Labor Economics

Chapter 1
Labor Supply
(Complete)

Pierre Cahuc, Stéphane Carcillo and André Zylberberg
In this chapter, we will:

- See how people make choices between consumption, leisure and household production
- Understand how the labor supply curve results from the combination of substitution effects and income effects
- Learn what the wage elasticities of labor supply are
- Explore when and why people decide to retire
- Learn the principles guiding the econometrics of labor supply and the main empirical results
- Provide an overview of the results of macro- and micro-empirical studies on labor supply elasticities
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Introduction

- Since 1900, in the United States, many changes in worked hours led to a significant increase in the labor force.
- As an individual disposes a limited amount of time, (s)he has to choose to allocate between paid work and leisure.
- Wage constitutes an important factor in the choice.
- Labor supply depends on trade-offs within family because it is the counterpart of the “household production”.
- A better understanding of labor supply behaviors allows to understand the consequences of the tax system.
Introduction

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Summary and conclusion
Facts about labor supply (1) - Basic definitions

- The labor force is composed of all persons employed or looking for a job.
- To be considered unemployed, people must:
  - Be without work
  - Be available for work
  - Be seeking for work
- The participation rate is the ratio of the labor force to the population in age of working (generally more than 15 years old).
- The employment rate is the ratio of the number of employed people to the working age-population.
- The unemployment rate is the ratio of the number of unemployed people to the labor force.
Facts about labor supply (2) - The trend in the amount of time worked

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Table 1: Hours worked annually per person and real hourly wages in the manufacturing sector. Source: Maddison (1995) for 1870, 1913, 1938 and OECD data for 1997 and 2011.
Facts about labor supply (3) - The trend in the amount of time worked

**Figure 1.1**
Amount of time worked annually in 7 OECD countries over the period 1970–2011 (total number of hours worked during the year divided by the average number of persons of working age).

Source: OECD Labor Force Statistics.
Facts about labor supply (4) - The trend in the amount of time worked

- Table 1 shows that labor productivity grows since the 1870s.
- In fact, before the agricultural and industrial revolution, productivity had varied very little for several centuries.
- Figure 1.1 shows the variation of the annual amount of time worked in different countries.
- However, these global trends in time worked are difficult to interpret because it depends on many factors.
Facts about labor supply (5) - The evolution of participation rates

**Figure 1.2**
The evolution in civilian labor force participation rates in the United States, Europe, and Japan for persons 15 years of age and older, 1956–2010.

Source: OECD Annual Labor Force Statistics.
Facts about labor supply (6) - The evolution of participation rates

**Figure 1.3**
The evolution in civilian labor force participation rates of men in the United States, Europe, and Japan for persons 15 years of age and older, 1956–2010.

Source: OECD Annual Labor Force Statistics.
Facts about labor supply (7) - The evolution of participation rates

Figure 1.4
The evolution in civilian labor force participation rates of women in the United States, Europe, and Japan for persons 15 years of age and older, 1956–2010.

Source: OECD Annual Labor Force Statistics.
Facts about labor supply (8) - The evolution of participation rates

**Figure 1.5**
The incidence of low-paying jobs among women in the United States, Japan, and the United Kingdom. Low pay is defined as less than two thirds of the gross median earnings of all full-time workers.

Source: OECD Earnings Statistics.
Facts about labor supply (8 bis) - The evolution of participation rates

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<td>2010</td>
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**Table 2**: Civilian labor force participation rates of women aged 16 and over, classified by their marital status, in the United States. Source: Ehrenberg and Smith (1994, Table 6.1, p. 165) for 1900, 1950 and 1988, and Census Bureau for 2010.
Figure 1.6

Source: OECD Labor Force Statistics.
Facts about labor supply (10) - Leisure and home production

**Figure 1.7**
Work, leisure, and home hours per week in the United States 1900–2005.

Source: Francis and Ramey (2009).
Facts about labor supply (11) - Leisure and home production

