decisions. In Section 4 we draw conclusions on innovator's profits and patterns of entry and diffusion, while Section 5 concludes. All proofs are presented in the appendix.

2 Model

We analyze a discrete-time game that lasts infinitely many periods of length $\Delta > 0$ each. The time variable is denoted by $t = 0, \Delta, 2\Delta, ...$ All players have the same per-period discount factor δ^{Δ}. We will focus on the case in which Δ is positive but converges to zero, i.e., the continuous-time limit of the game.

The game involves one innovator and $n-1 \geq 2$ (ex ante identical) potential

[5] Only a few papers consider entry timing games that display both preemption and waiting motives as play unfolds: Sahuguet (2006) and Park and Smith (2008) for instance.

imitators. Prior to the start of the game, the innovator has discovered a new technology. The imitators can then decide in each period whether to imitate or stay out of the market an additional period. We consider the dynamics of imitation of this technology. The cost of imitation depends on the strategic choices made by the innovator and the imitators who previously entered. In any period t, we refer to the players who have already entered the market as "insiders", whereas we refer to the imitators who have not yet entered as the "outsiders".

The innovator at time $t = 0$ and the imitators upon entry need to decide whether to invest in protection. Protection technologies are characterized by two parameters c_i and c_p. We denote $c_p > 0$ for the one-time cost that needs to be incurred to achieve protection. In any period, if the innovator and all insiders incurred the protection cost $c_p > 0$, the outsiders who decide to enter need to incur imitation cost $c_i > 0$. This one-time cost c_i gives instantaneous access to the same technology (see Henry and Ruiz-Aliseda (2012) for stochastic protection where a firm who pays c_p is uncertain of whether her technology can be successfully imitated by an imitator who pays c_i). However, if one of the insiders did not pay c_p upon entry, then imitation becomes costless for all outsiders. We assume that the costs c_p and c_i remain fixed throughout the game, in particular they are independent of the number of firms active in the market.[6]

In each period, an outsider can therefore choose among three actions:

- to imitate and pay the protection cost, an action denoted by p
- to imitate and not pay the protection cost, an action denoted by u
- not to imitate and wait another period, an action denoted by w

Per-period profits depend on the number of firms who have entered. We denote π_j for the per-period individual profit if $j \in \{1, ..., n\}$ firms (including

[6] see Henry and Ruiz-Aliseda (2012) section 5 for a micro foundation for c_p based on a model of researcher mobility.

the innovator) hold the technology.[7] Denoting the rate at which profits are discounted by firms by $r > 0$, let $\Pi_j \equiv \pi_j / r$ represent the value of a perpetual stream of discounted profits collected by a firm when a total of $j \in \{1, ..., n\}$ firms hold the technology and no further entry takes place. We assume π_j and thus Π_j are decreasing, with $\pi_n > 0$.

We mostly focus, in particular in Section 3, on the case in which $\Pi_n > c_i$. This corresponds to a situation where all firms will eventually enter the market: even if $n - 1$ firms are already on the market and all the insiders and the innovator paid the protection cost c_p, imitation is still profitable. Note that this is a priori the worst-case scenario for innovation in the absence of legal protection since the protection technology does not offer much of a guarantee. At the end of the section, we consider the case $\Pi_n < c_i$ and show that the result are very similar, up to a notational change.

We allow for mixed strategies and focus on symmetric Markov Perfect Equilibria (MPE), where the state corresponds to the number of firms who hold the technology. The focus on symmetric (mixed-strategy) equilibria can appear restrictive. However, as Farrell and Saloner (1988) and Bolton and Farrell (1990) convincingly argue, decentralized coordination mechanisms involving anonymous players cannot be properly captured by asymmetric equilibria in which (asymmetric) roles are very well defined among players. In addition, play based on mixed strategies can be interpreted as play arising in a game in which each player has private information about some disturbance affecting her final payoff.[8] Coordination failures occur under this interpretation not because of randomization but because players have incomplete information about others' payoffs.

Given our restriction on Markovian play, we use the following notation throughout. Thus, at the start of a period with k outsiders left to enter, we

[7] To avoid introducing several effects that would obscure the message of the paper, we assume that the flow profits earned do not depend on whether the protection cost was incurred or not. In other words, making the technology harder to reverse engineer does not directly affect the willingness to pay of consumers or production costs. However, the mechanics of the model would be preserved if this additional feature was introduced.

[8] This is the well-known purification argument in Harsanyi (1973).

denote:

- the expected discounted profits of an insider by I_k
- the expected discounted profits of an outsider by O_k

3 The dynamics of protection and imitation

In this section, we mainly focus on the characterization of the equilibria, in the case where $\Pi_n > c_i$ (we consider the other case at the end of the section). To help the reader through the arguments, we first sketch the shape of the equilibrium. The finiteness of the pool of potential imitators allows us to use backward induction when solving the infinite-horizon game, so we explain the reasoning by working backwards as well.

In the final subgames, when many firms are already active, the protection cost c_p appears large compared to the expected profits on the market. The next entrant will thus enter without any protective measure, thereby creating an incentive for the remaining imitators to delay imitation in the hope of free riding on the efforts of the next to enter. We actually find a critical number of outsiders J such that if the number of outsiders is strictly less than J, they engage in a waiting game and delay entry.

In earlier subgames with at least J outsiders, there is an incentive for imitators to enter quickly, preempt the others by protecting their technologies and benefit from the subsequent imitation delay. However, there is a risk of miscoordination were all imitators to enter simultaneously. This creates the conditions for a preemption game. In such a game, at least one outsider will enter right away (and several could in fact enter simultaneously). If the number of outsiders is still not below J following this wave of entry, another preemption game is played, and so on and so forth until the number of outsiders is finally smaller than J. Overall, we see that the pattern is a series of preemption games followed by a waiting game. Below, we make these arguments formal.

3.1 Solving the subgames with less than three outsiders

We note that in any subgame in which at least one of the insiders did not pay the protection cost upon entry, all outsiders immediately imitate the technology at no cost. Thus in the following discussion, we exclusively focus on subgames in which all insiders paid c_p upon entry.

The last entrant

We begin our analysis by considering those subgames in which just one imitator is left to enter the market. Since $\Pi_n > c_i$, the last outsider enters immediately without paying c_p. The expected profit of an insider in such a subgame is $I_1 = \Pi_n$. The expected profit of the outsider is $O_1 = \Pi_n - c_i$.

Two imitators left to enter

We now consider the subgames with only two outsiders. The outsider who enters first needs to incur cost c_i, but knows that, regardless of whether or not she pays the additional protection cost, the remaining outsider will enter immediately. It is then clear that action p (entering and paying the protection cost) is strictly dominated.

Therefore, the first entrant does not choose protection, and the second entrant incurs no imitation cost. This creates the conditions for a waiting game in which both players mix between entering without paying the protection cost and waiting. Both players prefer to be the second entrant, but also do not want to wait excessively as they lose profits every period. As is standard in such games (if stationary), in the limit when Δ converges to zero, the entry time of each imitator converges to an exponential distribution.

Lemma 1 *In subgames with two outsiders, the only symmetric MPE is such that both outsiders mix between actions u (entering without paying the protection cost) and w (waiting another period). As Δ converges to zero, the entry time of each outsider converges to an exponential distribution with parameter λ_2, where $\lambda_2 \equiv r(\Pi_n - c_i)/c_i$. The expected profit of each outsider is $O_2 =$*

$\Pi_n - c_i$*, whereas each of the insiders expects to gain* $I_2 = \mu_2 \Pi_{n-2} + (1-\mu_2)\Pi_n$*, where* $\mu_2 \equiv r/(r + 2\lambda_2)$*.*

The expected payoff of an outsider at the beginning of these subgames is $O_2 = \Pi_n - c_i$ since she is indifferent between all entry times, including entering immediately. On the contrary, the insiders expect significant profits since they will earn per-period profits π_{n-2} until the time of first entry, which is exponentially distributed (with hazard rate $2\lambda_2$).[9]

Three imitators left to enter

Before studying the complete dynamics, it is useful to understand in detail the resolution of subgames with three outsiders left. The results will partially extend to the case with more than three outsiders, but with some important differences highlighted in section 3.2. All players know that in any period, if a single outsider enters and pays the protection cost, the remaining two imitators will play a waiting game. In such a game, we established in Lemma 1 that insiders earn expected profits of $I_2 = \mu_2 \Pi_{n-2} + (1 - \mu_2)\Pi_n$.

Thus, we first note that, if $I_2 - c_p - c_i \leq \Pi_n - c_i$, playing action p is (weakly) dominated by u, that is, outsiders will never pay the protection cost. The condition can be equivalently expressed as $c_p \geq c_2^* \equiv \mu_2(\Pi_{n-2} - \Pi_n)$. According to the same logic as in the previous section, the three imitators will then engage in a waiting game. We show in Lemma 2 below that the individual entry time then follows an exponential distribution with parameter $\lambda_3 \equiv r(\Pi_n - c_i)/(2c_i)$.

On the contrary, if $c_p < c_2^*$, preemptively entering and paying the protection cost becomes very attractive if the two other outsiders do not enter. There is however a risk of coordination failure were all outsiders simultaneously to enter and pay c_p. This creates the conditions for a preemption game described in Lemma 2. As the time between two consecutive periods shrinks, outsiders essentially mix between p and w, that is, between entering with protection

[9] Indeed, the entry time of each individual imitator follows an exponential distribution with parameter λ_2, and thus the time of first entry is exponentially distributed with parameter $2\lambda_2$, since it is the minimum of two exponential random variables.

and waiting. Entry occurs almost instantaneously with probability one, and simultaneous entry of several outsiders occurs with positive probability.

