Monetary Shocks Under Incomplete Markets

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Abstract

This paper provides a systematic quantification of the short-run effects of monetary policy shocks under incomplete markets. Our framework of analysis is the benchmark flexible-price neoclassical growth model with infinitely-lived and ex ante identical individuals, which we augment with i. uninsurable idiosyncratic labor income shocks; ii. a liquidity motive for holding real money balances (via a money-in-the-utility specification); and iii. aggregate shocks to the rate of money growth. We calibrate the model so as to match the historical inflation process as well as the broad features of the cross-sectional distributions of monetary and nonmonetary assets in the US economy. Our main finding is that, even though market incompleteness has a moderate impact on the response of aggregates to the shocks (relative to the complete-market case), this results from composition effects that mask a great deal of cross-household redistribution and heterogeneities in individual portfolio adjustments.

JEL codes : E21; E32; E41

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1 Introduction

In this paper, we undertake a quantitative investigation of the short-run effects of monetary shocks for model economies with incomplete markets. Since the seminal work of Bewley (1980), Huggett (1993) and Aiyagari (1994), the existence of uninsurable individual risks has been largely viewed as crucial for understanding the patterns of consumption, savings and portfolio allocation of individuals. More recently, the incomplete markets literature has investigated the role of those idiosyncractic risks for the aggregate economy by considering their interplay with macroeconomic shocks. In particular, earlier work has quantified the importance of incomplete markets for the real business cycles (Krusell and Smith, 1998), unemployment dynamics (Krusell et al., 2010), portfolio

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choices (Krusell and Smith, 1997), fiscal policies (Heathcothe, 2005; Challe and Ragot, 2011) and the welfare effect of real fluctuations (Storesletten et al., 2001; Krebs, 2003; Krusell et al., 2009).

Thus far, the study of incomplete-market models has largely bypassed the analysis of the real effects of monetary policy shocks. This is all the more surprising that, first, market incompleteness is known to significantly alter the effect of monetary growth in the long-run steady state, due to heterogeneities in asset and money holdings across households (Erosa and Ventura, 2002; Algan and Ragot, 2010); and second, the presence of unsinsurable individual risks and borrowing constraints gives room to potentially important redistributive effects of monetary shocks in the short run, which are well identified both empirically and theoretically. For example, Doepke and Schneider (2006) show that there is a huge amount of heterogeneity in nominal asset positions among US households, implying that a moderate inflation episode would have large redistributive effects. From a theoretical point view, heterogeneities in asset holdings are naturally rationalized by the inability of agents to perfectly insure against idiosyncratic shocks (e.g., to labour incomes, trading opportunities, production possibilities, preferences, etc.). In this context, monetary shocks that affect either the cross-sectional distribution of nominal wealth or the price level or both redistribute real wealth across households and have nonneutral effects on allocations (see Scheinkman and Weiss, 1986; Berentsen et al., 2005; Algan et al., 2011.) While the emphasis on wealth redistribution as a key source of short-run monetary nonneutrality can be traced back at least to Friedman (1968), it has not yet made its way into quantitative business cycle analysis with heterogenous agents.

This paper fills this gap by providing a systematic quantification of the short-run effects and channels of monetary policy shocks under incomplete markets. The starting point of our analysis is the benchmark neoclassical growth model with infinitely-lived and ex ante identical individuals, which we augment with i. uninsurable labor income risk a la Bewley-Hugget-Aiyagari; and ii. a liquidity motive for holding real money balances, introduced via a money-in-the-utility function specification (MIU). These features imply that households hold nonnegative quantities of both real and nominal assets in equilibrium, which they use to partly self-insure against idiosyncratic income changes in the face of incomplete consumption insurance. Finally, aggregate shocks to the growth rate of the money supply affect the courses of nominal prices and the inflation rate, both of which influence the cross-sectional distribution of real money demands and the aggregate supply of real balances. Since our focus is on how market incompleteness and wealth redistribution affect the model's response to monetary shocks relative to the full-insurance case, we entirely abstract from other frictions, notably nominal rigidities.

Because our investigation is primarily quantitative, a prerequisite for a meaningful analysis of the impact of monetary shocks is that the model reproduce a realistic cross-sectional distribution of money holdings among the households in the first place (in addition to reproducing a realistic cross-sectional distribution of capital holdings). We show that this requirement necessitates departing from the functional form commonly used to parameterize money demand in monetary models with representative agent (e.g., Chari et al., 1996, 2000). As discussed extensively in Ragot (2011), the empirical cross-sectional distribution of money holdings in the US is close to that of financial assets and very different from that of individual consumption levels. This property cannot follow from the usual assumption that the elasticity of substitution between real money balances and consumption is constant, because constant elasticity implies that money holdings are proportional to consumption and hence inequalities in money holdings mirror inequalities in consumption. We thus introduce a more general utility function that nests the constant-elasticity specification as a special case, but also accommodates a non constant elasticity and hence allows changes in individual consumption to be associated with more than proportional changes in real money holdings. This function allows us to reproduce the broad features of the joint distribution of money holdings and consumption in the US, whilst at the same time being consistent with the observation that, at the individual level, higher wealth is associated with greater absolute money holdings but lower money holdings relative to total wealth. Given the key role of our assumed utility function in this study, we are careful to gauge its importance in the transmission of monetary shocks via systematic comparison with the constant-elasticity case.

Our analysis allows us to disentangle three potentially important channels of monetary non-neutrality under incomplete markets. First, a monetary shock is generically associated with an immediate redistribution of wealth across individuals who have heterogenous marginal propensities to consume out of wealth; in particular, if inflation hurts cash-rich households to the benefits of cash-poor ones and the latter have higher marginal propensity to consume, then this instantaneous redistributive effect tends to raise current consumption.[1]

[1] This channel was originally identified by Scheinkman and Weiss (1986). Algan et al. (2011) construct a tractable Bewley model that incorporates this channel, which they label the "intratemporal inflation tax" (as opposed to the intertemporal tax working through inflation expectations.) Note that the impact of money growth shocks crucially

The second effect is an intertemporal redistributive effect that is operative as soon as money growth shocks are persistent. If, for example, a money growth shock is associated with a redistribution of wealth from the rich to the poor, then a persistent shock is associated with the anticipation that this redistribution will prevail in the future, thereby deterring cash-rich households from holding real balances and urging them to buy both consumption goods and real assets (i.e., claims to the capital stock) instead.

The third effect is a portfolio composition effect that comes from the presence of borrowing-constrained households and their reaction to future inflation, relative to that of unconstrained households. Unconstrained households hold both claims to the capital stock and real money, and always rebalance their portfolio towards the former and away from the latter when expected inflation rises –and hence return from holding real money balances falls.[2] In contrast, households facing a binding borrowing constraint hold at least some money units (due to the complementarity between money and consumption) but no claim to the capital stock, and so do not enjoy the same portfolio rebalancing option. It follows that the overall impact of a persistent money growth shock is scaled by the proportion of households facing a binding borrowing constraint in the economy.[3]

We measure the relative contributions of those three channels by comparing our benchmark economy's response to aggregate shocks to those produced by alternative economies wherein some of these channels are made inoperative by construction. For example, imposing i.i.d. money growth shocks (rather than suitably parameterized persistent shocks) allows us to hold expected inflation constant, thereby making the second and third channels inoperative and thus isolating the intratemporal redistributive effect of the shock (the first channel.) Similarly, considering a representative-agent economy allows us to isolate the effect of expected inflation on the demand for real balances by unconstrained households; by difference, this provides a measure of the role of constrained households in modifying the response to the shock (the third channel.) As discussed above, we also systematically compare the responses of our economies with and without the assumption

depends on which agents receive the newly-issued money. Under lump-sum transfers, money growth shocks typically redistribute real wealth from cash-rich rich, low-marginal propensity to consume households towards cash-poor, high marginal propensity households, which goes towards raising aggregate consumption.

[2] See Cooley and Hansen (1989) for a detailed analysis of this intertemporal inflation tax within a representative-agent economy.

[3] The only tradeoff faced by constrained households is between holding cash and consuming. A rise in expected inflation may even lead them to demand more real balances if the intertemporal income effect dominates the intertemporal substitution effect. See Algan and Ragot (2010) for a full discussion of this point.

that the elasticity of substitution between consumption and real money holdings is constant.

Our general finding is that incorporating uninsurable individual risks into a monetary business cycle model leads to quantitatively important departure from the complete-market set-up. First, a positive, persistent money growth shock leads to a fall in consumption and an increase in saving and investment in the complete-market economy. Moving from complete to incomplete markets with a standard utility function (with constant elasticity of substitution between consumption and real balances) divides roughly by two the real short-run effect of the shock on aggregates, and also reverts the long-run effect. Second, moving from this simple utility function to one with non-constant elasticity of substitution (so as to match the empirical distribution of money holdings) increases by roughly 45% the short- and long-run impacts of persistent monetary shocks on aggregates, both under complete and incomplete markets. As a result, moving from the complete-market economy with simple utility function to our benchmark economy with incomplete markets and non-constant elasticity of substitution lowers the short run impact of a positive, persistent money growth shocks by 70%, and reverts the long-run impact of the shock.