**Figure 1.8**
Work, leisure, and home hours per week of men in the United States 1900–2005.

Source: Francis and Ramey (2009).
Facts about labor supply (12) - Leisure and home production

**Figure 1.9**
Work, leisure, and home hours per week of women in the United States 1900–2005.

Source: Francis and Ramey (2009).
The choice between consumption and leisure (1) - The basic model

Preferences

- The trade-off between consumption and leisure is based upon the utility function of each individual
  - $U(C, L)$ denotes the individual’s utility function, where $C$ and $L$ designate the consumption of goods and that of leisure
  - $L_0$ designates the total amount of time that an individual disposes
  - $h = L_0 - L$ is the length of time worked
- The set of consumption and leisure by which the consumer obtains a given level of utility $\bar{U}$, is called an *indifference curve*. It is described in figure 1.10
The choice between consumption and leisure (2) - The basic model

**Figure 1.10**
An indifference curve, where $C =$ consumption of goods and $L =$ leisure.
The choice between consumption and leisure (3) - The basic model

The indifference curves satisfy some properties:

▶ Each indifference curve corresponds to a higher level of utility, the farther out the curve is from the origin
▶ Indifference curves do not intersect
▶ The indifference curves are negatively sloped
  ▶ The slope defines *The marginal rate of substitution between consumption and leisure*
  ▶ It represents the quantity of goods that a consumer must renounce in exchange for an hour of supplementary leisure for her level of satisfaction to remain unchanged
▶ The marginal rate of substitution between consumption and leisure diminishes with leisure time
The choice between consumption and leisure (4) - The basic model

*Choices*

- The budget constraint of an agent is:

\[ C + wL \leq R_0 \equiv wL_0 + R \]

- \( w \) is the real hourly wage
- \( wh = w(L_0 - L) \) represents total income
- \( R \) is the income that an individual may acquire outside the labor market

- Thus the problem of the consumer becomes:

\[
\max_{\{C,L\}} U(C, L) \text{ subject to the budget constraint } C + wL \leq R_0
\]
The choice between consumption and leisure (5) - The basic model

The interior solutions

- Utilizing the Lagrangian and calculating the FOCs, the solution is:

\[
\frac{U_L(C^*, L^*)}{U_C(C^*, L^*)} = w \quad \text{and} \quad C^* + wL^* = R_0 \quad (2)
\]

- Figure 1.11 shows an example of a possible solution
The choice between consumption and leisure (6) - The basic model

**Figure 1.11**
The trade-off between consumption $C$ and leisure $L$. 
The choice between consumption and leisure (7) - The basic model

*The reservation wage*

- The marginal rate of substitution at point A is called the *reservation wage*, which is defined by:

  \[ w_A = \frac{U_L(R, L_0)}{U_C(R, L_0)} \]

- The reservation wage depends on the form of the function \( U \) and on the level of non-wage income \( R \)
- It determines the condition of *participation* in the labor market
- If the current wage falls below the reservation wage, a worker will not participate in the labor market
The choice between consumption and leisure (8) - The properties of labor supply

**Substitution effect and income effect**

- The properties of the supply of individual labor result from the combination of a substitution effect and two income effects
- By definition,
  - Leisure is a *normal good* if its demand rises with $R_0$
  - Leisure is an *inferior good* if its demand decreases with $R_0$
- The consequence of an increase in non-earned income are represented in figure 1.11 by the shift from point $E$ to point $E'$
The choice between consumption and leisure (9) - The properties of labor supply

**Figure 1.12**
The effects of a wage increase.
The choice between consumption and leisure (10) - The properties of labor supply

- Figure 1.12 shows the change in consumer’s equilibrium when the wage increases from $w$ to $w_l$.
- From the initial equilibrium $E$, the substitution effect moves it to $E'$. The substitution effect thus implies a reduction of leisure.
- From the equilibrium $E'$, the income effect shifts the equilibrium to $E''$. Thus the substitution effect and the income effect work to produce the same result.
- If leisure is a normal good, when the wage increases the substitution effect increases the labor supply while the income effect decreases it.
The choice between consumption and leisure (11) - The properties of labor supply