Lemma 2 *In subgames with three outsiders, as* Δ *converges to zero:*

(i) If $c_p \geq c_2^*$*, the three outsiders mix between actions* u *and* p*. The entry time of each outsider converges to an exponential distribution with parameter* λ_3*, where* $\lambda_3 \equiv r(\Pi_n - c_i)/(2c_i)$*. Furthermore, the expected profit of each outsider is* $O_3 = \Pi_n - c_i$*, whereas each of the insiders expects to gain* $I_3 = \mu_3 \Pi_{n-3} + (1 - \mu_3)\Pi_n$*, where* $\mu_3 \equiv r/(r + 3\lambda_3)$*.*

(ii) If instead $c_p < c_2^*$*, the three outsiders start playing a preemption game as soon as this subgame begins. The limiting distribution is such that outsiders play* w *and* p *with a probability bounded away from zero, and the payoff of the outsiders converges to* $O_3 = \Pi_n - c_i$*, whereas the payoff of the insiders converges to* $I_3 = \phi_3(1)I_2 + (1 - \phi_3(1))\Pi_n$*, where* $\phi_3(1)$ *is the probability of a single outsider entering.*[10]

Lemma 2 has a very natural interpretation. If the protection cost is relatively high, it will not be paid upon entry, and therefore all outsiders wait in the hope that one of them will move first without paying c_p. On the contrary, if the protection cost is low enough, it will be incurred upon entry. The problem is then one of coordination. All outsiders would like to be the only firm to enter and then enjoy payoff I_2 while the others engage in a waiting game, but no one has an interest in paying the protection cost if other outsiders choose to enter at the same time.

3.2 Subgames with more than three imitators left to enter

The ideas uncovered in the subgames with three outsiders partially extend to the subgames with a larger number of outsiders. *However*, it becomes more technically and conceptually challenging since a sequence of preemption games can now occur and the probabilities of entry are thus defined recursively. In

[10] See expression (7) in the appendix for the specific formula for $\phi_3(1)$.

the three-firm case, there was a single preemption and game and we could thus derive probabilities explicitly and take the limit for small values of Δ. We now take a different approach and consider a continuous time approximation of equilibrium play and show below that it is arbitrarily close to actual play. We obtain the following result:

Proposition 1 *When $\Pi_n > c_i$, in the continuous time limit of the game, the symmetric MPE exhibits the following properties: there exists a number of entrants $J \in \{3, ..., n\}$ such that, if the innovator initially paid c_p:*

1. *At least J outsiders quasi-instantaneously imitate and pay the protection cost.*

2. *The remaining outsiders, if there are more than one, delay imitation for a random length of time and do not pay for protection upon imitation*

3. *After one of them enters without paying the protection cost, all the remaining outsiders immediately imitate.*

The overall shape of entry is very similar to the three-firm case. In particular, there exists a critical number of outsiders J such that as long as the number of outsiders is greater or equal than J, they try to preempt each other, whereas they play a waiting game as soon as the number of outsiders crosses this threshold. In Section 4, we focus on the implications of these results. Below we explain the different building blocks behind Proposition 1.

In what follows, let $c_k^* \equiv \mu_k(\Pi_{n-k} - \Pi_n)$, where $\mu_k \equiv r/(r + k\lambda_k)$. We show in the following lemma that in the subgame with $k \geq 3$ outsiders, if $c_p \geq c_{k-1}^*$, players mix between waiting and entering without protection and the entry time is exponentially distributed. A key part of the induction argument is that $\{c_k^*\}_{k=2}^{n-1}$ is a monotonically increasing sequence.[11] This implies that, when $c_p \geq c_{k-1}^*$, if one outsider chooses to enter by paying the protection cost, the

[11] Note that $\mu_k = (k-1)c_i/(k\Pi_n - c_i)$ is increasing in k, since $c_i < \Pi_n$ implies that $d\mu_k/dk = c_i(\Pi_n - c_i)/(k\Pi_n - c_i)^2 > 0$. Taking into account that both μ_k and $\Pi_{n-k} - \Pi_n$ are positive, the fact that Π_{n-k} and μ_k are both increasing in k then yields that $c_2^* < c_3^* < ... < c_{n-1}^*$.

$k-1$ remaining outsiders will then engage in a waiting game, since $c_p > c^*_{k-2}$. Intuitively, the incentive to avoid paying the protection cost becomes more intense as fewer imitators remain inactive: as k decreases, the profit stream to be earned following entry becomes relatively smaller and the waiting game is expected to last less (note that Π_{n-k} is increasing in k, whereas $k\lambda_k$ is decreasing).

Lemma 3 *In the subgame with $k \in \{3, ..., n-1\}$ outsiders, if $c_p \geq c^*_{k-1}$, the k outsiders mix between actions u and w. The entry time of each outsider converges as Δ goes to zero to an exponential distribution with parameter $k\lambda_k$, where $\lambda_k \equiv r(\Pi_n - c_i)/((k-1)c_i)$. The expected profit of each outsider is $O_k = \Pi_n - c_i$, whereas each of the insiders expects to gain $I_k = \mu_k \Pi_{n-k} + (1-\mu_k)\Pi_n$, where $\mu_k \equiv r/(r + k\lambda_k)$.*

We now consider the more complex case with $k \geq 3$ outsiders and $c_p < c^*_{k-1}$. It is essential for our purposes to define J, the critical number of outsiders such that a waiting game is played if the number of outsiders is *strictly less* than J (i.e., in subgames in which the number of imitators left to enter equals $2, ..., J-1$). Formally, we have $J = \inf\{k \geq 3 : c_p < c^*_{k-1}\}$, where $J = n$ if it is not well defined. Note that J is a step function of c_p ranging from 3 to n. We will now show that for $k \geq J$, a series of preemption games takes place. A priori, the players mix between the three available actions, w, p and u.[12]

Recall that we are interested in equilibria where players can react instantaneously to each others actions, i.e in situations where the time Δ between successive play is negligible.[13] In what follows, we will not be deriving the exact play in a symmetric equilibrium for small values of Δ but consider a continuous-time approximation of equilibrium play that is arbitrarily close to the true outcome.

[12] We will show when proving Lemma 4 that action u is chosen, but with vanishing probability as Δ goes to zero.

[13] As emphasized by Fudenberg and Tirole (1991) when dealing with preemption games, a continuous-time version of the game cannot be directly used, and one is forced either to use approximations based on discrete-time games or to properly expand strategy spaces to accommodate for such approximations, as done by Fudenberg and Tirole (1985).

More specifically, the approach will be the following. For a given period length Δ, let $\rho_{a,k}(\Delta) \geq 0$ be the probability with which each outsider plays action $a \in \{w, p, u\}$ when k outsiders are left to enter. Also, let $V_{a,k}(\Delta)$ denote the outsider's payoff from choosing action a given that the $k-1$ other players are mixing over actions with probability $\rho_{a,k}(\Delta)$ in all subgames with k outsiders. In equilibrium, the mixing probabilities $\rho_{a,k}(\Delta)$ for $a \in \{w, p, u\}$ must be such that outsiders are indifferent between all three actions and such that these are indeed probabilities (i.e. $V_{p,k}(\Delta) = V_{u,k}(\Delta) = V_{w,k}(\Delta)$, $\rho_{a,k}(\Delta) \in (0, 1)$ and $\sum_{a\in\{w,p,u\}} \rho_{a,k}(\Delta) = 1$). What we will do is to solve for the solution of this system for $\Delta = 0$, what we call the continuous-time approximation of the equilibrium,[14] and we will show that this solution exists and is unique. Given that the value functions $V_{a,k}(\Delta)(a \in \{w, p, u\})$ are continuous in Δ and in the probabilities, this will be a close approximation of the equilibrium outcome for small enough values of Δ.

To illustrate further this method, consider the case of three players solved in the proof of Lemma 2. In that case we solved explicitly, for a small fixed value of Δ, for the probabilities $\rho_{a,3}(\Delta)$, $a \in \{w, p, u\}$ (see (5) and (6) in the appendix). We see from the solution presented in the proof of Lemma 2, that taking the limit of all the probabilities as Δ converges to zero (as we did) leads to the same solution as directly solving the system consisting of equations (2)-(4) for $\Delta = 0$, as was to be expected due to the continuity of the system. From now on, $\rho_{a,k}$ and $V_{a,k}$ shall respectively denote $\rho_{a,k}(\Delta)$ and $V_{a,k}(\Delta)$ for $\Delta = 0$.