Related literature. Our paper relates to a vast literature that evaluates the impact of monetary growth on aggregates under heterogenous cash holdings. From a theoretical point of view, our analysis follows Bewley (1980), Scheinkman and Weiss (1986), Kehoe et al. (1992), Imrohoroglu (1992), Akyol (2004) and Algan and Ragot (2011). Those papers share with the present contribution the emphasis on uninsurable labor market risk as potentially relevant for the transmission of monetary policy. However, all these contributions except for Scheinkman and Weiss (1986) focus on the impact of long-run inflation on aggregates, while we are chiefly interested in the economy's response to money growth shocks. Our framework also differs substantially from that in Scheinkman and Weiss (1986), who ignore capital accumulation or the role of persistent aggregate shocks. The non-neutrality of inflation working through wealth redistribution has also been explored within search-theoretic models, in which households face idiosyncratic trading opportunities and need cash to facilitate future trades (Green and Zhou, 1999; Camera and Corbae, 1999; and Molico, 2006.) Erosa and Ventura (2002) as well as Albanesi (2006) also focus on the effect of long-run inflation under heterogenous cash holdings.

Doepke and Schneider (2006b) quantify the redistributive effect of monetary shocks on macroeconomic aggregates by using an overlapping-generations model with exogenous heterogeneity across ages and productivity (see also Heer and Maussner, 2011.) An important difference with our approach is that they exclude labor market risks, the key source of heterogeneity in asset holdings in our economy. The redistributive channels through which monetary shocks affect the economy in Doepke and Schneider are linked to life-cycle effects in saving behavior, while they are due to precautionary saving in our setup.

The paper is organized as follows. Section 2 introduces the model and constructs its recursive equilibrium. Section 3 calibrates the model. Section 4 presents our results, both at the steady state and with aggregate uncertainty. Section 5 concludes the paper.

2 The Model

The model is a version with aggregate shocks of the framework originally developed by Algan and Ragot (2010). While their work is concerned with the effect of long-run mean inflation on capital accumulation and macroeconomic aggregates, the present paper focuses on how the redistributive effects of inflation shocks affect those variables in the short run. The key difference with the framework of Aiyagari (1994) and Krusell and Smith (1998) is that real money balances enter households' utility function, so that households hold both money and claims to the capital stock in equilibrium.

2.1 Preferences

Households are infinitely-lived and in constant mass equal to 1. They share identical and additively time-separable preferences over sequences of consumption, $c \equiv \{c_t\}_{t=0}^{\infty}$, and real money holdings, $m \equiv \{m_t\}_{t=0}^{\infty}$. Thus, they maximize

$$U(c, m) = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, m_t), \tag{1}$$

where $\beta \in (0, 1)$ is the subjective discount factor, E_t denotes expectations conditional on the information set at date t, and u is the instant utility function. As discussed above, matching a realistic joint cross-sectional distribution of consumption, real balances and claims to the capital stocks requires considering a parametric instant utility function that is more general than that commonly used in the MIU literature. More specifically, we assume that

$$u\left(c_t, m_t\right) = \frac{1}{1-\lambda}\left(\omega c_t^{1-\rho} + (1-\omega)\, m_t^{\theta(1-\rho)}\right)^{\frac{1-\lambda}{1-\rho}} .\rho, \theta, \lambda > 0. \tag{2}$$

When $\theta = 1$, (2) becomes a standard homothetic utility function such as that used by Chari et al. (1996, 2000) and Algan and Ragot (2010), among others. In this case, the interest-rate elasticity of real money demand is $1/\rho$, while the intertemporal elasticity of substitution between consumption and real balances is $(1-\rho)/(\lambda-\rho)$. However, a important limitation of the homothethic specification is that it implies a strict proportionality between individual real money holdings and individual consumption levels (for any given values of the nominal interest rate and the inflation rate), making it impossible to reproduce the highly unequal distribution of money holdings that is observed in US data. Our benchmark economy will thus have $\theta \neq 1$.

2.2 Idiosyncratic Uncertainty

In every period, households are subject to idiosyncratic labor income shocks. Labor productivity can take three different values $e_t \in E$, $E = \{e^l, e^m, e^h\}$ with $e^l < e^m < e^h$ and where e^h stands for 'high productivity', e^m for 'medium productivity', and e^l for 'low productivity'. Each household's productivity evolves according to a first-order Markov chain with the 3×3 transition matrix Π. We denote by p^* the vector of stationary ergodic probabilities and normalize productivity levels so that the mean of the invariant distribution is one, i.e., $\sum p_i^* e_i = 1$. Given a population of measure one, we can interpret p^* as describing the distribution of the population across productivity states. It follows that the effective aggregate labor supply is equal to $\sum p_i^* e_i \bar{n} = \bar{n}$, where $\bar{n}$ stands for labour hours per period.

2.3 Production Technology

Markets are competitive. In every period t, the representative firm uses aggregate capital $K_t \in \mathbb{R}_+$ and households' labor to produce $Y_t \in \mathbb{R}_+$ units of a single good with the aggregate technology

$$Y_t = f(K_t, \bar{n}) = K_t^{\alpha} \bar{n}^{1-\alpha}.$$

Capital depreciates at the constant rate $\delta \in (0, 1)$ and accumulates according to the law of motion

$$K_{t+1} = I_t + (1-\delta) K_t. \tag{3}$$

where I_t denotes aggregate investment. Perfect competition in the markets for the representative firm's inputs implies that the real interest rate, r_t, and the real wage, w_t, are given by:

$$r_t = \alpha K_t^{\alpha-1} \bar{n}^{1-\alpha} - \delta, \; w_t = (1-\alpha) K_t^{\alpha} \bar{n}^{-\alpha}.$$

2.4 Monetary policy

At each date t, the government chooses the growth rate of the money stock, γ_t. Let us denote by Δ_t the quantity of newly issued money at date t (relative to the stock of nominal money at the end of date $t-1$), by Ω_t the aggregate supply of real balances at date t, and by $\Pi_t = P_t/P_{t-1} = 1 + \pi_t$ the gross inflation rate between date $t-1$ and date t. In real term, the quantity of newly issued money can be written as:

$$\Delta_t/P_t = \gamma_t P_{t-1} \Omega_{t-1}/P_t = \gamma_t \Omega_{t-1}/\Pi_t. \tag{4}$$

On the other hand, the dynamics of real money balances is given by:

$$\Omega_t = (1+\gamma_t) \frac{\Omega_{t-1}}{\Pi_t}. \tag{5}$$

There are several possible ways to model money creation in heterogeneous-agent economies. It could for example result from open market operations, which in their simplest form would amount to giving the newly-issued money to the government. Alternatively, the newly-issued pieces of currency could be targeted towards specific households, which would by construction generate sizeable redistributive effects. In what follows, it is assumed that the new money is distributed equally to every household in a lump-sum manner. This choice is the most natural from a theoretical point of view, as any deviation from that benchmark can be thought as a fiscal transfer across households. Moreover, there is no clear empirical evidence that money creation is targeted towards specific households.

2.5 The households' problem

We assume that markets are incomplete, so that households cannot write insurance contracts contingent on their labor income. Moreover, they face borrowing constraints and are thus prevented from using private loans to fully smooth out individual income fluctuations. Each household i

maximizes its expected lifetime utility (1) subject to the following constraints:

$$c_t^i + k_{t+1}^i + m_t^i = a_t^i + w_t e_t^i \bar{n} + \gamma_t \frac{\Omega_{t-1}}{\Pi_t}, \tag{6}$$

$$k_{t+1}^i \geq 0, \;\; m_t^i \geq 0, \;\; \text{and } c_t^i \geq 0, \tag{7}$$

where k_{t+1}^i and m_t^i denote the claims to the capital stock and the real money balances held by household i at the end of date t, and where

$$a_t^i = (1 + r_t) \, k_t^i + \frac{m_{t-1}^i}{1 + \pi_t}$$

is the household's financial wealth at the beginning of date t. In (7), the presence of the borrowing constraint is reflected in the fact that capital and money holdings must be nonnegative at all times, while no other assets (i.e., private bonds) can be issued by the households.

From the households' objective and constraints, we find that their optimal asset demands, m_t^i and k_{t+1}^i, must satisfy the following first order conditions:

- Money:

$$u_c\left(c_t^i, m_t^i\right) - u_m\left(c_t^i, m_t^i\right) = \beta E_t \left[\frac{u_c\left(c_{t+1}^i, m_{t+1}^i\right)}{1 + \pi_{t+1}} \right]. \tag{8}$$

- Capital:

$$\begin{aligned} \text{Either } u_c\left(c_t^i, m_t^i\right) &= \beta E_t \left[(1 + r_{t+1}) \, u_c\left(c_{t+1}^i, m_{t+1}^i\right) \right] \text{ and } k_{t+1}^i > 0, \\ \text{or } \; u_c\left(c_t^i, m_t^i\right) &> \beta E_t \left[(1 + r_{t+1}) \, u_c\left(c_{t+1}^i, m_{t+1}^i\right) \right] \text{ and } k_{t+1}^i = 0. \end{aligned} \tag{9}$$

The instant utility function (2) implies that $u_m\left(c_t^i, 0\right) = \infty$, so the demand for real balances is always interior. In contrast, the demand for capital may be corner (i.e., $k_{t+1}^i = 0$), in which case the household would like to raise current consumption by borrowing against future income, but is prevented from doing so by a (binding) borrowing constraint.The solution to the households' problem provides sequences of functions $m_t(a, e)$, $k_t(a, e)$ and $c_t(a, e)$, $(a, e) \in \mathbb{R}_+ \times \{e_1, e_2, e_3\}$, where a and e denote individual beginning-of-period asset wealth and productivity, respectively. To better understand the implications of our assumed period utility function with non-constant elasticity of substitution (i.e., (2)), consider the optimal trade-off between consumption and real money holdings by an unconstrained household (so that $k_{t+1}^i > 0$ in (9)) and abstract from aggregate shocks momentarily. From (8)–(9), we find the relation between money holdings and consumption to be:

$$m_t^i = A\left(r_{t+1}, \pi_{t+1}\right)\left(c_t^i\right)^{\frac{\rho}{1-\theta(1-\rho)}}, \tag{10}$$

where $A\left(r_{t+1}, \pi_{t+1}\right)$ is a coefficient whose value depends on the returns on the two assets and the deep parameters of the utility function. In the constant-elasticity case (i.e., $\theta = 1$), we have $m_t^i = A\left(r_{t+1}, \pi_{t+1}\right) c_t^i$, that is, real money demand is strictly proportional to consumption, so that cross-sectional inequalities in these two variables mirror each other. For individual money holdings to increase more than proportionally following an increase in individual consumption, so that money be more unequally distributed than consumption (as is observed in the data), one needs $\rho / \left(1 - \theta\left(1-\rho\right)\right) > 1$ (whether θ must lie above or below 1 for this inequality to hold depends on the value of ρ.)