- The Hicksian or compensated elasticity is:
  \[ \eta_H = \frac{w}{\hat{h}} \frac{d\hat{h}}{dw} \]

- The Marshallian or non-compensated elasticity is:
  \[ \eta_M = \frac{w}{h^*} \frac{dh^*}{dw} \]

- These two elasticities are linked by the Slutsky equation:
  \[ \eta_M = \eta_H + \frac{wh^*}{R_0} \eta_{R_0} \]

  - Where \( \eta_{R_0} \) represents the Marshallian elasticity of labor supply with respect to potential income
  - This equation shows that Marshallian elasticity is to be interpreted as the sum of the substitution effect, represented by the Hicksian elasticity, and the income effect
The choice between consumption and leisure (12) - The properties of labor supply

The shape of the labor supply curve

**Figure 1.13**
The individual labor supply.
Extensive margin, Intensive margin, and aggregate labor supply

- The wage exerts two distinct effects on labor supply by influencing
  - The decision to work or not, called the extensive margin
  - The number of hours supplied by every person who does decide to work, is called the intensive margin

- The supply of labor denoted $h(w, R)$ is a decreasing function of non-earned income

- We define $\bar{R}$ by $h(w, \bar{R}) = 0$, i.e., only individuals whose non-earned income is inferior to $\bar{R}$ work

- We consider a large population in which individuals have different non-earned income represented by the cdf $\Phi$
The choice between consumption and leisure (14) - The properties of labor supply

- The aggregate labor supply is:

\[ L_A(w) = \int_0^{\bar{R}} h(w, R) d\Phi(R). \]

- The derivative of the aggregate labor supply with respect to the wage \( w \) is:

\[ \int_0^{\bar{R}} \frac{\partial h(w, R)}{\partial w} d\Phi(R) + h(w, \bar{R}) \Phi'(\bar{R}) \frac{d\bar{R}}{dw}. \]

- The first term represents the changes in the intensive margin. The second term represents the changes in the extensive margin.

- As \( h(w, \bar{R}) = 0 \), it means that small variations in wages have an impact on the aggregate supply of labor felt solely through changes at the intensive margin.
Labor supply with household production and within the family (1) - Household production

The consumer’s program

- Goods consumed may be purchased, in quantity $C_M$, or produced domestically, in quantity $C_D$, with $C = C_D + C_M$
- We will denote:
  - $L_0$ the total endowment of time available
  - $h_D$ the paid working time, $h_M$ household working time
  - $C_D = f(h_D)$ the domestic production function
  - $L = L_0 - h_D - h_M$

- The optimal solutions of consumption and leisure are:
  \[
  \frac{U_L(C^*, L^*)}{U_C(C^*, L^*)} = w = f'(h_D^*) \quad \text{and} \quad C^* + wL^* = \tilde{R}_0 \quad (7)
  \]
  - With the potential income, $\tilde{R}_0 = R_0 + f(h_D^*) - wh_D^*$
  - The agent thus has an interest in devoting his working time to household activities to the extent that the marginal productivity of an hour of this type of work is superior to an hour’s wage
Labor supply with household production and within the family (2) - Household production

Elasticity of labor supply

► The system of equations (7) allows us to write the optimal demand for leisure in the form $L^* = \Lambda(w, \tilde{R}_0)$. Differentiating this equality with respect to $w$, we get:

$$\frac{dL^*}{dw} = \Lambda_1 + \Lambda_2 \frac{d\tilde{R}_0}{dw} \text{ with } \frac{d\tilde{R}_0}{dw} = L_0 - h^*_D$$