We formally show in the proof of Lemma 4 below that the symmetric MPE can be approximated for small enough values of Δ by an equilibrium where the action of entering without protection is played with essentially zero probability, i.e., $\rho_{u,k} \approx 0$. Thus, in the approximation we consider, the players will essentially mix just between actions w and p. We denote $\rho_k \equiv \rho_{p,k}$ for the individual probability of entry (so we have $\rho_{w,k} \equiv 1 - \rho_k$). Given k outsiders,

[14] Formally, what we mean by (continuous-time) approximation of the equilibrium is a set of admissible mixing probabilities $\rho_{a,k}$ $(a \in \{p, u, w\})$ satisfying the following property: for any (small) $\epsilon > 0$, there exists Δ_ϵ such that $\Delta < \Delta_\epsilon$ implies that $|\rho_{a,k}(\Delta) - \rho_{a,k}| < \epsilon$, where $\rho_{a.k}(\Delta)$ is the exact equilibrium play.

the payoff to choosing action p is given by

$$V_{p,k} = \sum_{l=0}^{k-1} C_{k-1}^{l} (\rho_k)^l (1-\rho_k)^{k-1-l} I_{k-1-l} - c_p - c_i,$$

where $C_{k-1}^{l} = \binom{k-1}{l}$ denotes the binomial coefficient indexed by $k-1$ and l. The value to an outsider of paying the protection cost when entering depends on how many other outsiders simultaneously enter. If l other outsiders enter, the outsider participates in the next period as an insider in a subgame with $k-1-l$ outsiders. Her expected gain in this case is thus I_{k-1-l} (the value of being an incumbent with $k-1-l$ outsiders).

Each of the k outsiders will mix between p and w so as to leave others indifferent between these two actions, which yields that

$$V_{p,k} = \Pi_n - c_i,$$

since it can be shown that an outsider's payoff to waiting is $V_{w,k} = \Pi_n - c_i$ for $\Delta = 0$. Letting $\overline{I}_{k-1-l} \equiv I_{k-1-l} - \Pi_n$ and

$$F_k(\rho) \equiv \sum_{l=0}^{k-1} C_{k-1}^{l} \ \rho^l (1-\rho)^{k-1-l} \ \overline{I}_{k-1-l},$$

the indifference condition can be equivalently written as:

$$F_k(\rho_k) = c_p. \tag{1}$$

Thus, we have in subgames with k imitators left to enter (and such that $c_p < c_{k-1}^*$) that the approximate mixing probability (provided it exists) must solve $F_k(\rho_k) = c_p$. Largely inspired by Vettas (2000), we now exploit the recursive nature of the problem and the properties of $F_k(\cdot)$. We show that the symmetric MPE of the game can be approximated for small values of Δ by an equilibrium such that outsiders mix between actions p and w with strictly positive probabilities. Furthermore, in this approximation, the probability of playing action p in equilibrium decreases as the number of outsiders decreases.

The main properties of the $F_k(\cdot)$ functions, for $k \in \{J, ..., n-1\}$, are presented in Figure 1.

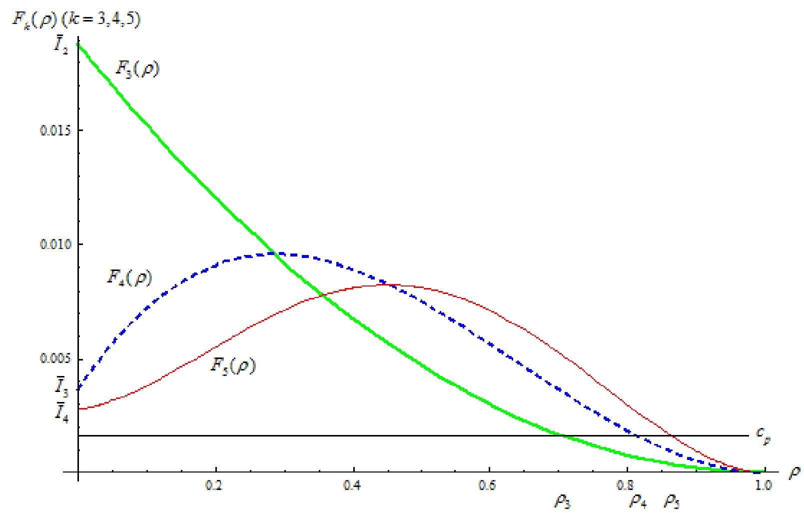

Figure 1: $F_5(\rho)$ (solid curve), $F_4(\rho)$ (dashed curve) and $F_3(\rho)$ (thick solid curve) plotted for $n = 6$, $c_p = 0.0017$, $c_i = 0.02$ and $\Pi_j = (j+1)^{-2}$ ($J = 3$ under these assumptions)

It holds that $F_J(\rho)$ is strictly decreasing in ρ, with $F_J(0) > c_p > F_J(1)$. There is thus clearly a unique solution to $F_J(\rho) = c_p$, namely ρ_J. This is intuitive: when $k = J$, following entry by at least one outsider, preemptive motives disappear and a waiting game is played thereafter (by definition of J). The length of such a waiting game is determined by the number of other outsiders who enter. Given our previous finding that the continuation payoff of an insider is lower in a waiting game played by fewer outsiders, the best scenario is if no one else enters ($\rho = 0$), whereas the worst scenario is if everyone else enters ($\rho = 1$). The randomization performed in equilibrium is somewhere in between.

For $k > J$, the pattern is slightly different. In these cases $F_k(\rho)$ is not everywhere decreasing in ρ. Unlike the case in which $k = J$, larger ρ does not make lower continuation payoffs more likely. Indeed, it can be shown (see proof of Lemma 4) that the continuation payoff of an insider (net of Π_n) has an inverted-U shape as a function of k: $\bar{I}_{n-1} < \bar{I}_{n-2} < ... < \bar{I}_J < \bar{I}_{J-1}$ and

$\overline{I}_{J-1} > \overline{I}_{J-2}... > \overline{I}_0 = 0$. So $F_k(\rho)$ also has an inverted-U shape as a function of ρ. Furthermore, we can show that $F_k(0) > c_p > F_k(1)$, for all $k > J$.

There is additional structure that can be exploited. In particular, $F_{k+1}(\cdot)$ starts off below $F_k(\cdot)$, reaches its maximum when crossing $F_k(\cdot)$ and then remains above $F_k(\cdot)$. A direct consequence is that the equilibrium ρ_k is increasing in k, an intuitive property. In these preemption games, players want to rush to enter to become one of the insiders during the waiting game that will likely follow. There is however a risk of excessive entry ex post. In a subgame where many players have already entered, and hence k is close to J, this risk becomes particularly severe, and the players in equilibrium therefore chose to enter with a lower probability. The following lemma formalizes all these ideas.

Lemma 4 *In subgames with $k \in \{J, ..., n-1\}$ outsiders, if $c_p < c^*_{k-1}$, then, for small enough Δ, the symmetric MPE can be approximated by the following equilibrium:*

(i) Outsiders mix only between actions p and w, and the probability ρ_k of playing p is uniquely given by the solution to $F_k(\rho_k) = c_p$.

(ii) ρ_k is increasing in k.

(iii) Quasi-instantaneous entry by at least one outsider occurs.

We are now in a position to show that the results of Proposition 1 partially extend to the case where $c_i \geq \Pi_n$, a situation that for large enough values of n approximates free entry. In particular, there exists a critical value J' such that if the number of outsiders is larger or equal to J', outsiders mix between actions p and w and at least J' quasi instantaneously enter. The main difference is that if the number of outsiders is less than J', no further entry takes place whereas in the case $c_i < \Pi_n$, all players play a waiting game and eventually enter. We derive the value of J' below.

In situations where $c_i \geq \Pi_n$, action u is always dominated by w if all insiders have paid c_p, since playing u yields payoff $\Pi_n - c_i \leq 0$. Letting $c'_k \equiv \Pi_{n-k} - c_i$ in what follows, J' is then defined by $J' \equiv \inf\{k \geq 3 : c_p < c'_{k-1}\}$ (with $J' = n$ if the definition is vacuous), so that subgames with $k \leq J' - 1$ outsiders exhibit no further entry.[15] Lemmas 1-4 are then directly applicable by simply letting $c_i = \Pi_n$ and redefining c_k^* and J as $c_k' \equiv \Pi_{n-k} - c_i$ and J', respectively. Hence, the case in which $c_i \geq \Pi_n$ corresponds to that in which imitation delays in subgames without preemption features are infinitely long. Proposition 1 then applies accounting for this new notation and the fact that the imitation delay after the initial preemptive imitation phase is infinite.

4 Using complexity: returns to R&D and diffusion of innovations

We can use the results of the previous section to draw important implications on incentives to innovate and patterns of entry of imitators. We first show that our model and the mechanism we consider can explain why innovation flourished in certain sectors even when patents were not available. Second, by examining how profits of innovators vary with the characteristics of technical protection we can shed some light on the need for regulation of reverse engineering. Finally we show our results can have important implications for the patterns of entry in innovative industries as well as the diffusion of innovations.

4.1 High profits outside patents

Even though we consider an environment a priori very unfavorable to innovators, where in particular imitation is instantaneous, our first result shows that profits of innovators can be very high even when patents are not available or are not chosen.

Proposition 2 *In the continuous-time limit of the game, the equilibrium payoff of the innovator can be arbitrarily close to* $\Pi_1 - \Pi_2$*. This happens for technologies that are such that* $c_p \downarrow \Pi_2$ *and either* $c_i \geq \Pi_n$ *or* $c_i \uparrow \Pi_n$*.*

Interestingly, the maximum profit $\Pi_1 - \Pi_2$ is attained for a protection technology that is expensive ($c_p = \Pi_2$) and potentially does not perform very

[15] Since it holds that $\Pi_{n-(J'-1)} - c_i > c_p \geq \Pi_{n-(J'-2)} - c_i$ by definition of J'.

well (c_i close but less than Π_n).