2.6 Market Equilibria

Define $\mu_t : \mathbb{R}_+ \times \left\{e^h, e^m, e^l\right\} \rightarrow \mathbb{R}_+$ as the joint cross-sectional distribution of wealth and individual productivity at the beginning of period t. The market-clearing conditions in the money and capital markets are given by:

$$\int\int m_t^i(a_t, e_t) d\mu\left(a_t, e_t\right) = \Omega_t, \tag{11}$$

$$\int\int k_t(a_t, e_t) d\mu\left(a_t, e_t\right) = K_{t+1}. \tag{12}$$

By Walras law, the goods market clear when both the money and the capital markets clear.

It is worth noting at this stage that introducing money and a market-clearing condition of the form of (11) raises specific computational difficulties, relative to conventional market-clearing conditions in heterogenous-agent models. More specifically, households must base their optimal consumption plans on their beginning-of-period asset wealth. In models with capital and/or public debt only, the beginning-of-period cross-sectional distribution of wealth is entirely determined by portfolio decisions made in the previous period. In monetary models like ours, a key component of beginning-of-period asset wealth is real money, whose value is affected by nominal prices that are determined in the current period (see (5), where current inflation, and hence current nominal prices, determine the supply of real balances in the current period.) Hence, the value of the price level that clears the money market at any given date must be solved jointly with the households' optimal portfolio decisions about capital and real money balances.

2.7 Definition of the recursive equilibrium

Since we are considering a recursive equilibrium in which the aggregate state changes over time, we must include in the individual value functions both the aggregate stock of capital and the aggregate stock of real money. Hence, given prices, the recursive problem of an individual household can be written as:

$$v\left(a_t, e_t; \gamma_t, M_{t-1}, K_t\right) = \max_{m_t, c_t, k_{t+1}} u\left(c_t, m_t\right) + \beta E_t\left[v\left(a_{t+1}, e_{t+1}; \gamma_{t+1}, M_t, K_{t+1}\right) | e_t, \gamma_t\right], \tag{13}$$

subject to (6)–(7). Following Krusell and Smith (1998) and much of the subsequent literature, we posit that households are able to successfully forecast the dynamics of the aggregate state by means of (log-) linear laws of motion involving only the first moments of the distributions of the relevant endogenous state variables. That is, these laws of motion approximately follow

$$\ln\left(M_t\right) = a_1(\gamma) + a_2(\gamma)\ln\left(K_t\right) + a_3(\gamma)\ln\left(M_{t-1}\right), \tag{14}$$

$$\ln\left(K_{t+1}\right) = b_1(\gamma) + b_2(\gamma)\ln\left(K_t\right) + b_3(\gamma)\ln\left(M_{t-1}\right). \tag{15}$$

The solution to (13) produces individual decision rules for consumption as well as holdings of real balances and claims to the capital stock, which we denote by $g_c(a_t, e_t; \gamma_t, M_{t-1}, K_t)$, $g_m(a_t, e_t; \gamma_t, M_{t-1}, K_t)$ and $g_k(a_t, e_t; \gamma_t, M_{t-1}, K_t)$, respectively. The law of motion of the distribution of beginning-of period real wealth is denoted by H. For a given set of individual policy rules, this law of motion can be written as

$$\mu_{t+1} = H(\mu_t, \gamma_t, \gamma_{t+1}).$$

As usual, the cross-sectional distribution of wealth at date $t+1$ depends on the same distribution at date t and on the date t aggregate shock, γ_t. As explained above, it also depends on the price level at date $t+1$ and thus on the realized value of the money growth shock, γ_{t+1}.

Definition of the recursive equilibrium. *The recursive equilibrium of this economy is defined by a law of motion H of the joint distribution μ, a set of optimal individual policies and value functions $\{g_c, g_m, g_k, v\}$, a set of pricing functions $\{\pi, r, w\}$, and a set of law of motions for K and M such that:*

1. *Given $\{\pi, r, w\}$, the exogenous transition matrices for the exogenous shocks e and γ, the law of motions for K and M, $\{g_c, g_m, g_k\}$ solve the household's problem;*

2. *The money and capital markets clear;*

3. *The law of motion $H(\mu_t, \gamma_t, \gamma_{t+1})$ is generated by the optimal decisions $\{g_c, g_m, g_k\}$, the law of motions for K* and *M and the transition matrices for the shocks.*

2.8 Numerical solution

As mentionned above, we use the same approach as in Krusell and Smith (1997, 1998), who summarize the cross-sectional distribution of wealth with a finite set of moments and approximate the transition for the aggregate laws of motion using a simulation procedure. However, rather than using Monte Carlo simulations to generate an updated cross-sectional distribution, we use the grid-based simulation procedure proposed by Young (2010), which keeps track of the mass of households at a fine grid of wealth levels. This allows us to get rid of the cross-sectional sampling variations in the Monte Carlo simulation procedure. However, the grid-based procedure slightly complicates the numerical solution to our model. Indeed, the updating of the wealth distribution should now take account of the fact that the wealth distribution at the beginning of the current period not only depends on past decisions but also on the current inflation rate. So, for a given arbitrary inflation rate Π, we determine the individual policy functions $g_m(., \Pi)$ and $g_k(., \Pi)$ as in Krusell and Smith (1997) *and* the wealth distribution $\mu(a, e; \Pi)$, given last period's wealth distribution $\mu\left(a, e; \Pi^*_{t-1}\right)$ and policy functions $g_m(., \Pi^*_{t-1})$ and $g_k(., \Pi^*_{t-1})$, where Π^*_{t-1} stands for last period's equilibrium inflation rate. It is then possible to find the value of Π that clears the money market. A more detailed description of the algorithm is provided in the Appendix.

Before presenting the results, it is important to assess the accuracy of the aggregate laws of motion. We did so by calculating the maximum absolute forecast error ten period ahead, that is, the difference between the predicted values of K and M using the aggregate laws of motion and the supposed 'true' values that come out of the simulation using individual policy functions. Every ten periods, we update the values of K and M with the corresponding supposed true values, since we are mostly interested in the short-run effects of aggregate shocks. Using a 10,000-period simulation, we found for the benchmark model that the maximum cumulative absolute forecasting errors are 0.28% for M and 0.82% for K.[4] The accuracy is thus satisfactory.

[4] The average errors are 0.018% and 0.052% for M and K, respectively.

3 Parametrization

3.1 Description of the economies

Our benchmark economy is one with i. persistent money growth shocks, as summarized by a (2-state) Markov chain parameterized to match the historical evidence on the persistence of inflation; and ii. a instant utility function featuring non-constant elasticity of substitution between real balances and consumption, which will help us match the empirical cross-sectional distribution of money holdings. In order to precisely identify and disentangle the various channels of monetary nonneutrality at work in our model and the way they interact, we compare our benchmark economy with suitably chosen alternatives.

First, we systematically compare our model with non-constant elasticity of substitution between money and consumption to the more common constant-elasticity specification. For ease of exposition, in the remainder of the paper we shall simply refer to the former and the latter as the "elaborate" and "simple" utility functions, respectively. This comparison will allow us to assess how the matching of a realistic distribution of money holdings matters for the predicted impact of aggregate money growth shocks. Second, we shall compare our benchmark model with the complete-market model, where the self-insurance motive for holding assets is shut down. This will allow us to precisely measure the specific contribution of the redistribution of wealth to the overall impact of a monetary shock, as opposed to the direct portfolio effect based on changes in the expected return on holding cash. Third, we work out the aggregate implications of our model when money growth shocks are i.i.d. This economy will produce i.i.d. inflation rates and hence constant *expected* inflation. Consequently, the nonneutrality of money coming from changes in expected inflation is shut down, so that the effects of the shock have to come from the contemporaneous redistribution of wealth.