► As $f'(h^*_D) = w$ implies that $dh^*_D/dw = 1/f''(h^*_D)$, the identity $h^*_M = L_0 - h^*_D - L^*$ entails:

$$\frac{dh^*_M}{dw} = -(\Lambda_1 + \Lambda_2 L_0) + \left[\Lambda_2 h^*_D - \frac{1}{f''(h^*_D)}\right]$$

► The term $-(\Lambda_1 + \Lambda_2 L_0)$ represents the impact of a variation in the wage on the supply of wage labor for a given amount of household activity
Labor supply with household production and within the family (3) - Intrafamilial decisions

The family has considerable influence on the behavior of its members. The analysis of family choices has been developed along two different lines:

▶ The unitary approach considers a family composed of two persons whose preferences are presented by $U(C, L_1, L_2)$
▶ This representation reveals that the unitary representation of the household implies that the distribution of non-earned incomes has no importance
▶ This is known as income pooling which signifies that it is not necessary to know which member of the couple is the beneficiary of transfer income
▶ However, this last consequence is questionable
Labor supply with household production and within the family (4) - Intrafamilial decisions

*The collective model*

- This model starts from the principle that household choices must arise out of individual preferences.
- From the empirical point of view, the collective model has the advantage of not adopting the hypothesis of “income pooling.”
- Chiappori (1992) shows that this formulation of the decision-making process within a household allows us to deduct individual consumption, using the individual supplies of labor and the total consumption of the household, which are observable entities.
Labor supply with household production and within the family (5) - Intrafamilial decisions

*The added worker effect*

- Models of intrafamilial choice throw a revealing light on decisions to participate in the labor market
- The members’ choices are interdependent, and an individual’s fluctuations in income will affect not only her own income but also that of her family members
- In principle, a fall in wages may entail an increase in the labor force by spurring additional workers to enter the market
- From the empirical point of view, this *added worker effect* seems to be small
The dynamic theory of labor supply gives a central role to the possibility of substituting for consumption and leisure over time.

A dynamic model of labor supply

- Consumer makes his choice over a “life cycle”
- We assume that the utility function is temporally separable. Hence, it is written:
  \[
  \sum_{t=0}^{T} U(C_t, L_t, t)
  \]
- The influence of past consumption on the utility of the current period is neglected.
- Besides, training increases the human capital and raises the wage-earning prospects, so there must be trade-offs among leisure, working time and time dedicated to training.
In this model, we assume the opportunity to save, with $r_t$ the real interest rate.

The evolution of the assets of the consumer is described by:

$$A_t = (1 + r_t)A_{t-1} + B_t + w_t(1 - L_t) - C_t \quad (9)$$

- $A_t$ designates the consumer’s assets
- $B_t$ designates his income apart from wages

This equation signifies, at each period $t$

- The increase in wealth is due to income from wage labor, $w_t(1 - L_t)$, to income $r_tA_{t-1}$ from savings, and to other income $B_t$
- Consumption $C_t$ for the period has to be deducted from these gains
- The non-earned income $R_t$ for the period $t$ is equal to $B_t + r_tA_{t-1}$
Optimal solutions and demands in Frisch’s sense

- The consumer attempts to maximize his intertemporal utility subject to the budget constraint (9). The FOCs are:
  \[ U_C(C_t, L_t, t) = \nu_t \text{ and } U_L(C_t, L_t, t) = \nu_t w_t \]
  \[ \nu_t = (1 + r_{t+1})\nu_{t+1} \]

- Limiting ourselves to interior solutions, the optimal consumption and leisure are implicitly written as:
  \[ C_t = C(w_t, \nu_t, t) \text{ and } L_t = L(w_t, \nu_t, t) \]

- This equation shows that the supply of labor at date \( t \) depends on the current wage and the multiplier \( \nu_t \), which is the marginal utility of wealth
According to the FOCs, successive iterations of the logarithms of equation of $\nu_t$ entail:

$$\ln \nu_t = -\sum_{\tau=1}^{\tau=t} \ln(1 + r_\tau) + \ln \nu_0$$

A priori, the value of $\nu_0$ depends on all the wages received by an individual during his lifetime.