The intuition behind this result for $c_i < \Pi_n$ is as follows: the fact that the protection technology is expensive ($c_p \geq \Pi_2$) means that, upon entry, imitators do not use it. Thus, as soon as the first imitator enters, the knowledge necessary to reproduce the technology enters the public domain and all remaining imitators enter for free. This creates a strong incentive for all imitators to try to free-ride on other imitators' efforts. The incentive to enter first converges to zero when c_i approaches Π_n, and thus waiting is infinitely long and the innovator's payoff converges to Π_1. Of course, she has to pay a protection cost that, in the most favorable case, is equal Π_2. For such a technology, the payoff to the innovator is thus high even in environments with no legal protection.[16]

Our model can thus explain the evidence in the surveys mentioned in the introduction and why innovation flourished in certain sectors where patent protection was not available and investment to increase complexity were feasible. Consider the software industry. Until recently, software was not covered by patents and it was common for inventors to obfuscate the code: in other words, transforming the readable source code into code difficult to use directly. Today, various techniques are available and appear to be relatively cheap (low c_p). But this was not always the case and our theory could help explain why, even though patents did not apply, this was nevertheless an industry characterized by relentless innovation (see Boldrin and Levine (2007)).

There is unfortunately no data available on the level of c_p for different technologies. It is however interesting to pay attention to the sectors in which patents are judged to be least effective according to surveys of managers (Cohen et al. 2000), such as electronic components and semiconductors (the software industry is not part of the survey). These are also the sectors in which protection technologies can be most commonly observed. For instance, hardware obfuscation is a technique by which the description or the structure of electronic hardware is modified to intentionally conceal its functionality,

[16] The case $c_i \geq \Pi_n$ is equivalent to $c_i \uparrow \Pi_n$ as in both cases the imitation delays are infinitely long.

making it significantly more difficult to reverse engineer.[17] We also observe technologies for sale that allow protection of integrated circuits from reverse engineering.[18]

4.2 On the need for regulation of reverse engineering

As stated in Samuelson and Scotchmer (2002), "reverse engineering has always been a lawful way to acquire a trade secret as long as acquisition of the known product is by a fair and honest means". The authors nevertheless emphasize that attempts have been made in certain industries to legally restrict the use of reverse engineering. The most prominent example is the introduction of the Semiconductor Chip Protection Act (SCPA) in 1984 designed to protect chip layout from reverse engineering.

Samuelson and Scotchmer (2002) argue that lead time before reverse engineers can enter and costliness of reverse engineering provide natural protection, but when these barriers disappear, legislation might be needed. In fact, it is argued that one of the reasons for the introduction of SCPA was a fall in the cost of copying (a fall in c_i in the terminology of our model). Our paper complements this analysis by introducing dynamic considerations and introducing another dimension that can evolve through time, the cost of technical protection c_p, an aspect previously neglected. We in fact show that when dynamics of protection are considered, lags and high cost of copying are not necessary to guarantee profits since in our model we have no lags and we consider small values of c_i. The dynamic reasons we unveil is that imitators endogenously want to keep barriers to entry high and they also want to free ride on each other's copying efforts.

As the result of the previous subsection suggests, a high value of c_p can actually have a positive effect on the innovator's profits and thus reduce the need for legislation restricting reverse engineering. In this section we characterize

[17] There are other techniques, some cryptography-based.

[18] An example is given by the United States patent 7128271, described as "a semiconductor integrated circuit having a reverse engineering protection part that can be easily implemented".

more fully the shape of profits as a function of the characteristics of technical protection. We confirm that a drop in c_p can decrease profits. In other words it is possible the SCPA was driven not only by a drop in c_i, but possibly also by a simultaneous drop in c_p.

Let us first examine how the profits of the innovator vary with the characteristics of the protection technology. We plot in Figure 2, for a given value of c_i, how the innovator's payoff (net of the protection cost) varies with c_p. The pattern observed in Figure 2 is typically found for different specifications of parameters. First, when c_p is less than c_2^*, the net profits of the innovator initially increase and then decrease with c_p until c_p reaches c_2^*. Second, the behavior in the following intervals (c_{k-1}^*, c_k^*) (for $3 \leq k \leq n-2$) is as follows: the profits of the innovator decrease and reach Π_n as c_p approaches the upper bound of the interval. Third, when c_p goes above that upper bound we observe a discrete upward jump. Finally, when c_p is above c_{n-2}^*, net profits are linearly decreasing in c_p.

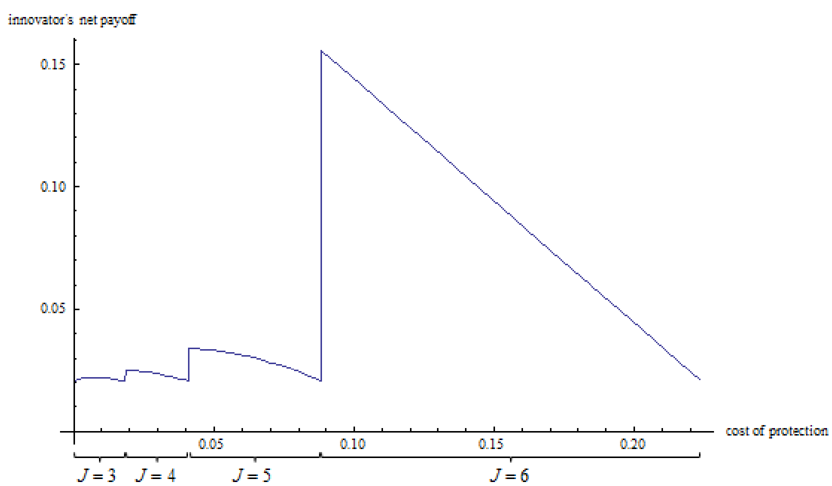

Figure 2: $I_5 - c_p$ plotted against c_p for $n = 6$, $c_i = 0.02$ and $\Pi_j = (j+1)^{-2}$

The key to understanding these effects is to notice that within an interval (c_{k-1}^*, c_k^*), the value of J (the critical number of firms such that a waiting game starts) remains fixed. Within such an interval, an increase in c_p generates two opposing forces. On the one hand, clearly an increase in c_p directly decreases the payoff of the innovator as he has to pay a higher protection cost. On the other hand, an increase in c_p decreases the probability of excessive entry in the preemption phase.

The second effect can be understood as follows. When c_p is smaller than the upper bound of the interval, outsiders imitate in each period of the preemption phase with some probability. Excessive entry above J thus happens with positive probability, at a cost for the innovator. As c_p increases, outsiders in any preemption subgame coordinate their actions better and better. As c_p gets close to the upper bound of the interval, the mixing probability in any subgame of the preemption phase converges to zero as the gains from entering before the others becomes small. The probability that at least two outsiders simultaneously enter in the same period converges even faster to zero. Thus, the outsiders perfectly coordinate their entry, and in equilibrium exactly J enter quasi instantaneously in a sequential manner.[19] So, as c_p increases, this second effect increases the profits of the innovator because of the lower risk of miscoordination when entering.

Figure 2 suggests that the first effect dominates only at the start of the first interval; in the other intervals, the second effect dominates. Note that if $c_p \geq c^*_{n-1}$, there is no preemption phase so only the first effect plays a role and profits linearly decrease with c_p. The previous discussion also allows us to explain why profits converge to Π_n at the upper bounds of the intervals (c^*_{k-1}, c^*_k). Because of the almost perfect coordination we highlighted when c_p approaches c^*_k, the gross profits of the innovator I_{n-1} are equal to the profits when k outsiders remain, I_k. Since $c^*_k = I_k - \Pi_n$, $I_k - c_p$ and hence $I_{n-1} - c_p$ both go to Π_n if c_p is close to the upper limit of the interval.

Figure 2 also clearly illustrates a discrete upward jump in the innovator's net payoff as c_p passes just above c^*_k. Although the value of c_p is quite similar on both sides of the threshold, the critical value J increases by one unit as soon as this threshold is crossed. This discretely increases I_{J-1}, which means that imitation delays involve more firms and hence being an insider is more valuable.

[19]Note that in Fudenberg and Tirole (1985) coordination failures do not occur on the equilibrium path even if players randomize independently, as in our case when c_p is close to the upper bound of the interval.

This favors the innovator, but the downside is that the potential entrants find entry more attractive and choose to enter with higher probability than when c_p is just below c_k^*, thus leading to more miscoordination. However, as our previous discussion suggests, this faster rate of imitation does not completely dissipate the greater profit triggered by the unit increase in J.

Some of the results highlighted in Figure 2 are stated formally in the following lemma:

Lemma 5 *Fix an interval* (c_{k-1}^*, c_k^*) *for* $2 \leq k \leq n-1$ *(with* $c_1^* \equiv 0$ *and* $c_{n-1}^* \equiv \infty$*). Then the following properties hold:*

(i) As c_p *converges to* c_k^* *from below (*$2 \leq k \leq n-2$*), net profits for the innovator converge to* Π_n *from above*

(ii) As c_p *converges to 0 from above, net profits for the innovator converge to* Π_n *from above*

(iii) As c_p *decreases starting from* c_{n-2}^**, net profits for the innovator fall linearly*

The pattern we uncover confirms that the parameter c_p should be a relevant dimension when considering legislation to regulate reverse engineering. It suggests that, on average, higher values of c_p can increase profits of firms. Of course, Figure 2 is characterized by discontinuities in the shape of profits, confirmed in the result of Lemma 3. In fact, this figure corresponds to the profits of the innovator for a particular product/technology. Policy makers would be interested more in the effect of c_p at the sector level. It would thus be a reasonable approach to take the average of profits of innovators over a large number of products/technologies within a industry with varying values for c_i. This would presumably smooth out the previously mentioned discontinuities.