3.2 Parameter values

Deep parameters common to all model specifications. Table 1 presents the parameters that are common to all economies under investigation. The time period is a quarter. Following Chari et al. (2000), our benchmark value for the utility parameter λ is 1. The capital share is set to $\alpha = 0.36$, and the depreciation rate to 0.025. Finally, labor supply is constant for all

Table 1: Parameter values and simulation targets common across economies

Parameter set outside the model					
Preferences	λ	1.00	*Production*	α	0.360
				δ	0.025
				$\bar{n}$	0.300
Productivity	e^l	3.940			
	e^m	0.849			
	e^h	0.213			
	$\pi_{l,l}$	0.975			
	$\pi_{m,m}$	0.992			

households and set to 0.3.[5] The individual productivity states and the transition probabilities across states are calibrated as follows. Following Domjei and Heathcote (2004), we use a Markov chain with three states, zero probabilities to transit between extreme states (i.e., $\pi_{h,l} = \pi_{l,h} = 0$), and an equal probability to reach any of the extreme states when in the intermediate state (i.e., $\pi_{m,h} = \pi_{m,l}$). The transition matrix is then fully identified once $\pi_{l,l}$, $\pi_{m,m}$ and $\pi_{h,h}$ are set, and we set $\pi_{l,l} = \pi_{h,h} = 0.9750$, $\pi_{m,m} = 0.9925$. Finally, the ratios of productivity are set to $e^h/e^m = 4.64$ and $e^m/e^l = 3.99$. This process yields an autocorrelation of the real wage equal to 0.91 and a standard deviation of the innovation term equal to 0.22 at annual frequency, in line with the data.

Deep parameters that vary across model specifications. Table 2 gathers the parameters that will vary across model specifications so as to always match the same steady state targets as in the baseline case. Our key targets are i. the Gini of the money distribution, ii. the money to GDP ratio, iii. the interest rate elasticity of money, and iv. the capital-output ratio. The monetary aggregate that we consider is M2, which best corresponds to the notion of "liquid assets" in the Survey of Consumer Finances. Note that the Gini coefficient of this distribution is as high as 0.85. The quarterly value of M2 over GDP is 0.52 for the period 1982-2005. Estimates of the interest elasticity of money demand applied to the M2 aggregate vary from 0.11 to 1, depending on the estimation method being used and the period of estimation (see, e.g., Ireland, 2001, and Holman, 1998). We target the relatively low value of 0.2, which is close to its post-Volcker empirical counterpart. Finally, we target a capital-output ratio of 12 at quarterly frequency (see, e.g., Cooley, 1995.)

Table 2 provides the preference parameters that best match those targets for the four economies

[5] It is straightforward to introduce elastic labor supply using GHH preferences, as in Heathcote (2005).

Table 2: Parameter values varying across economies

	Economy			
	Simple utility		Elaborate utility	
	Complete-market economy	Incomplete-market economy	Complete-market economy	Incomplete-market economy
Preferences				
β	0.9943	0.99	0.9943	0.99
ρ	5.00	5.00	15.00	15.00
ω	0.24	0.24	0.017	0.48
θ	1	1	0.2857	0.2857

under consideration. Since the simple utility function cannot match the distribution of money holdings, the preference parameters in this case are set to match the other targets only. In the case of the elaborate utility function, we also aim at being as close as possible to the empirical Gini of the money distribution.

Monetary policy parameters. We model the dynamics of monetary conditions between 1982Q1 and 2005Q4 as a two-state, first-order Markov chain. More specifically, we estimate this chain using CPI-inflation and extract the inflation levels that prevail in the 'high-' versus 'low-inflation' regimes, as well as the probabilities to transit between those regimes. We then use the two inflation rates to parameterize money growth –our exogenous aggregate state– in each regime. There are at least two alternative ways of estimating the dynamics of monetary conditions. One would be to directly estimate a Markov chain for the money supply. However, doing so would have led us to miss the average inflation rate that prevailed over the period due to substantial low-frequency movements in the velocity of money. Since the inflation rate is an important determinant of money demand and the transmission of monetary shocks in our model, it is crucial that the latter produces an average inflation rate that is consistent with the data. Another way to proceed would have been to treat the estimated inflation rates in both regimes as exogenous forcing variables in our model and to let the money supply adjust to exactly produce such rates in equilibrium; however, doing so would make inflation exogenous, whereas we are also interested in the endogenous response of inflation to monetary shocks under incomplete markets. Our approach, which consists of imposing the money growth rates that correspond exactly to the inflation rates of each regime, can be seen as striking a balance between these two alternatives.

The parameters of the stochastic process for the money growth rate are shown in Table 3, with the estimated transition probabilities corresponding to the autocorrelated case. As explained above, we also experiment the impact of i.i.d. money growth shocks in our model economy; even though not directly relevant empirically, the i.i.d. experiment is instructive as it maintains the contemporaneous wealth redistribution induced by money growth shocks whilst eliminating changes in expected money growth –the only source of nonneutrality in flexible-price, complete-markets economies (Walsh, 2010, chapter 2.)

Table 3: Parameters of the stochastic process of the money supply for various economies

Money growth rate		
$\gamma_1 = 0.64\%$	$\gamma_2 = 1.17\%$	
Transition probabilities		
	IID	Autocorrelated
	$\Pi_{1,1} = 0.5$	$\Pi_{1,1} = 0.944$
	$\Pi_{1,1} = 0.5$	$\Pi_{2,2} = 0.889$

4 Results

4.1 Equilibrium distribution and laws of motion

Before studying the impact of money growth shocks in our economy, we check that the latter reproduces the broad features of the US wealth distribution. Unlike earlier studies, we seek to match the distributions of two components of total wealth: money wealth and nonmonetary wealth. Given our focus on portfolio allocations by the households, our empirical counterparts from the Survey of Consumer Finances (SCF 2004) are the following. First, we use the "liquid assets" component of financial wealth as a measure of households' money wealth (as argued above, this roughly corresponds to the assets belonging to the M2 monetary aggregate). Liquid assets in the SCF are essentially made of money market accounts, checking accounts, saving accounts and call accounts. Second, we compute the distribution of nonmonetary wealth by removing liquid assets from the financial assets held by the households in the SCF. From the SCF, nonmonetary wealth refers to bonds, stocks, life insurance, retirement plans and other managed financial assets. Table

4 compares the properties of those two distributions with those generated by the model, under the parameter configuration specified in Tables 1-3.

Given our parametric utility function, all households hold some money in our model, even though the amount being held may be very small. However, many households are not wealthy enough to hold both money and nonmonetary assets: they are “constrained”, in the sense of endogenously choosing not to hold capital –not in the sense of holding zero wealth.

The benchmark model predicts a fairly high Gini index for the distribution of nonmonetary assets (0.76), only slightly underestimating its empirical counterpart (0.82.) Moreover, the model does a fairly good job at matching the lower tail of the distribution of nonmonetary assets. Perhaps unsurprisingly, the model underestimates the nonmonetary wealth share of the top 1%, which is predicted to be 8.99% while it is 34.30% in the data. This flaw is common to many models that only use idiosyncratic income risk to generate wealth dispersion and ignore, for example, entrepreneurship (see, e.g., Quadrini, 2000.)

The empirical measure of the share of households facing a binding borrowing constraint heavily depends on which indicator is chosen. Using information on the number of borrowing requests which were rejected in the SCF, Jappelli argued that up to 19% of families are liquidity-constrained. However, using updated SCF data, Budria Rodriguez et al. (2002) reported that only 2.5% of the households have zero wealth, which might better correspond to our theoretical borrowing limit. Obviously, this figure does not mean that only these households are liquidity-constrained. In particular, Budria Rodriguez et al. (2002) also report that 6% of households have delayed their debt repayments for two months or more, which could be used as another proxy for liquidity constraints. In this respect, that our elaborate utility implies that 8% of the households face a binding borrowing constraint can be considered as reasonable value, and notably one that prevents us from over-estimating the effect of the constraint on the non-neutrality of inflation.

Appendix B shows the aggregate laws of motion for money and capital. As in Krusell and and Smith (1998), we find the first-order moments of the distributions to yield an almost perfect prediction of prices (based on the R-square statistics.)

Table 4: Wealth distribution

	Data	Economy* Simple utility	Elaborate utility
Distribution of nonmonetary assets			
Gini	0.82	0.77	0.76
Share of constrained households	[2%, 20%]	0.42	0.08
Fraction of total asset held by			
Bottom 20%	0.00	0.00	0.01
Bottom 40%	0.20	0.00	0.05
Top 20%	84.70	80.60	79.27
Top 10%	71.20	55.02	53.61
Top 1%	34.30	9.32	8.99
Distribution of money holdings			
Gini	0.85	0.31	0.72
Fraction of total money held by			
Bottom 20%	0.00	15.61	0.72
Bottom 40%	0.00	26.92	2.81
Top 20%	88.20	41.52	78.19
Top 10%	76.46	24.72	56.93
Top 1%	39.49	3.37	11.44
Capital/GDP	12.00	11.71	11.86
Money/GDP	0.52	0.52	0.52

* The model properties are averages over a 10,000 period simulation.

4.2 I.I.D. money growth shocks

We first investigate the effect of monetary shocks under incomplete markets under the assumption that money growth shocks are i.i.d. (and denote the corresponding transition matrix across aggregate states as T^{IID}.) This is the most natural process to start with since, as argued above, monetary shocks have no effect on aggregates when shocks are i.i.d. and markets are complete. Hence, while not realistic, i.i.d. shocks are interesting theoretically as they allow us to isolate a transmission channel that is purely related to market incompleteness and borrowing constraints.