This model shows that it is useful to distinguish the impact of temporary variations in the wage from the impact of permanent wage variations.
Frischian, Hicksian, and Marshallian Elasticities of labor supply

- The Frischian elasticity represents the impact of a modification of the wage at date $t$ on the supply of labor on the same date, assuming that the marginal utility of wealth remains constant.
  
  - It is useful to measure the impact of a transitory wage variation, which has a negligible impact on wealth.

- Marshallian elasticity measures the total impact of a wage variation on labor supply, taking into account variability in the marginal utility of wealth.

- Hicksian elasticity measures the variation in labor supply, on the assumption that the level of intertemporal utility remains constant.
The relation between the Marshallian, $\eta_M$, and Frischian, $\eta_F$, elasticities is:

$$\eta_M = \eta_F + \frac{wh}{\Omega} \eta_{\Omega} (1 - \gamma \eta_{\Omega}) \quad (14)$$

- $h$ and $w$ represent respectively the labor supply and the wage for the current period
- $\Omega$ represents present intertemporal wealth
- $\gamma = \frac{-V_{\Omega}}{V_{\Omega \Omega \Omega}}$, where $V$ designates the indirect intertemporal utility function. Parameter $\gamma$ corresponds to the elasticity of intertemporal substitution, equal to the inverse of Arrow-Pratt risk aversion

The relation (14) shows that the impact of a wage variation may be broken down into an intertemporal substitution effect, measured by Frischian elasticity, and a wealth effect represented by the other term

One has $\eta_F \geq \eta_H \geq \eta_M$. In the absence of income effect, the three elasticities are identical
Transitory wage changes versus permanent wage changes

We suppose that:

- The real interest rate, \( r \), is constant
- The consumer is receiving no exogenous income
- Her instantaneous utility is:

\[
U(C_t, L_t, t) = (1 + \rho)^{-t} \left( \ln C_t + \frac{\eta}{\eta-1} L_t^{\eta} \right) \quad \eta > 1, \ \rho \geq 0
\]

The Frischian demand functions are given by:

\[
L_t = \left[ \frac{1}{v_0 w_t} \left( \frac{1 + r}{1 + \rho} \right)^t \right]^\eta \quad \text{and} \quad C_t = \frac{1}{v_0} \left( \frac{1 + r}{1 + \rho} \right)^t
\]

The intertemporal budget constraint of the consumer is:

\[
\sum_{t=1}^{T} (1 + r)^{-t} (C_t + w_t L_t) = \sum_{t=1}^{T} (1 + r)^{-t} w_t
\]
The value of $\nu_0$ is defined as:

$$
\sum_{t=1}^{T} (1 + \rho)^{-t} \left\{ 1 + \left[ \left( \frac{1 + r}{1 + \rho} \right)^{-t} \nu_0 w_t \right]^{1-\eta} - \left( \frac{1 + r}{1 + \rho} \right)^{-t} \nu_0 w_t \right\} = 0
$$

The multiplier $\nu_0$ is affected very little by changes in a particular wage (transitory shock). On the other hand, it is affected by a change that affects all wages: what we have then is a modification of the wage profile, or a permanent shock.

A permanent shock (all wage are multiplied by the same scalar) has no impact on labor supply because the income effect and the substitution effect cancel out each other.

Agents adjust their supply of labor in response to temporary changes in wage.
Social security and private pensions

- Most countries in the OECD have put in place pension systems, public and private, enabling workers to receive income when they retire from the labor market.
- Every individual has the opportunity to supplement the public retirement payout with private pensions, contributions to which are negotiated between employer and employee at the moment the labor contract is signed.
- The system of public and private pensions creates incentives for workers to take their retirement earlier or later.
Figure 1.15
Net replacement rate from pensions (as a percentage of individual income), at the level of the median wage.