4.3 Entry and diffusion dynamics

Proposition 1 also has clear implications for entry dynamics of imitating firms in environments where innovators choose to protect through technical complexity. There is initial entry of a certain number of firms followed by a delay and then bunched entry of the remaining imitators. We discuss later in the section how Proposition 1 can also have consequences for the path of diffusion of innovations.

For our theory to be an interesting guide for empirical work on dynamic entry of imitators, it is useful to establish some comparative statics on the length of the delay depending on the characteristics of the protection technologies.

Proposition 3 *The expected delay until all imitators enter the market is*

1. *increasing for small variations in* c_i
2. *weakly increasing in* c_p

Consider first the effect of c_i, the cost of imitation of a protected technology. This parameter affects directly the speed of entry in the waiting game played by k outsiders (given in lemma 3 by $kr(\Pi_n - c_i)/((k-1)c_i)$), but also affects the critical value J that determines the expected number of firms playing this waiting game. To isolate the first effect, we consider only small variations that do not affect the value of J.[20] It is then clear that delay increases in c_i: the higher the imitation cost, the bigger the incentives to free ride on other's efforts.

The effect of c_p is slightly more intricate, we do not prove it formally but can show it with numerical simulations and we provide some intuition as follows. The cost of protection c_p does not affect directly the speed of entry in the waiting game. However it affects it indirectly through the effect on J and on the probabilities of entry. In particular, as c_p increases, J, the benchmark value such that a waiting game starts, also weakly increases: paying the protection cost is less attractive and thus players start more quickly the waiting game. The final step of the argument is to note that an increase in the number of outsiders playing the waiting game increases the delay: the aggregate speed of entry of course mechanically increases in k, but each outsider strategically takes this into account and actually overcorrects in the sense that equilibrium

[20] For a continuous distribution for c_i, the set of values for c_i such that J changes is of zero measure.

aggregate entry rate decreases. Overall, an increase in c_p seems to increase the delay.

We provide some basic evidence coherent with our story. We match at the sector level the data on the popularity of patents, secrecy and lead time in Cohen et al. (2000) to the data on average entry rates in Dunne et al. (1988). We find a significant correlation of -.55 between average yearly entry and the popularity of secrecy, but no correlation with the popularity of patents or lead time.

This evidence is coherent with the result of Proposition 3 on the positive effect of c_p on delay, provided that the popularity of secrecy partly reflects an increased use of complexity as a protection mechanism and thus potentially a higher value of c_p (we saw in the previous section that higher values of c_p can lead to higher profits for an innovator choosing complexity as a protection mechanism). Of course this is just weakly suggestive evidence, and it would be more convincing to show directly the link between entry and c_p, but such data is unfortunately not available to the best of our knowledge.[21]

We conclude this section by discussing the implications of Proposition 1 for the diffusion of innovations. Hoppe (2002) in a survey on the patterns of diffusion, reports that a stylized fact in the literature is that "the adoption of new technology is in general anything but instantaneous". If we view diffusion of innovations as a process of imitation, Proposition 1 provides a foundation for this delay. The empirical literature on diffusion is in fact not explicit about what is the process of adoption of a technology, whether it is purchasing from the inventor or whether it comes through imitation.

Our theory thus proposes an alternative explanation for this delay based on strategic motives. It adds to quite a rich literature on the topic (see survey by Hoppe (2002)). There is however a distinctive feature that allows to empirically distinguish our theory from alternative ones: we predict delay followed

[21] It would also have been more appropriate to show correlations between entry and the popularity of complexity, but the data in Cohen et al. (2000) does not include complexity and for surveys that do (such as CIS survey), we do not have the data on sectorial variations. Note however that the correlation with lead time is close to zero and that it is thus quite likely that the correlation with secrecy is a sign of the effect of complexity.

by bunched entry but also a certain mass of entry at the beginning. This distinguishes our results from seminal work on adoption of new technologies in oligopoly when a firm does not need to imitate another one so as to enter a new market.[22] In particular, a stationary variant of Fudenberg and Tirole (1985) would lead all firms to enter simultaneously at the outset of the game. Diffusion happens in their setting because the environment is not stationary and the cost of adopting a new technology exogenously decreases over time. In our stationary environment, diffusion still arises with non negligible probability, the reason being the incentive that imitators have to wait until one of them enters without paying c_p. Imitation-driven dynamics therefore have different fundamentals from those that arise in the absence of imitation, even if outcomes seem similar. We believe that a proper understanding of diffusion processes should consider whether firms are imitating other ones or whether they are using technological suppliers.

5 Conclusion

In this paper we show that the dynamic investment in complexity can generate high rents for the innovators. Surprisingly, the protection technologies that yield the highest returns for the innovator are expensive and do not protect very well. We also show that our model has implications for the path of diffusion of innovations.

We believe our model and results could be the basis for interesting empirical work. At the very least it underlines the need for more comprehensive data on two dimensions. First, little is still known on the cost of reverse-engineering inventions, and how these costs vary by industry. Second, little information is available on protection technologies, be it their cost or the level of protection they confer. Although there is a large body of anecdotal evidence showing that technological protection is commonly used, there is no systematic measurement allowing for more detailed empirical analysis.

[22] See Hoppe (2002) for a description of papers in this large literature and how they relate to each other.

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For Online Publication: Appendix

Proof of Lemma 1. Fudenberg and Tirole (1991) show that the unique symmetric equilibrium of the discrete-time war of attrition with short period lengths converges to the unique symmetric equilibrium of the war of attrition in continuous time. This leads us to prove the result using the continuous-time version of the game directly.

Counting from the date at which the subgame is first reached, let us consider the expected payoff of an outsider if she chooses to imitate at time τ_2 given that the other outsider chooses her imitation time according to an atomless and gapless distribution $F_2(\cdot)$ with full support on $[0, \infty)$ and density $f_2(\cdot)$. Given that the other firm has made an unknown draw from $F_2(\cdot)$, a firm who enters at τ_2 expects to gain

$$\widehat{V}_2(\tau_2) = \int_0^{\tau_2} \Pi_n e^{-rs} dF_2(s) + \int_{\tau_2}^{\infty} (\Pi_n - c_i) e^{-r\tau_2} dF_2(s).$$

In a mixed-strategy Nash equilibrium, the firm should be indifferent among all possible imitation times, which formally means that we should have that $d\widehat{V}_2(\tau_2)/d\tau_2 = 0$ for all $\tau_2 \geq 0$. Straightforward differentiation using the fact that $\int_{\tau_2}^{\infty} dF_2(s) = 1 - F_2(\tau_2)$ yields that

$$\frac{d\widehat{V}_2(\tau_2)}{d\tau_2} = e^{-r\tau_2}[c_i f_2(\tau_2) - r(\Pi_n - c_i)(1 - F_2(\tau_2))].$$

Letting $h_2(\tau_2) \equiv f_2(\tau_2)/(1 - F_2(\tau_2))$ denote the hazard rate of $F_2(\cdot)$ and equating $d\widehat{V}_2(\tau_2)/d\tau_2$ to zero yields that the hazard rate is constant and equal to $h_2(\tau_2) = r(\Pi_n - c_i)/c_i$, so $F_2(\tau_2) = 1 - e^{-\lambda_2 \tau_2}$, where $\lambda_2 \equiv r(\Pi_n - c_i)/c_i$. Given that a probability distribution is exponential if and only if its hazard rate is constant, the individual entry time follows an exponential distribution with parameter $\lambda_2 = r(\Pi_n - c_i)/c_i$.

Furthermore, since a firm is indifferent among all the pure strategies played with positive density, the expected gain of an outsider converges to $O_2 = \Pi_n - c_i$ (payoff to imitating immediately).