4.2.1 Individual Policy Rules

Figure 1 displays the individual policy rules when the money growth rate is γ_1 (the policy rules when $\gamma = \gamma_2$ are very similar) and for the average levels of capital K and money M. The policy rules for total wealth, nonmonetary assets, money and consumption are decomposed for each level of productivity e^h, e^m and e^l. For example, the third panel of Figure 1 reports the individual policy rules for nonmonetary assets. The policy rule lies above the 45-degree line for the most productive household (with productivity e^h) and below the 45-degree line for the other two types (with productivity e^m or e^l). This implies that the former accumulate nonmonetary assets for self-insurance purposes whereas the latter dis-save to smooth individual consumption. For medium- and low-productivity households, the policy rule displays a kink at low levels of wealth; this is because at such wealth levels these households choose to hold money but no nonmonetary assets. The second and fourth panels show the policy rules for money holdings and consumption, which roughly display the same pattern as the policy rule for nonmonetary assets. The more productive the household (holding wealth constant), or the wealthier the household (holding productivity constant), the higher are individual consumption and money holdings. The close connection between the policy rules for consumption and money holdings stems from the complementarity between the two, a direct implication of our assumed instant utility function.

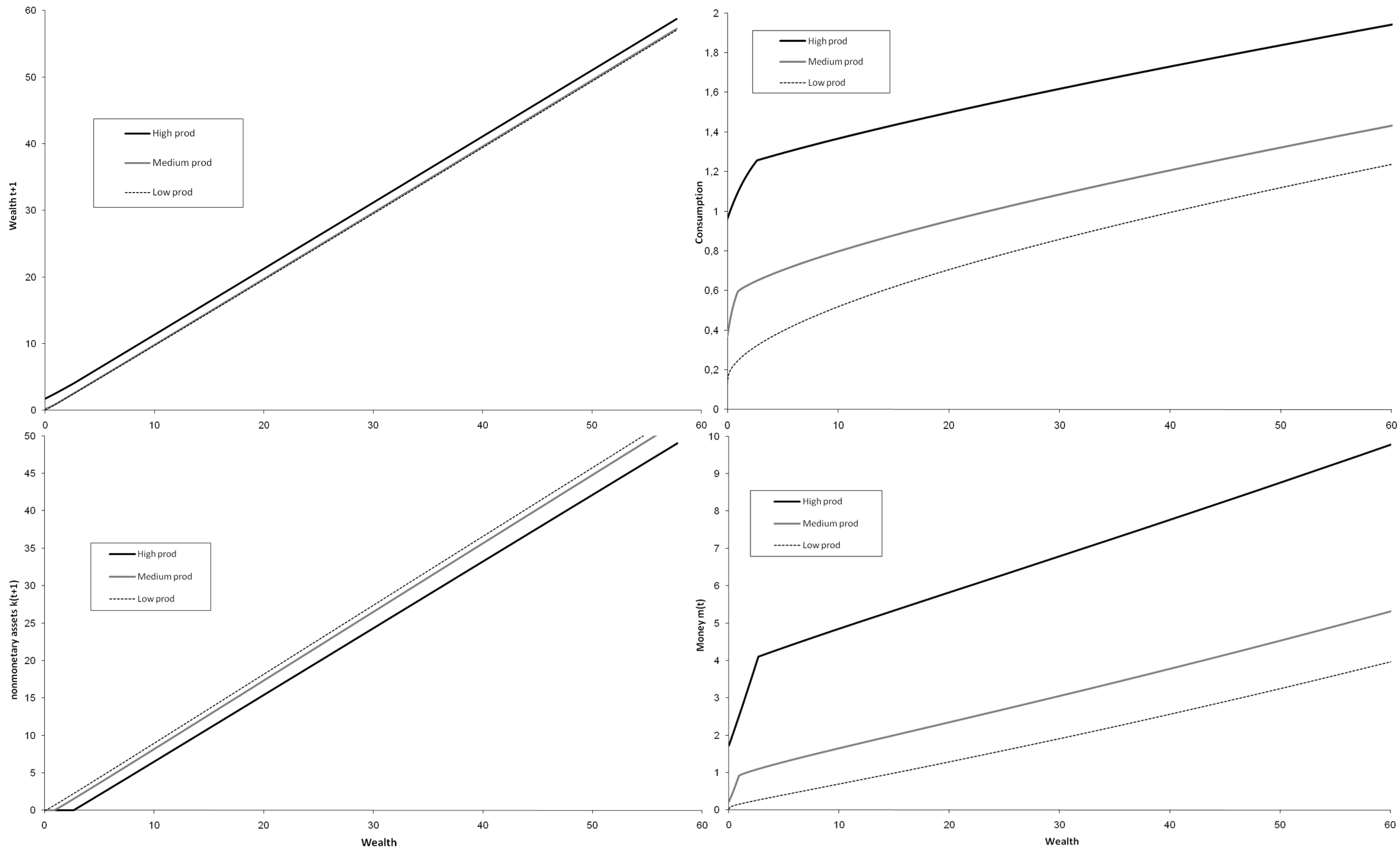

Figure 1: Individual policy rules as a function of total wealth for $\gamma = \gamma_1$.

4.2.2 Aggregate Results

To disentangle the effects of a positive money growth shock in this setting, we perform the following experiment.[6] We first solve the stochastic model under i.i.d money growth shocks; the implied laws of motion of the wealth distribution are given in Appendix B. We then hold monetary growth at its low value $\gamma_1 = 0.64\%$ for a long period of time ($T = 600$). Aggregate consumption, real money balances and the capital stock converge toward C_0, M_0 and K_0, respectively, the value of which are used as initial conditions for our experiment. At date 1, the economy is hit by a positive money growth shock, with money growth jumping from γ_1 to $\gamma_2 = 1.17\%$. At the time of the shock, aggregate consumption and real money balances are changed to C_1 and M_1, respectively, while the capital stock (which is predetermined) changes to K_1 one period later. Our measures of of the short-run effects of the policy shock are simply the instant proportional changes in the relevant aggregates, i.e., $\Delta C_1 \equiv (C_1 - C_0)/C_0$, $\Delta M_1 \equiv (M_1 - M_0)/M_0$ and $\Delta K_1 \equiv (K_1 - K_0)/K_0$. To measure the long-run impact of the shock, we hold monetary growth at $\gamma = \gamma_2$ for a long period of time until the economy converges to the new long-run values of consumption, money and capital holdings, denoted by C_∞, M_∞ and K_∞, respectively. The long-run effects of the shock are then given by $\Delta C_\infty = (C_\infty - C_0)/C_0$, $\Delta M_\infty = (M_\infty - M_0)/M_0$ and $\Delta K_\infty = (K_\infty - K_0)/K_0$. Although the probability that this particular history of aggregate states will occur is low, this experiment is natural as it allows to precisely identify the short- and long-run impact of a money growth shock on aggregates. Table 5 shows the results for both the elaborate and the simple utility function. Tables 6 and 7 decompose the responses to monetary shock by productivity types and wealth levels, again under both specifications of households' instant utility function.

The first row of Table 5 shows the short- and long-run impact of a money growth shock for our benchmark economy, in which households preferences are characterized by the elaborate utility function and under the assumption that the budget of the government is balanced (that is, money creation is not used to finance government spendings.) On impact, aggregate consumption rises by 0.15% and real money balances fall by 0.06%; after one period, the capital stock falls by a small amount, 0.009%. These aggregate changes are best understood by looking at their decomposition across wealth levels and productivity types, as is reported in Table 6. More specifically, by looking at the two ends of the wealth distribution (bottom and top 10%), we see that the aggregate effects of

[6] The impact of a negative money growth shock is roughly the mirror image of that of a positive shock.

the shock result from the composition of the reaction of the poor and the rich to the redistribution of wealth caused by the shock. Under lump-sum money injections, the intratemporal inflation tax redistributes real wealth from wealthy households, whose money holdings are relatively high, towards poorer households, whose money holdings are initially low. Since poor households have a comparatively larger marginal propensity to consume out of wealth, this redistribution raises aggregate consumption at the time of the shock. This effect also shows up in the average change in consumption by productivity type. The less productive households enjoy a consumption boom on average, while the more productive ones cut down their consumption. This reflects the fact that low-productivity households are more numerous amongst wealth-poor households, while high-productivity households are over-represented amongst the wealth-rich.

At the individual level, changes in real money holdings track changes in consumption, a reflection of the complementarity between those two inputs of the utility function. However, the consumption fall for wealthy households is associated with a proportionally larger drop in their real money demand, relative to the rise in money holdings experienced by poor households. The composition of these two effects leads to a mild fall in aggregate real money balances overall. Note that this negative net effect is a direct implication of our utility function with nonconstant elasticity of substitution, which implies that a fall in consumption by rich households is associated with a relatively large fall in their real money holdings, while a rise in consumption by poorer households entails a more moderate rise in their real money holdings.

The short-run effects of the money growth shock are reverted in the long-run (again, a hypothetical situation wherein households experience a long spell of high money growth while having expected the latter to be i.i.d.). In particular, aggregate consumption, money holdings and capital all eventually fall. In the long run, the evolution of those variables is governed by the negative wealth effects incurred by wealthy households in every period. This leads to a fall in their holdings of both monetary assets and claims to the capital stock and, ultimately, to a drop in aggregate output and consumption.