Option value in the life-cycle model

- We consider a person employed on date $\tau$ who decides to retire on date $s \geq \tau$
- We suppose that the agent does not work at all after date $s$
- Let us denote:
  - $V_\tau(s)$ the welfare of the consumer when he decides to retire at date $s$
  - $T_m$ the legal age of retirement
- The agent chooses his retirement date, denoted $s^*$, by solving:

$$\max_s V_\tau(s) \quad \text{subject to constraint} \quad T_m \geq s \geq \tau$$
Life cycle and retirement (12) - Economic analysis of the decision to retire

- This way of estimating the process of ending one’s working life leads us to examine the *option value* attached to the decision not to take retirement right now.

- Supposing that the decision is irreversible, if $s^* = \tau$, the agent stops working immediately.

- If $s^* > \tau$ the agent continues to work and reconsiders his decision at age $(\tau + 1)$.

- The option value of not retiring today is thus equal to $V_\tau(s^*) - V_\tau(\tau)$.

- The agent continues to work if the option value is positive and retires if it is negative.
Life cycle and retirement (13) - Economic analysis of the decision to retire

*The impact of eligibility rules*

- Gustman et al. (1994) show that individuals with the highest pensions are those who retire soonest. Conversely, workers under financial pressure to put off their retirement do in fact extend their working lives.

- It is possible that, for some reasons of efficiency, firms may offer pension plans that privilege a sooner retirement. Such firms therefore attract workers who have a strong inclination to retire early.

- To eliminate this *endogenous* bias, numerous studies analyze the behavior of workers in the face of *unanticipated* changes in their retirement conditions.
Life cycle and retirement (14) - Economic analysis of the decision to retire

- Lumsdaine et al. (1990) studied the case of an American firm where employees had the opportunity to retire early. They found that the new arrangement more than tripled the rate of leaving of the most advantaged workers.
- The importance of financial incentives, direct or indirect, is confirmed by most research.
- Jones (2001) estimated that pushing the age of eligibility for Medicare back from 65 to 67 years delays retirement by around 27 days between 60 and 69 years of age.
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Summary and conclusion
Estimation of the structural parameters of labor supply model (1)

- The main problem confronting empirical analysis of labor supply is that the correlation between the pertinent financial incentives and the number of hours worked does not necessarily indicate a causal relation.

- The method frequently adopted is to compare the behavior of persons who belong to a “treated” group affected by an exogenous change in their income with the behavior of other persons belonging to a “control” group who are unaffected by the same change.

  - A first approach is to evaluate the impact of the financial incentive, without estimating the parameters of a theoretical model (see chapter 12).
  
  - The second one is to estimate the parameters of a theoretical model, especially the different elasticities of Frisch, Marshall, and Hicks, so as to be able to extrapolate the obtained results to other situations.
Estimation of the structural parameters of labor supply model (2) - Elasticities

The basic equation and the specification of control variables

- The basic equation of empirical models of labor supply relates hours $h_t$ worked by a given individual at hourly wage $w_t$ at each date $t$. It generally takes the form of a double-log-linear relation:

$$\ln h_t = \alpha_w \ln w_t + \alpha_R R_t + x_t \theta + \epsilon_t \quad (20)$$

- $R_t$ is a measure of income other than the current wage
- $x_t$ is a vector of dimension $(1, n)$ describing the $n$ individual characteristics
- $\theta$ is a vector of dimension $(n, 1)$ comprising $n$ parameters to be estimated
- $\alpha_w$ and $\alpha_R$ are also parameters to be estimated
- $\epsilon_t$ designates a random term reflecting individual heterogeneity that is not observed
Estimation of the structural parameters of labor supply model (3) - Elasticities

*Estimating Frischian Elasticity*

- Substituting \( \ln v_0 + \rho t \) for \( R \) in equation (20) and taking this equation in first-differences gives:

\[
\Delta \ln h_t = \rho + \alpha_w \Delta \ln w_t + \Delta x_t \theta + \Delta \varepsilon_t
\]

- This equation allows us to estimate the impact of a transitory change in the wage
- It does not allow us to evaluate the impact of a change in the overall wage profile, because this change causes the marginal utility of wealth to vary
Estimation of the structural parameters of labor supply model (4) - Elasticities

*Estimating Hicksian and Marshallian elasticities*

- We have seen that the optimality of labor supply always implies equality between the marginal rate of substitution between consumption and leisure and the current wage at each date.