We have shown that both outsiders make independent draws from an exponential distribution with the same hazard rate λ_2, so the time $\widehat{\tau}$ of first entry must be exponentially distributed with parameter $2\lambda_2$. The expected payoff for an insider is therefore given by:

$$I_2 = \int_0^\infty \left(\int_0^{\widehat{\tau}} \pi_{n-2} e^{-rs} ds + \int_{\widehat{\tau}}^\infty \pi_n e^{-rs} ds \right) 2\lambda_2 e^{-2\lambda_2 \widehat{\tau}} d\widehat{\tau}.$$

Integrating and letting $\mu_2 \equiv r/(r + 2\lambda_2)$ yields that:

$$I_2 = \mu_2 \Pi_{n-2} + (1 - \mu_2)\Pi_n.$$

■

Proof of Lemma 2. (i) As indicated in the main text, action p is weakly dominated if $c_p \geq c_2^*$, so the outsiders mix every period between u and w. Counting from the date at which the subgame is first reached, suppose that two outsiders draw their time of imitation with an unprotected technology using an atomless and gapless distribution function $F_3(\cdot)$ with full support on $[0, \infty)$. Denoting these (random) draws by s and s', we have that the expected payoff of an outsider if she imitates at time τ_3 with probability one (conditional upon no other outsider imitating earlier) is

$$\begin{aligned} \widehat{V}_3(\tau_3) &= \int_0^{\tau_3} \Pi_n e^{-rs} f_3(s)(1 - F_3(s)) ds + \int_0^{\tau_3} \Pi_n e^{-rs} f_3(s')(1 - F_3(s')) ds' \\ &\quad + (1 - F_3(\tau_3))^2 (\Pi_n - c_i) e^{-r\tau_3}. \end{aligned}$$

Because it must hold that $d\widehat{V}_3(\tau_3)/d\tau_3 = 0$ for all $\tau_3 \geq 0$, straightforward computations show that we must have $h_3(\tau_3) \equiv f_3(\tau_3)/(1 - F_3(\tau_3)) = r(\Pi_n - c_i)/(2c_i)$. Hence, $F_3(\tau_3) = 1 - e^{-\lambda_3 \tau_3}$, where $\lambda_3 \equiv r(\Pi_n - c_i)/(2c_i)$. Each outsider expects to gain $O_3 \equiv \Pi_n - c_i$ (since $\widehat{V}_3(\tau_3) = \Pi_n - c_i$ for $\tau_3 = 0$). In turn, the fact that the time at which imitation takes place is exponentially distributed with parameter $3\lambda_3$ yields that the payoff expected by the insiders is

$$I_3 = \mu_3 \Pi_{n-3} + (1 - \mu_3)\Pi_n,$$

where $\mu_3 \equiv r/(r+3\lambda_3)$.

(ii) We now consider the case $c_p < c_2^*$. In principle, firms will mix using the three actions available to each of them, namely w, p and u. We denote $\rho_{a,k} \geq 0$ the probability with which one of the outsiders plays action a when k outsiders remain to enter. We let $V_{a,k}$ denote the outsider's payoff when following action $a \in \{w, p, u\}$. In a mixed-strategy equilibrium in which outsiders play stationary strategies, we must have that $V_{w,3} = V_{p,3} = V_{u,3}$, where

$$V_{p,3} = \rho_{w,3}^2(\pi_{n-2}\Delta + I_2\delta^\Delta) + 2\rho_{w,3}(1-\rho_{w,3})(\pi_{n-1}\Delta + \Pi_n\delta^\Delta) + (1-\rho_{w,3})^2\Pi_n - c_i - c_p \tag{2}$$

$$V_{u,3} = \rho_{w,3}^2(\pi_{n-2}\Delta + \Pi_n\delta^\Delta) + 2\rho_{w,3}(1-\rho_{w,3})(\pi_{n-1}\Delta + \Pi_n\delta^\Delta) + (1-\rho_{w,3})^2\Pi_n - c_i \tag{3}$$

and

$$V_{w,3} = \rho_{w,3}^2(V_{w,3}\delta^\Delta) + 2\rho_{w,3}\rho_{p,3}O_2\delta^\Delta + (\rho_{w,3} + \rho_{p,3} + 1)\rho_{u,3}\Pi_n\delta^\Delta + \rho_{p,3}^2(\Pi_n - c_i)\delta^\Delta. \tag{4}$$

Because $V_{p,3} = V_{u,3}$, it holds after using the fact that $\rho_{w,3} \geq 0$ that

$$\rho_{w,3} = \sqrt{\frac{c_p}{(I_2 - \Pi_n)\delta^\Delta}}. \tag{5}$$

Using the working hypothesis that $c_p < c_2^* \equiv I_2 - \Pi_n$ yields that $\frac{c_p}{(I_2 - \Pi_n)\delta^\Delta} < \delta^{-\Delta}$, so $\rho_w < 1$ for $\Delta > 0$ close enough to zero.

Because $\rho_{u,3} = 1 - (\rho_{w,3} + \rho_{p,3})$ and $O_2 = \Pi_n - c_i$, the expression for $V_{w,3}$ can be rewritten as follows:

$$V_{w,3} = \frac{(1-\rho_{w,3}^2)\Pi_n - \rho_{p,3}(\rho_{p,3} + 2\rho_{w,3})c_i}{\delta^{-\Delta} - \rho_{w,3}^2}.$$

Equating $V_{u,3}$ and $V_{w,3}$ yields the value for $\rho_{p,3} \geq 0$ after some manipulations:

$$\rho_{p,3} = \sqrt{\frac{c_i - (1-\delta^\Delta)B\Pi_n - \Delta\rho_{w,3}(1-\delta^\Delta\rho_{w,3}^2)C}{\delta^\Delta c_i}} - \rho_{w,3}, \tag{6}$$

where $B = \delta^{\Delta}(2-\rho_{w,3})\rho_{w,3}^3 + (1-\rho_{w,3})^2$ and $C = 2\pi_{n-1}(1-\rho_{w,3}) + \rho_{w,3}\pi_{n-2}$.

Using the fact that $I_2 - \Pi_n = \mu_2(\Pi_{n-2} - \Pi_n)$, we find for small $\Delta > 0$ that

$$\rho_{w,3} \approx \sqrt{\frac{c_p}{\mu_2(\Pi_{n-2} - \Pi_n)}},$$

$$\rho_{p,3} \approx 1 - \sqrt{\frac{c_p}{\mu_2(\Pi_{n-2} - \Pi_n)}},$$

and

$$\rho_{u,3} \approx 0,$$

that is, action u is played with positive but vanishing probability.

We now determine payoffs. To make exposition notationally simpler, let us normalize to zero the date at which the subgame with three outsiders starts. Given m periods of play between time 0 and some fixed time $t > 0$, it holds that the probability that no outsider has imitated and protected her technology once time t has elapsed is $(\rho_{w,3})^{3m} = (\rho_{w,3})^{3t/\Delta}$ (since $m = t/\Delta$), which converges to zero as Δ converges to zero for any arbitrarily chosen $t > 0$. We then must have that there is probability one that at least one outsider will imitate and protect her technology (almost) instantaneously. In words, outsiders correlate their actions as Δ goes to zero even though they randomize independently.

We conclude the proof by characterizing the probability distribution over entry outcomes at (normalized) time 0 as well as equilibrium payoffs. Because the probability of no entry at any point in time is $(1-\rho_{p,3})^3$, it holds that the probability that at least one outsider enters is $1-(1-\rho_{p,3})^3$. Conditional upon at least one outsider entering, we then have that

$$\phi_3(3) = (\rho_{p,3})^3/(1-(1-\rho_{p,3})^3),$$

$$\phi_3(2) = 3(1-\rho_{p,3})(\rho_{p,3})^2/(1-(1-\rho_{p,3})^3),$$

and

$$\phi_3(1) = 3(1-\rho_{p,3})^2\rho_{p,3}/(1-(1-\rho_{p,3})^3), \tag{7}$$

where $\phi_k(l)$ denotes the probability that $l \geq 1$ outsiders enter simultaneously at 0 given that there are $k \geq l$ of them. We finally observe that an outsider's continuation payoff at the beginning of these subgames is approximately $O_3 = \Pi_n - c_i$ (since $V_{p,3} = V_{u,3} = V_{w,3} \approx \Pi_n - c_i$ for small enough $\Delta > 0$). Since $I_1 = I_0 = \Pi_n$, the expected payoff earned by an insider is approximately

$$I_3 = \phi_3(1)I_2 + (1 - \phi_3(1))\Pi_n.$$

■

Proof of Proposition 1. Proposition 1 directly follows from Lemmas 1-4. ■

Proof of Lemma 3. We prove the result by induction. Lemma 2 established the result for $k = 3$, so it only remains to prove that it holds for $k \geq 4$ whenever it is true for $k - 1$. So suppose that the result holds for $k - 1$, and consider the subgames with k outsiders when $c_p \geq c^*_{k-1}$.

Let us focus on an outsider's incentive to play p. Since $c^*_j < c^*_{k-1}$ (see proof in main text) for all $j < k - 1$, he knows when choosing action p that p being simultaneously chosen by $l \geq 0$ other imitators will result in the remaining outsiders playing a waiting game (by the induction hypothesis). Clearly, the highest payoff that can be achieved is the one attained when no other outsider enters simultaneously, i.e., when $l = 0$. Thus, the highest payoff she can obtain by entering and paying the protection cost is $I_{k-1} - c_p - c_i = \mu_{k-1}\Pi_{n-k+1} + (1 - \mu_{k-1})\Pi_n - c_p - c_i$. Since $c_p \geq c^*_{k-1}$ implies $I_{k-1} - c_p - c_i < \Pi_n - c_i$, it then follows that no outsider must be willing to enter by paying the protection cost in subgames with k outsiders.

The k outsiders will therefore mix between waiting and entering without protection. Counting from the date at which the subgame is first reached, let us suppose that the outsiders draw their time of imitation with an unprotected technology using an atomless and gapless distribution function $F_k(\cdot)$ with full support on $[0, \infty)$. We then have that the expected payoff of an outsider if she imitates at time τ_k with probability one (conditional upon no other outsider imitating earlier) is

$$\widehat{V}_k(\tau_k) = (k-1)\int_0^{\tau_k} \Pi_n e^{-rs} f_k(s)(1-F_k(s))ds + (1-F_k(\tau_k))^{k-1}(\Pi_n - c_i)e^{-r\tau_k}.$$

In order for such an outsider to be indifferent between all the possible imitation times, it is easy to show that we must have that $F_k(\tau_k) = 1 - e^{-\lambda_k \tau_k}$, where $\lambda_k \equiv r(\Pi_n - c_i)/((k-1)c_i)$. Each of the outsiders expects to gain $O_k \equiv \Pi_n - c_i$ (since $\widehat{V}_k(\tau_k) = \Pi_n - c_i$ for $\tau_k = 0$). In addition, because the time at which the first imitation takes place is exponentially distributed with parameter $k\lambda_k$, the expected profit of an insider is given by

$$I_k = \mu_k \Pi_{n-k} + (1-\mu_k)\Pi_n,$$

where $\mu_k \equiv r/(r + k\lambda_k)$. ■

Proof of Lemma 4. As explained in the main text, we solve for the approximation of the equilibrium outcome, taking directly the solution for a time period of length $\Delta = 0$. We show this result in a number of steps

Step 1: $\rho_{u,k} = 0$.