The second row of Table 5 shows the results when the transfers from the government towards private households do not depend on the money shock. In this case, the budget of the government is not balanced as before, and we simply assume that the government spends its extra resources on public consumption. This case isolates the specific redistributive effects of the inflation tax since the amount of wealth redistribution is not affected by real money growth. In this situation, the increase in monetized government spendings that follows from a positive money growth shock induces substantial negative wealth effects. As a consequence, all aggregates fall both in the short- and the long-run.

The third and fourth rows of Table 5 show the results in the case of the simple instant utility function (e.g. with constant elasticity of substitution between consumption and real money holdings). As already discussed, this economy displays less inequalities in money holdings than under the elaborate utility function. Consequently, a money growth shock has less redistributive effects under the balanced-budget rule, and the impact on aggregate consumption is milder. The utility function also overturns the impact of the shock on total real money balances. Under the simple utility function money demand is roughly proportional to consumption for all individuals, so an increase in consumption directly translates into an increase in real money demand.

To summarize, the wealth heterogeneity that results from the presence of uninsurable labor income risk implies that money growth shocks have real effect even when expected money growth is held constant (by the i.i.d. assumption.) Moreover, the size of those effects depend on the extent of the inflation tax (balance-budget vs. constant transfer fiscal rule) as well as the cross-sectional distribution of money holdings (which is indexed by the degree of complementarity between money holdings and consumption.)

4.3 Persistent money growth shocks

We now consider the case where the rate of money growth is autocorrelated and given by the transition matrix T^{AR}. Unlike in the i.i.d. case, under persistent shocks an increase in the rate of money growth changes households expected inflation (e.g., a jump from low to high money growth raises expected inflation), which alters individual portfolio decisions and hence equilibrium aggregates. To quantify the effects of persistent money growth shocks, we perform an experiment similar to that in the previous section; namely, money growth is held at the low rate γ_1 until the economy converges to a low-inflation equilibrium; the economy then switches to the high rate γ_2 and stays there for a long period of time. All along, households form expectations according to the transition matrix T^{AR}. The short-run effects of the shock are measured by the immediate proportional adjustments of the relevant aggregates, and the long-run effects by the value that these

Table 5: Responses to Monetary shocks: IID Shocks (in percent)

		Short run effect			Long run effect		
		ΔC_1	ΔM_1	ΔK_1	ΔC_∞	ΔM_∞	ΔK_∞
		Elaborate utility function					
1.	*Balanced government budget*	0.150	-0.058	-0.009	-0.115	-3.927	-1.247
2.	*Constant transfers*	-0.054	-0.122	-0.085	-1.676	-4.802	-3.503
		Simple utility function					
3.	*Balanced government budget*	0.034	0.026	-0.002	-0.049	-0.102	-0.513
4.	*Constant transfers*	-0.093	-0.085	-0.085	-1.661	-1.557	-2.095

Note: Short run and long run effects are calculated when the economy switches from of a stationary equilibrium economy for $\gamma_1 = 0.64\%$ to an economy hits by a high value of monetary shock $\gamma_2 = 1.17\%$. We calculate the changes in the aggregate X in the short run as $\frac{X_1 - X_0}{X_0}$ and in the long run $\frac{X_\infty - X_0}{X_0}$, where X_0 is the stationary equilibrium value when $\gamma = \gamma_1$, X_1 is the new value just after the shock, and X_∞ is the long run stationary equilibrium value when $\gamma = \gamma_2$.

aggregates eventually take once the economy has eventually settled in the high-inflation regime. Table 8 summarizes the impact of a persistent money growth shock under complete and incomplete markets, for both the elaborate and the simple utility functions.

The first row of Table 8 shows the effect of an increase in the rate of money growth in the complete-market economy when preferences are given by the elaborate utility function. On impact, consumption falls by 0.33% and aggregate money demand by 2%, while the capital stock increases by 0.02% (after one period.) The reason is as follows: after a jump from low to high money growth, households expect this higher growth to persist and hence future inflation to be higher on average. This leads them to lower their money holdings, and also their consumption due to the complementarity between the two. The overall effect of the shock is a portfolio shift towards nonmonetary assets (i.e., the capital stock), with the aim of raising *future* consumption (a version of the 'Tobin effect' of inflation on portfolio choice.) In the long run, the higher capital accumulation that prevails in the high money growth regime translates into higher aggregate consumption and lower real money balances.

The second row of Table 8 shows the impact of a positive money growth shock in the incomplete-

Table 6: Response to monetary shock by wealth level and productivity type: Simple utility function, IID shocks (in percent)

	Balanced government budget						Constant transfers					
	ΔC_1	ΔM_1	ΔK_1	ΔC_∞	ΔM_∞	ΔK_∞	ΔC_1	ΔM_1	ΔK_1	ΔC_∞	ΔM_∞	ΔK_∞
Bottom 10%	1.445	1.941	3.356	4.768	7.329	23.637	-38.735	-60.557	110.458	-28.266	-44.783	70.832
Top 10%	-0.077	-0.148	-0.114	-0.655	-1.111	-0.877	-0.026	-0.067	-0.055	-2.032	-5.259	-4.681
e^h	-0.025	-0.095	-0.018	-0.519	-0.965	-0.608	-0.047	-0.119	-0.131	-1.485	-3.984	-3.692
e^m	0.037	0.046	0.005	0.005	0.057	-0.489	-0.058	-0.127	-0.063	-1.743	-5.690	-3.401
e^l	0.316	0.426	0.051	1.831	2.533	-0.058	-0.031	-0.079	-0.033	-1.641	-7.611	-3.388

Note: Short run and long run effects are calculated when the economy switches from of a stationary equilibrium economy for $\gamma_1 = 0.64$ to an economy hits by a high value of monetary shock $\gamma_2 = 1.17$. We calculate the changes in the aggregate X in the short run as $\Delta X_1 = \frac{X_1 - X_0}{X_0}$ and in the long run as $\Delta X_\infty = \frac{X_\infty - X_0}{X_0}$, where X_0 is the stationary equilibrium value when $\gamma = \gamma_1$, X_1 is the new value just after the shock, and X_∞ is the long run stationary equilibrium value when $\gamma = \gamma_2$.

Table 7: Response to monetary shock by wealth level and productivity type: Elaborate utility function, IID shocks (in percent)

	Balanced government budget						Constant transfers					
	ΔC_1	ΔM_1	ΔK_1	ΔC_∞	ΔM_∞	ΔK_∞	ΔC_1	ΔM_1	ΔK_1	ΔC_∞	ΔM_∞	ΔK_∞
Bottom 10%	2.432	4.035	16.711	10.911	32.342	103.797	0.275	-0.281	-4.566	-0.136	-1.111	-6.855
Top 10%	-0.095	-0.318	-0.102	-1.936	-5.865	-3.776	-0.026	-0.067	-0.055	-2.032	-5.259	-4.681
e^h	-0.057	-0.226	-0.059	-1.151	-4.055	-1.809	-0.047	-0.119	-0.131	-1.485	-3.984	-3.692
e^m	0.134	0.140	0.013	0.051	-3.653	-0.919	-0.058	-0.127	-0.063	-1.743	-5.690	-3.401
e^l	1.492	0.151	0.077	2.801	-6.833	-1.217	-0.031	-0.079	-0.033	-1.641	-7.611	-3.388

Note: Short run and long run effects are calculated when the economy switches from of a stationary equilibrium economy for $\gamma_1 = 0.64$ to an economy hits by a high value of monetary shock $\gamma_2 = 1.17$. We calculate the changes in the aggregate X in the short run as $\Delta X_1 = \frac{X_1 - X_0}{X_0}$ and in the long run as $\Delta X_\infty = \frac{X_\infty - X_0}{X_0}$, where X_0 is the stationary equilibrium value when $\gamma = \gamma_1$, X_1 is the new value just after the shock, and X_∞ is the long run stationary equilibrium value when $\gamma = \gamma_2$.

markets case. On impact, consumption falls by 0.07% and total money demand by 2.44%, while the capital stock increases slightly. The effects on consumption and capital are much weaker than under complete markets. The milder fall in consumption stems from the redistributive effect of the shock, which goes towards raising aggregate consumption (just as in the i.i.d. case). In the long run, the switch from low to high money growth under incomplete markets leads to a reduction in aggregate consumption, money demand and the capital stock. These long-run effects have been identified and discussed in Algan and Ragot (2010); essentially, under incomplete markets the redistributive effect of inflation deters self-insurance, since inflation is a real transfer from the money-rich to the money-poor.

The third and fourth rows of Table 8 show the impact of the money growth shock when preferences are characterized by the simple utility function. In the complete-markets case, the short-run effects of the shock on aggregates are qualitatively similar than under the elaborate utility function, but about a third lower in magnitude; this is due to the difference in the complementarity between consumption and real balances across the two economies. In the incomplete-market case, consumption and real balances fall on impact by 0.12 and 1.78%, respectively, while the capital stock rises by a very small amount. The long-run effects are again qualitatively similar, but quantitatively smaller in magnitude, essentially because the simple utility function generates less inequalities in money holdings and hence a milder redistributive effect of the shock.

To summarize, under the elaborate utility function, incomplete markets and borrowing constraints divide by two the short-run effects of a persistent money growth shock on aggregates, and lead to a reversion of its long-run effects (relative to the complete-markets case.) On the other hand, moving from the simple to the elaborate utility function raises the impact of a persistent money growth shock by roughly 45%. As a result, moving from the complete-market economy with simple utility function (the basic representative-agent monetary model) to the economy with incomplete markets and elaborate utility function (our benchmark model) leads to a reduction in the real short-run effects of a persistent money growth shocks of about 70%. Table 9 decomposes the individual responses of consumption, money holdings and capital holdings to the shock by productivity types and wealth levels under the simple and the elaborate utility functions.