- This property suggests a two-stage resolution of the consumer’s program:
  1. In the first stage, the agent maximizes her utility at date $t$ subject to the static budget constraint $C_t = R_t + w_t h_t$, where $R_t = (1 + r_t)A_{t-1} + B_t - A_t$.
  2. In the second stage, the consumer selects the optimal path for her assets $A_t$. 
At the conclusion of the first stage, the consumer attains a level of indirect utility $V(\mathcal{R}_t, t)$

In the second phase, she selects the optimal path for her assets $A_t$ by solving the program:

$$\max \left\{ \sum_{t=0}^{T} V(\mathcal{R}_t, t) \right\} \text{ subject to } \mathcal{R}_t = (1 + r_t)A_{t-1} + B_t - A_t$$

Substituting $R_t = C_t - w_t h_t$ in equation (20) gives

$$\ln h_t = \alpha_w \ln w_t + \alpha_R (C_t - w_t h_t) + x_t \theta + \varepsilon_t$$

We can estimate the Marshallian elasticity, $\alpha_w = \partial \ln h / \partial \ln w$, and the Hicksian elasticity $\eta_H = \alpha_w - \alpha_R w h$
Estimation of the structural parameters of labor supply model (6) - An instructive example of a life-cycle-consistent approach

- The method of ordinary least squares (OLS) is a flawed method
- The endogeneity of wages is an issue: unobserved confounding variables can influence the hours of work and wages
- The same problem applies to the relationship between hours and non-earned income
Estimation of the structural parameters of labor supply model (7) - An instructive example of a life-cycle-consistent approach

- Another approach consists in finding situations with exogenous changes in incomes
- Blundell et al. (1998) applied the life-cycle-consistent approach to married women in the United Kingdom during a period when the tax rates fell
- The fact that some working individuals have been exempt from any direct impact of these reforms due to the progressive nature of the tax system yields the opportunity to construct a suitable control group
Estimation of the structural parameters of labor supply model (8) - An instructive example of a life-cycle-consistent approach

- The basic idea of Blundell et al. (1998) is to net out the endogenous changes from wage variations
- The authors first group the individual data by cohort and education
- They construct group means of hours and wages
- Separately, they calculate the means for each group over all periods and the means for each period over all groups
- Then they subtract these groups and period means from the group means calculated in each period
- After this operation, unobserved time-invariant group factors that could influence wage levels and that could also be related to hour levels are eliminated
Estimation of the structural parameters of labor supply model (9) - An instructive example of a life-cycle-consistent approach

*Estimation of the structural parameter with difference-in-differences*

- We consider the basic semi-log equation which leaves aside non-earned income for notational simplicity:

\[ h_{it} = \alpha + \alpha_w \ln w_{it} + \epsilon_{it} \]

- We imagine that tax reform implemented at date \( t \) affected the treated group and the control group differently and that the effect of the treatment transits through net wages.

- We assume that in the absence of the policy change, the means of hours would have evolved in the same way over time in both groups:

\[ \mathbb{E} [\epsilon_{it} | g, t] = \eta_g + m_t \text{ for all } g \text{ and } t \]  

(A1)

- \( \eta_g \) is a time-invariant group effect and \( m_t \) a period effect common to all groups.
Estimation of the structural parameters of labor supply model (10) - An instructive example of a life-cycle-consistent approach