We show this result by induction. For $\Delta = 0$, we have

$$V_{u,J} = \Pi_n - c_i$$

and

$$\begin{aligned} V_{w,J} \;&=\; \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0]\; V_{w,J} + \\ &\sum_{m=1}^{J-1} \sum_{l=0}^{J-1-m} \Pr[X_{w,J} = J-1-l-m, X_{p,J} = l, X_{u,J} = m]\; \Pi_n + \\ &\sum_{l=1}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0]\; O_{J-l}, \end{aligned}$$

where $\Pr[X_{w,k}, X_{p,k}, X_{u,k}]$ denotes the probability that $X_{w,k}$ outsiders choose w, $X_{p,k}$ outsiders choose p and $X_{u,k}$ outsiders choose u. We know, that for all $k < J$, a waiting game is played and, according to Lemma 3, $O_k = \Pi_n - c_i$, so the system of equations can be rewritten as

$$V_{u,J} = \Pi_n - c_i$$

and

$$\begin{aligned} V_{w,J} &= \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0]\ V_{w,J} + \\ & (1 - \Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0])\ \Pi_n - \\ & \sum_{l=1}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0]\ c_i \end{aligned}$$

In a mixed-strategy equilibrium, an outsider must be indifferent between all actions played with positive probability, so we must have $V_{u,J} = V_{w,J}$, which implies that

$$\sum_{l=1}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0]\ /\ (1-\Pr[X_{w,J} = J-1, X_{p,J} = 0, X_{u,J} = 0]) = 1.$$

This holds if and only if

$$\sum_{l=0}^{J-1} \Pr[X_{w,J} = J-1-l, X_{p,J} = l, X_{u,J} = 0] = 1,$$

hence we get that $\rho_{u,J} = 0$. Furthermore, this implies that $O_J = \Pi_n - c_i$, and the property is therefore true for $k = J$. The reasoning follows exactly the same lines for larger values of k. We can therefore use the notation adopted in the main text where $\rho_k \equiv \rho_{p,k}$

Step 2: ρ_k is the unique solution to $F_k(\rho_k) = c_p$.

Consider first the "last preemption game", i.e., the subgame where J outsiders are left to enter. As shown in the main text, the indifference between actions p and w is defined by

$$F_J(\rho_J) = c_p,$$

where

$$F_J(\rho) = \sum_{l=0}^{J-1} C_{J-1}^l \ \rho^l (1-\rho)^{J-1-l} \ \overline{I}_{J-1-l}.$$

Note that following entry by at least one outsider, a waiting game is played (by definition of J). The speed is determined by the number of other outsiders who enter. Note that according to Lemma 3, $\overline{I}_{J-1-l} = \mu_{J-1-l}(\Pi_{n-(J-1-l)} - \Pi_n) = c^*_{J-1-l}$. We showed previously that c^*_k is an increasing function of k. So we have $\overline{I}_{J-1} > \overline{I}_{J-2} > ... > \overline{I}_0$, and it can be immediately observed that $F_J(\rho)$ is a strictly decreasing function of ρ. Indeed, increasing ρ shifts the distribution to states where the payoff is lower.

Furthermore, $J = \inf\{k \geq 3 : c_p < c^*_{k-1}\}$ implies that $F_J(0) = \overline{I}_{J-1} = c^*_{J-1} > c_p$. Since $F_J(1) = \overline{I}_0 = 0$ and $F_J(\rho)$ is a continuous and strictly decreasing function, it then follows that the equation $F_J(\rho) = c_p$ has a unique solution $\rho_J \in (0, 1)$.

We now work recursively with $F_{k+1}(\rho)$ for $k \geq J$. We use the following key properties of $F_{k+1}(\rho)$ proven below:

- Property 1:
$$\frac{\partial F_{k+1}}{\partial \rho}(\rho) = (\frac{k}{1-\rho})(F_k(\rho) - F_{k+1}(\rho)).$$
- Property 2:
$$\frac{\partial F_{k+1}}{\partial \rho}(0) > 0.$$
- Property 3:
$$\overline{I}_k = F_{k+1}(\rho_k).$$

From Properties 1 and 2, we can conclude that $F_{k+1}(\rho)$ is increasing at zero, reaches a maximum when $F_{k+1}(\rho)$ and $F_k(\rho)$ cross and is then decreasing. Furthermore, we know that $F_{k+1}(1) = \overline{I}_0 = 0$. So to establish that $F_{k+1}(\rho) = c_p$ has a unique solution it is sufficient to show that $F_{k+1}(0) > c_p$. To prove it, note that we have $F_{k+1}(0) = \overline{I}_k$, and Property 3 implies that $\overline{I}_k = F_{k+1}(\rho_k)$, so it holds that $F_{k+1}(0) = F_{k+1}(\rho_k)$. Because $F_{k+1}(\rho)$ is increasing at zero according to Property 2, the unique maximum must be reached somewhere between 0 and ρ_k. According to Property 1, we know that $F_{k+1}(\rho) > F_k(\rho)$ for $\rho \geq \rho_k$, and therefore $F_{k+1}(\rho_k) > F_k(\rho_k)$. Taking into account that $F_{k+1}(0) = F_{k+1}(\rho_k)$, as we just showed, and that $F_k(\rho_k) = c_p$, it follows that $F_{k+1}(0) > c_p$.

Step 3: (i) follows directly from steps 1 and 2. We also showed above that $F_{k+1}(\rho) > c_p$ for $\rho \in (0, \rho_k)$, so we must that have $\rho_k < \rho_{k+1}$, which proves (ii). Finally (iii) can be shown as in the proof of Lemma 2.

To conclude the proof we show that properties 1-3 state above do hold:

Property 1 We have that

$$F_k(\rho) = \sum_{l=0}^{k-1} C_{k-1}^{l}\ (\rho)^l (1-\rho)^{k-1-l}\ \overline{I}_{k-1-l}$$

and

$$F_{k+1}(\rho) = \sum_{l=0}^{k} C_{k}^{l}\ (\rho)^l (1-\rho)^{k-l}\ \overline{I}_{k-l}. \tag{8}$$

So we can establish that

$$\begin{aligned} F_k(\rho) - F_{k+1}(\rho) &= \sum_{l=0}^{k-1} C_{k-1}^{l}\ (\rho)^l (1-\rho)^{k-1-l}\ \overline{I}_{k-1-l} - \sum_{l=0}^{k} C_{k}^{l}\ (\rho)^l (1-\rho)^{k-l}\ \overline{I}_{k-l} \\ &= \sum_{l=1}^{k} C_{k-1}^{l-1}\ (\rho)^{l-1} (1-\rho)^{k-l}\ \overline{I}_{k-l} - \sum_{l=1}^{k} C_{k}^{l}\ (\rho)^l (1-\rho)^{k-l}\ \overline{I}_{k-l} - (1-\rho)^k\ \overline{I}_k \end{aligned} \tag{9}$$

Consider

$$\begin{aligned} \frac{\partial F_{k+1}}{\partial \rho}(\rho) &= \sum_{l=0}^{k} C_{k}^{l}\ \left[l(\rho)^{l-1}(1-\rho)^{k-l} - (k-l)(\rho)^l (1-\rho)^{k-l-1} \right]\ \overline{I}_{k-l} \\ &= \sum_{l=0}^{k} C_{k}^{l}\ (\rho)^{l-1}(1-\rho)^{k-l-1}(l-k\rho)\ \overline{I}_{k-l} \\ &= \sum_{l=1}^{k} C_{k}^{l}\ (\rho)^{l-1}(1-\rho)^{k-l-1}(l-k\rho)\ \overline{I}_{k-l} - k(1-\rho)^{k-1}\overline{I}_k, \end{aligned} \tag{10}$$

so that

$$\frac{\partial F_{k+1}}{\partial \rho}(\rho) = \sum_{l=1}^{k} l C_k^l \ (\rho)^{l-1}(1-\rho)^{k-l-1} \ \overline{I}_{k-l} - k \sum_{l=1}^{k} C_k^l \ (\rho)^{l}(1-\rho)^{k-l-1} \ \overline{I}_{k-l} - k(1-\rho)^{k-1}\overline{I}_k.$$

Given that $C_{k-1}^{l-1} = lC_k^l/k$, using (9) yields:

$$\frac{\partial F_{k+1}}{\partial \rho}(\rho) = (\frac{k}{1-\rho})(F_k(\rho) - F_{k+1}(\rho)), \tag{11}$$

as claimed.