Table 8: Responses to Monetary shocks: Auto correlated shocks (in percent)

		Short run effect			Long run effect		
		ΔC_1	ΔM_1	ΔK_1	ΔC_∞	ΔM_∞	ΔK_∞
		Elaborate utility function					
1.	*Complete markets*	-0.334	-2.007	0.020	0.076	-0.958	0.800
2.	*Incomplete markets*	-0.066	-2.440	0.004	-0.049	-5.094	-0.528
		Simple utility function					
3.	*Complete markets*	-0.21	-1.85	0.013	0.041	-1.60	0.42
4.	*Incomplete markets*	-0.12	-1.78	0.007	-0.016	-1.79	-0.17

Note: Short run and long run effects are calculated when the economy switches from of a stationary equilibrium economy for $\gamma_1 = 0.64\%$ to an economy hits by a high value of monetary shock $\gamma_2 = 1.17\%$. We calculate the changes in the aggregate X in the short run as $\frac{X_1 - X_0}{X_0}$ and in the long run $\frac{X_\infty - X_0}{X_0}$, where X_0 is the stationary equilibrium value when $\gamma = \gamma_1$, X_1 is the new value just after the shock, and X_∞ is the long run stationary equilibrium value when $\gamma = \gamma_2$.

4.4 Unconditional correlation

We conclude our discussion on the impact of monetary shocks under incomplete markets by looking at the implied correlation between aggregates (see Table 10.) First, the inflation-consumption correlation is negative in all cases, and more that a third smaller in absolute value when we switch from the complete-market, simple utility case (CMSU) to the incomplete-market, elaborate utility case (IMEU), a change that comes both from the utility function and the incompleteness of asset markets. The inflation-investment correlation is positive in all cases except for the IMEU economy, a switch that comes mostly from the incompleteness of asset markets (rather than the utility function). The correlation between inflation and the consumption growth is negative for all economies and divided by almost 1.5 when moving from the CMSU to the IMEU, with this change in magnitude mostly coming from the incompleteness of asset markets. The standard deviation of aggregate variables is roughly the same for inflation, real money balances and investment. The standard deviation of consumption and capital are divided by more than three between CMSU and IMEU, an effect that comes both from the utility function and the structure of financial markets.

Table 9: Response to monetary shock by wealth level and productivity type: Simple and elaborate utility function, auto-correlated shocks

	Simple Utility						Elaborate utility					
	ΔC_1	ΔM_1	ΔK_1	ΔC_∞	ΔM_∞	ΔK_∞	ΔC_1	ΔM_1	ΔK_1	ΔC_∞	ΔM_∞	ΔK_∞
Bottom 10%	1.600	0.049	-5.444	3.477	3.569	13.973	3.161	2.019	2.010	10.095	26.456	52.291
Top 10%	-0.301	-1.958	-0.002	-0.685	-2.824	-0.582	-0.716	-3.078	0.050	-2.041	-7.195	-2.626
e^h	-0.290	-1.939	0.001	-0.550	-2.676	-0.221	-0.653	-2.951	0.079	-1.308	-5.543	-0.625
e^m	-0.100	-1.755	0.012	0.064	-1.602	-0.145	0.007	-1.844	-0.039	0.194	-4.454	-0.423
e^l	0.518	-1.231	0.001	1.830	0.365	0.031	1.918	-1.543	-0.001	2.870	-7.178	-1.074

Note: Short run and long run effects are calculated when the economy switches from of a stationary equilibrium economy for $\gamma_1 = 0.64$ to an economy hits by a high value of monetary shock $\gamma_2 = 1.17$. We calculate the changes in the aggregate X in the short run as $\Delta X_1 = \frac{X_1 - X_0}{X_0}$ and in the long run as $\Delta X_\infty = \frac{X_\infty - X_0}{X_0}$, where X_0 is the stationary equilibrium value when $\gamma = \gamma_1$, X_1 is the new value just after the shock, and X_∞ is the long run stationary equilibrium value when $\gamma = \gamma_2$.

Table 10: Correlations

	Economy			
	Simple utility		Elaborate utility	
	Complete-market economy	Incomplete-market economy	Complete-market economy	Incomplete-market economy
Correlations				
Inflation-Consumption	-0.64	-0.64	-0.56	-0.20
Inflation-Investment	0.63	0.63	0.59	0.17
Inflation-ΔC	-0.90	-0.82	-0.92	-0.63
Standard deviation				
Inflation	0.007	0.006	0.007	0.007
Money	0.022	0.021	0.022	0.032
Consumption	0.002	$4e^{-4}$	0.001	$6e^{-4}$
Investment	0.041	0.004	0.023	0.013

Note: The model properties are averages over a 10,000 period simulation. Inflation-ΔC stands for the correlation of inflation at date t and percentage change in consumption $(C_t - C_{t-1})/C_{t-1}$.

5 Conclusion

This paper has explored the implications of uninsurable labor income risk for the redistributive and aggregate effects of monetary shocks. Our benchmark economy features incomplete markets, borrowing constraints, as well as a form of the money-in-the-utility specification that is designed to reproduce the empirical cross-sectional distribution of money holdings observed in US data. Our analysis suggests that money growth shocks have a moderate effect on aggregates in this framework (relative to the complete-market case), but also that changes in aggregates in fact mask a great deal of wealth redistribution and heterogeneity in portfolio adjustments at the individual level.

In the current model, nonmonetary assets (i.e., claims to the capital stock) are inflation-indexed by construction, so the only asset whose value is directly affected by the inflation tax is fiat money. While this is a natural benchmark to start with, it clearly underestimates the redistributive effects of inflation shocks, since many nonmonetary assets (e.g., corporate bonds) are not indexed. This implies that the inflation tax that is responsible for the redistributive effects of monetary shocks under heterogenous cash holdings may in fact be much larger than when considering as nominal assets only a narrow monetary aggregate (such as M2.)

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A Numerical Algorithm

A.1 Overview of the Algorithm

The algorithm used to obtain the solution of the model is as follows.

1. Given laws of motion for real money balance and capital, defined by (14) and (15), solve the household's problem given by the equations (13), (6), (7) and $k_{t+1} > 0$.

2. Simulate the economy to approximate the equilibrium laws of motion for K and M. We use the grid-based simulation procedure proposed by Young (2008).

(a) Set an initial wealth/employment-efficiency distribution $\mu_0(a,e)$ that provides $p_0^{i,e}$ the mass of agents of employment-efficiency type e with wealth a_i at the ith wealth grid point for $i = 1, \cdots, N_{grid}$; an initial value for γ; and initial individual policy rules for k and m.[7]

(b) Find the inflation rate Π_t^* that achieves money market clearing *and* the associated wealth/employment-efficiency distribution $\mu(a_t, e_t; \Pi_t^*)$. In particular, we iterate on Π_t until the following condition is satisfied:

$$\int\int g_m(a_t, e_t; \gamma_t, M_{t-1}, K_t, \Pi_t) d\mu(a_t, e_t; \Pi_t) = \frac{(1+\gamma_t)\Omega_{t-1}}{\Pi_t} \tag{16}$$

where $g_m(a_t, e_t; \gamma_t, M_{t-1}, K_t, \Pi_t)$ is the policy function for real balance that depends explicitly on the value of Π_t that solves the following household's problem:

$$\tilde{v}(a_t, e_t; \gamma_t, M_{t-1}, K_t, \Pi_t) = \max_{m_t, c_t, k_{t+1}} u(c_t, m_t) + \beta E_t \left[v\left(a_{t+1}, e_{t+1}; \gamma_{t+1}, M_t, K_{t+1}\right) | s_t, \gamma_t \right], \tag{17}$$

where the value function v is the solution of the individual problem defined in step (1). An interesting point in this problem is that the current distribution μ depends on the current inflation rate Π_t. Thereby, we have a different distribution of wealth for each Π_t. The steps to find the equilibrium Π_t^* are detailed below.

(c) Given the decision rules $g_k(a_t, e_t; \gamma_t, M_{t-1}, K_t, \Pi_t^*)$, and $g_m(a_t, e_t; \gamma_t, M_{t-1}, K_t, \Pi_t^*)$, and $\mu(a_t, e_t; \Pi_t^*)$, calculate

$$\int\int g_m(a_t, e_t; \gamma_t, M_{t-1}, K_t, \Pi_t^*) d\mu(a_t, e_t; \Pi_t^*) = M_t$$

and

$$\int\int g_k(a_t, e_t; \gamma_t, M_{t-1}, K_t, \Pi_t^*) d\mu(a_t, e_t; \Pi_t^*) = K_{t+1}$$

d. Repeat steps (b) and (c) and obtain a long time series for K and M, of which the first part is discarded.

3. Use the time series obtained in step (2) to get the new equilibrium laws of motion for K and M.

[7] We use the wealth distribution and the associated policy rules that we get for the economy without monetary shock.

4. Compare the new equilibrium laws of motion for K and M with those used in step (1). If they are similar, stop. Otherwise, update the coefficients of the laws of motion, and go to step (1).