Properties 2 and 3 We have that

$$\begin{aligned}\frac{\partial F_k}{\partial \rho}(\rho) &= \sum_{l=0}^{k-1} C_{k-1}^l \ \left[l(\rho)^{l-1}(1-\rho)^{k-1-l} - (k-1-l)(\rho)^l(1-\rho)^{k-l-2}\right] \ \overline{I}_{k-1-l} \\ &= \sum_{l=1}^{k-1} C_{k-1}^l \ (\rho)^{l-1}(1-\rho)^{k-l-2} \left[l-(k-1)\rho\right] \ \overline{I}_{k-1-l} - (k-1)(1-\rho)^{k-2}\overline{I}_{k-1},\end{aligned}$$

so

$$\frac{\partial F_k}{\partial \rho}(0) = -(k-1)(\overline{I}_{k-1} - \overline{I}_{k-2}) \tag{12}$$

for $k \geq J+1$.

Denote now $\widehat{I}_k(\rho)$ for the expected payoff to an insider when there are k outsiders who choose to enter with probability ρ (the expectation being conditional upon at least one outsider entering). Then

$$\widehat{I}_k(\rho) = \sum_{l=1}^{k} C_k^l \ \frac{(\rho)^l(1-\rho)^{k-l}}{1-(1-\rho)^k} \ \overline{I}_{k-l}, \tag{13}$$

so straightforward manipulations yield:

$$\begin{aligned}(1-(1-\rho)^k)\ \widehat{I}_k(\rho) &= \sum_{l=1}^{k} C_k^l\ (\rho)^l(1-\rho)^{k-l}\ \overline{I}_{k-l} \\ &= \sum_{l=0}^{k} C_k^l\ (\rho)^l(1-\rho)^{k-l}\ \overline{I}_{k-l} - (1-\rho)^k\ \overline{I}_k \\ &= F_{k+1}(\rho) - (1-\rho)^k\ \overline{I}_k.\end{aligned}$$

If there existed a unique ρ_k satisfying $F_k(\rho_k) = c_p$, then we would have $\widehat{I}_k(\rho_k) = \overline{I}_k$, so using the previous equality for $\rho = \rho_k$ would yield

$$(1-(1-\rho_k)^k)\ \overline{I}_k = F_{k+1}(\rho_k) - (1-\rho_k)^k\ \overline{I}_k,$$

that is, an insider's expected payoff (net of Π_n) when k outsiders remain to enter would satisfy

$$\overline{I}_k = F_{k+1}(\rho_k) \tag{14}$$

if a unique ρ_k satisfying $F_k(\rho_k) = c_p$ existed.

Because we know that there exists a unique ρ_J satisfying $F_J(\rho_J) = c_p$, it simply remains to prove that $\frac{\partial F_k}{\partial \rho}(0) > 0$, that is, $\overline{I}_{k-1} < \overline{I}_{k-2}$ for $k \geq J+1$, which follows from working recursively on k as in Vettas (2000).[23] ■

Proof of Proposition 2. Let $c_i < \Pi_n$. If $c_p \geq \Pi_2$, then $J = n$ and

$$I_{n-1} - c_p = \mu_{n-1}\Pi_1 + (1-\mu_{n-1})\Pi_n - c_p.$$

Note that $I_{n-1} - c_p$ is decreasing in c_p for $c_p \geq \Pi_2$, whereas it increases in $c_i < \Pi_n$. In particular, $c_p \downarrow \Pi_2$ and $c_i \uparrow \Pi_n$ implies that $I_{n-1} - c_p$ converges to $\Pi_1 - \Pi_2$ from below (since $\mu_{n-1} \uparrow 1$). When $c_i \geq \Pi_n$, $c_p \geq \Pi_2$ implies that $J' = n$, so $I_{n-1} - c_p = \Pi_1 - c_p$, which implies that $I_{n-1} - c_p$ converges to

[23] Notice that expressions (4a), (5a) and (6)-(9) in Vettas (2000) are equivalent to expressions (1), (14), (8) for $\rho = 0$, (11), $F_k(1) = 0 < c_p$, and (12), respectively. Note that the expression that turns out to be equivalent in our setting to (10) in Vettas (2000) (namely, $F_{k+1}(0) > \overline{I}_k$) actually holds with equality, and hence it is redundant based on the expression in (8) evaluated at $\rho = 0$.

$\Pi_1 - \Pi_2$ as $c_p \downarrow \Pi_2$. ■

Proof of Lemma 5. Let $c_p \in [c^*_{J-2}, c^*_{J-1})$ for some integer J between 3 and n, and consider the subgames with $k \geq J$ outsiders.

(i) Working backwards, we first show that $c_p \uparrow c^*_{J-1}$ implies that one, and only one, of the $k \geq J$ outsiders enters, even though each of them chooses action p with negligible probability (i.e., $\rho_k \downarrow 0$ as $c_p \uparrow c^*_{J-1}$ for all $k \geq J$). Given $k = J$ outsiders, the probability that one outsider enters conditional upon at least one of them entering equals

$$\phi_J(1) = \frac{J\rho_J(1-\rho_J)^{J-1}}{1-(1-\rho_J)^J}.$$

Since $\rho_J \downarrow 0$ as $c_p \uparrow c^*_{J-1}$, it follows from L'Hôpital's rule that

$$\lim_{c_p \uparrow c^*_{J-1}} \phi_J(1) = 1 - \frac{(J-1)\rho_J}{1-\rho_J} = 1,$$

so one, and only one, outsider (out of the J existing ones) enters as $c_p \uparrow c^*_{J-1}$. This also implies that $\overline{I}_J = \overline{I}_{J-1}$, so it also follows that $\rho_{J+1} \downarrow 0$ as $c_p \uparrow c^*_{J-1}$, with $\lim_{c_p \uparrow c^*_{J-1}} \phi_{J+1}(1) = 1$ and $\overline{I}_{J+1} = \overline{I}_J = \overline{I}_{J-1}$. Iteration then yields for all $k \geq J$ that $\rho_k \downarrow 0$ as $c_p \uparrow c^*_{J-1}$, with $\lim_{c_p \uparrow c^*_{J-1}} \phi_k(1) = 1$ and $\overline{I}_k = \overline{I}_{J-1}$, so outsiders enter quasi-instantaneously by paying c_p in a sequential and coordinated manner, with each having the same probability of entry as any other outsider in any subgame in which $k \geq J$. Because these results hold for any arbitrary value of J and $\overline{I}_k = \overline{I}_{J-1} = c^*_{J-1}$ for all $k \geq J$, we have that $\lim_{c_p \uparrow c^*_{J-1}} (\overline{I}_{n-1} - c_p) = 0$ regardless of the value taken by J, so $I_{n-1} - c_p = \Pi_n$.

Having shown that the innovator's net profits $I_{n-1} - c_p$ converge to Π_n as $c_p \uparrow c^*_{J-1}$ for any integer value of J between 3 and n, we now prove that the convergence is from above by showing that the innovator's net profit has a negative derivative as $c_p \uparrow c^*_{J-1}$. In subgames in which $k \geq J$, we showed (see (13) noticing that $\widehat{I}_k(\rho_k) = \overline{I}_k$) that an insider's continuation payoff (net of

Π_n) satisfies the recursive equation

$$\overline{I}_k = \sum_{l=1}^{k} C_k^l \frac{(\rho_k)^l (1-\rho_k)^{k-l}}{1-(1-\rho_k)^k} \overline{I}_{k-l},$$

since at least one of them enters immediately. Noticing that ρ_k is an (implicit) function given by $F_k(\rho_k) = c_p$ and that $\rho_k \downarrow 0$ as $c_p \uparrow c^*_{J-1}$, we have that

$$\begin{aligned}\frac{\partial \overline{I}_k}{\partial c_p} &= \sum_{l=1}^{k} C_k^l \frac{(\rho_k)^l (1-\rho_k)^{k-l}}{1-(1-\rho_k)^k} \left(\frac{\partial \overline{I}_{k-l}}{\partial c_p}\right) + \\ &\quad \sum_{l=1}^{k} C_k^l \Big(\frac{l(\rho_k)^{l-1}(1-\rho_k)^{k-1-l}}{1-(1-\rho_k)^k} - \frac{k(\rho_k)^l(1-\rho_k)^{k-1-l}}{(1-(1-\rho_k)^k)^2}\Big) \overline{I}_{k-l} \left(\frac{\partial \rho_k}{\partial c_p}\right).\end{aligned}$$

Making (repeated) use of L'Hôpital's rule, it follows that

$$\begin{aligned}\left.\frac{\partial \overline{I}_k}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} &= C_k^1 \frac{1}{k} \left(\left.\frac{\partial \overline{I}_{k-1}}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}}\right) - \left(C_k^1 \frac{(k-1)}{2k} \overline{I}_{k-1} - C_k^2 \frac{1}{k} \overline{I}_{k-2}\right) \left(\left.\frac{\partial \rho_k}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}}\right) \\ &= \left(\left.\frac{\partial \overline{I}_{k-1}}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}}\right) - \frac{(k-1)(\overline{I}_{k-1} - \overline{I}_{k-2})}{2} \left(\left.\frac{\partial \rho_k}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}}\right),\end{aligned}$$

where

$$\left.\frac{\partial \rho_k}{\partial c_p}\right|_{c_p \uparrow c^*_{J-1}} = \left(\left.\frac{dF_k(\rho_k)}{d\rho_k}\right|_{\rho_k \downarrow 0}\right)^{-1} < 0,$$

since the derivative of $F_k(\rho)$ is negative for $\rho = \rho_k$. ■