A.2 Details on the Resolution of the Individual Problem

We have the following first order conditions

$$u_c(c_t, m_t) - u_m(c_t, m_t) = \beta E_t \left[\frac{v_a\left(a_{t+1}, e_{t+1}; \gamma_{t+1}, M_t, K_{t+1}\right)}{1+\pi_{t+1}} \right] \tag{18}$$

$$u_c(c_t, m_t) = \beta E_t \left[(1+r_{t+1}) v_a\left(a_{t+1}, e_{t+1}; \gamma_{t+1}, M_t, K_{t+1}\right) \right] \tag{19}$$

The last equation is true if $k_{t+1} > 0$. Given an initial guess for $v_a^0(.)$, the first derivative with respect to a of the value function, for each grid point, we will solve the individual problem defined by the previous FOC and the budget constraint (6). Given the solution at each grid point, we get a new $v_a^1(.) = u_c(c_t, m_t)$. If the new derivative of the value function is close to the old one, we have found an approximation of the fixed point, and we get $g_c^0(.)$, $g_k^0(.)$, and $g_m^0(.)$ as the solution of the problem. If not, we update $v_a^0(.) = v_a^1(.)$.

We have two distinct cases:

1) If $k_{t+1} = 0$, we solve the non linear equation in m_t:

$$u_c(c_t, m_t) - u_m(c_t, m_t) - \beta E_t \left[\frac{v_a^0\left(a_{t+1}, e_{t+1}; \gamma_{t+1}, M_t, K_{t+1}\right)}{1+\pi_{t+1}} \right] = 0$$

where $c_t = a_t + w_t e_t + \gamma_t \frac{M_{t-1}}{\Pi_t} - m_t$, and given $v_a^0\left(a_{t+1}, e_{t+1}; \gamma_{t+1}, M_t, K_{t+1}\right)$ where $a_{t+1} = \frac{m_t}{1+\pi_{t+1}}$. M_t, and K_{t+1} are given by the fixed aggregate laws of motion.

2) If $k_{t+1} > 0$, we find the solution for g_c, g_m, g_k, and v_a using nested bisection methods. First, we solve for m and k given a certain level of consumption using equation (18) and the budget constraint. Second, we solve the capital foc for c where the m and k are given by the previous step. More formally, for each grid points, we have the following steps:

1. For a given value of c, solve the following foc for m

$$u_c(c_t, m_t) - u_m(c_t, m_t) - \beta E_t \left[\frac{v_a^0\left(a_{t+1}, e_{t+1}; \gamma_{t+1}, M_t, K_{t+1}\right)}{1+\pi_{t+1}} \right] = 0$$

given $v_a^0\left(a_{t+1},e_{t+1};\gamma_{t+1},M_t,K_{t+1}\right)$ where $a_{t+1}=\left(1+r_{t+1}\right)k_{t+1}+\frac{m_t}{1+\pi_{t+1}}$, and k_{t+1} deduced from the budget constraint. M_t, and K_{t+1} are given by the fixed aggregate laws of motion.

2. Given k_{t+1}, and m_t found in the previous step, solve the following non-linear equation for c

$$u_c\left(c_t,m_t\right)-\beta E_t\left[\left(1+r_{t+1}\right)v_a^0\left(a_{t+1},e_{t+1};\gamma_{t+1},M_t,K_{t+1}\right)\right]=0$$

given $v_a^0\left(a_{t+1},e_{t+1};\gamma_{t+1},M_t,K_{t+1}\right)$ where $a_{t+1}=\left(1+r_{t+1}\right)k_{t+1}+\frac{m_t}{1+\pi_{t+1}}$, and M_t, and K_{t+1} are given by the fixed aggregate laws of motion.

A.3 Details to find Π_t^*

The following iterative sub-algorithm is implemented to find Π_t^*:

1. Given an arbitrary value of Π_t, solve problem (17), where the value function v is the solution of the individual problem defined in step (1). This problem generates decision rules $g_k(a_t,e_t;\gamma_t,M_{t-1},K_t,\Pi_t^*)$, and $g_m(a_t,e_t;\gamma_t,M_{t-1},K_t,\Pi_t^*)$ which depend explicitly on the value of Π_t.

2. For the same arbitrary value of Π_t used in the previous step, and given K_t, M_{t-1}, Π_{t-1}^* and the wealth distribution $\mu\left(a_{t-1},e_{t-1};\Pi_{t-1}^*\right)$ that we got from the previous simulation period, calculate the resulting current and updated wealth/employment-efficiency distribution by using the decision rules that we get in period $t-1$. Specifically for a given Π_t, for each wealth grid point a_i and each employment-efficiency type, calculate the new financial wealth such as

$$\begin{aligned}g_a(a_i,e;\gamma_t,M_{t-1},K_t,\Pi_t) &= g_k(a_i,e_{t-1};\gamma_{t-1},M_{t-2},K_{t-1},\Pi_{t-1}^*)R_t+\\ &\quad g_m(a_i,e_{t-1};\gamma_{t-1},M_{t-2},K_{t-1},\Pi_{t-1}^*)\Pi_t^{-1}\end{aligned}$$

and find the index I within the grid such that $g_a(a_i,e;\gamma_t,M_{t-1},K_t,\Pi_t)$ lies in $[\omega_I,\omega_{I+1}]$. Then, redistribute the current mass to the grid points ω_I and ω_{I+1} taking into account the employment-efficiency flows. In particular, if $p_t^{i,e}$ stands for the mass of agents of type e at the ith grid points at period t, let

$$p_t^{I,e}=p_t^{I,e}+g_{e_{-1}e}\frac{\omega_{I+1}-g_a(a_i,e;\gamma_t,M_{t-1},K_t,\Pi_t)}{\omega_{I+1}-\omega_I}p_{t-1}^{i,e}$$

and let

$$p_t^{I+1,e}=p_t^{I+1,e}+g_{e_{-1}e}\frac{g_a(a_i,e;\gamma_t,M_{t-1},K_t,\Pi_t)-\omega_I}{\omega_{I+1}-\omega_I}p_{t-1}^{i,e}$$

where $g_{e_{-1}e}$ stands for the mass of agents with employment status e that had employment status e_{-1} last period.

3. Check if equation (16) is verified. If not, repeat steps (1) and (2) until money market clears.

B Results on Aggregate law of Motions

B.1 IID shocks

We find the following laws of evolution for current aggregate money M_t and next-period capital K_{t+1}. Whatever the values of the growth rate of money, the coefficients associated with M_{t-1} and K_t are statistically significant at the 1 percent level.

Simple utility function

i) $\gamma = \gamma_1$

$$\ln M' = -0.0452 + 0.0794 \ln K + 0.8173 \ln M, \;\; R^2 = 0.9997 \tag{20}$$

$$\ln K' = 0.0342 + 1.0090 \ln K - 0.0643 \ln M, \;\; R^2 = 0.9999. \tag{21}$$

ii) $\gamma = \gamma_2$,

$$\ln M' = -0.0558 + 0.0900 \ln K + 0.7981 \ln M, \;\; R^2 = 0.9999 \tag{22}$$

$$\ln K' = 0.0341 + 1.0091 \ln K - 0.0645 \ln M, \;\; R^2 = 0.9999. \tag{23}$$

Elaborate utility function

i) $\gamma = \gamma_1$

$$\ln M' = -5.7112 + 2.6949 \ln K + -0.5667 \ln M, \;\; R^2 = 0.9997 \tag{24}$$

$$\ln K' = -1.7936 + 1.8468 \ln K - 0.4934 \ln M, \;\; R^2 = 0.9999. \tag{25}$$

ii) $\gamma = \gamma_2$,

$$\ln M' = -5.7961 + 2.7345 \ln K + -0.5886 \ln M, \;\; R^2 = 0.9999 \tag{26}$$

$$\ln K' = -1.7175 + 1.8114 \ln K - 0.4742 \ln M, \;\; R^2 = 0.9999. \tag{27}$$

B.2 Persistent shocks

We find the following laws of evolution for next-period aggregate money M' and capital K'. In all states of the world the real quantity of money and the aggregate capital stock are statistically significant to forecast the dynamics of capital and money.

Simple utility function

i) $\gamma = \gamma_1$

$$\ln M' = 0.5762 + 0.1324 \ln K - 0.0077 \ln M, \;\; R^2 = 0.9936 \tag{28}$$

$$\ln K' = 0.0123 + 0.9945 \ln K + 0.0025 \ln M, \;\; R^2 = 0.9912. \tag{29}$$

ii) $\gamma = \gamma_2$,

$$\ln M' = 0.0425 + 0.3277 \ln K + -0.0076 \ln M, \;\; R^2 = 0.9921 \tag{30}$$

$$\ln K' = 0.0558 + 0.9780 \ln K + 0.0024 \ln M, \;\; R^2 = 0.9919. \tag{31}$$

Elaborate utility function

i) $\gamma = \gamma_1$

$$\ln M' = -4.6057 + 2.0884 \ln K + 0.0041 \ln M, \;\; R^2 = 0.9987 \tag{32}$$

$$\ln K' = 0.0101 + 0.9949K + 0.0036 \ln M, \;\; R^2 = 0.9942. \tag{33}$$

ii) $\gamma = \gamma_2$,

$$\ln M' = -4.4188 + 2.0103 \ln K + 0.0031 \ln M, \;\; R^2 = 0.9981 \tag{34}$$

$$\ln K' = 0.0401 + 0.9836 \ln K + 0.0037 \ln M, \;\; R^2 = 0.9941. \tag{35}$$