- Assumption (A1) is known as the common trend assumption
- It assumes that the difference in average labor supply across groups, given the observables, remains unchanged over time
- This assumption is a key identifying assumption, which means that unobserved factors can vary across groups or over time for all groups but cannot vary differently with groups
- Using this assumption, and denoting by $\Delta$ the first difference operator, we get:

$$
\Delta \mathbb{E} [h_{it} | T, t] = \alpha_w \Delta \mathbb{E} \ln [w_{it} | T, t] + \Delta m_t \\
\Delta \mathbb{E} [h_{it} | C, t] = \alpha_w \Delta \mathbb{E} \ln [w_{it} | C, t] + \Delta m_t
$$
Estimation of the structural parameters of labor supply model (11) - An instructive example of a life-cycle-consistent approach

- Assuming that the average change in after-tax wages before and after the reform is different for the treatment group and the control group, the coefficient $\alpha_w$ is given by:

$$
\alpha_w = \frac{\Delta \mathbb{E}[h_{it} \mid T, t] - \Delta \mathbb{E}[h_{it} \mid C, t]}{\Delta \mathbb{E}[\ln w_{it} \mid T, t] - \Delta \mathbb{E}[\ln w_{it} \mid C, t]}
$$

- In this case, the difference-in-differences estimator $\hat{\alpha}_w$, which measures the causal effect of the policy change on hours worked before and hours worked after the introduction of the new policy on those affected by this change compared to those who do not, is

$$
\hat{\alpha}_w = \frac{\Delta h^T_t - \Delta h^C_t}{\Delta \ln w^T_t - \Delta \ln w^C_t}
$$

- An advantage of this estimator is that it deals with measurement errors
Estimation of the Structural parameters of labor supply
model (12) - An instructive example of a
life-cycle-consistent approach

<table>
<thead>
<tr>
<th>Group means:</th>
<th>Uncompensated wage</th>
<th>Compensated wage</th>
<th>Other Income</th>
<th>Group means:</th>
</tr>
</thead>
<tbody>
<tr>
<td>No children</td>
<td>4.493</td>
<td>0.140</td>
<td>0.000</td>
<td>32</td>
</tr>
<tr>
<td>Youngest child 0-2</td>
<td>4.105</td>
<td>0.205</td>
<td>-0.185</td>
<td>20</td>
</tr>
<tr>
<td>Youngest child 3-4</td>
<td>6.686</td>
<td>0.371</td>
<td>-0.173</td>
<td>18</td>
</tr>
<tr>
<td>Youngest child 5-10</td>
<td>2.777</td>
<td>0.132</td>
<td>-0.102</td>
<td>21</td>
</tr>
<tr>
<td>Youngest child 11+</td>
<td>3.260</td>
<td>0.130</td>
<td>-0.063</td>
<td>25</td>
</tr>
</tbody>
</table>
Main results in the literature (1) - Form of labor supply

- The study of Blundell et al. (1992) suggests that an individual’s labor supply takes the form of a hump-shaped curve.
- They used data from the expenditures of British families, focusing on the single mothers and differentiating those who have non-earned income greater than the median of the sample and those who have not. The results are presented in figure 1.16.
Main results in the literature (2) - Form of labor supply

**Figure 1.16**
The labor supply of single mothers.

Source: Blundell et al. (1992).
Main results in the literature (3) - Extensive and intensive margin elasticities

- Many studies found that extensive-margin labor supply elasticity is larger than intensive-margin labor supply elasticity.
- Two reasons explain this result: indivisible labor supply and optimization frictions.

*Indivisible labor supply*

- In this case, changes in tax or wage rates are compatible with large extensive-margin responses, even if they have little effect on hours conditional on employment.
**Main results in the literature (4) - Extensive and intensive margin elasticities**

*Optimization frictions*

- This may arise from organizational constraints internal to the firm, which make the adjustment of hours costly, or create the cost of finding another job better adapted to the worker’s desired timetable.
- Such adjustment costs may lead to underestimate the elasticity of labor supply at the intensive margin.
- To show it, we take the basic model and assume that working entails a fixed cost $F$. 
Main results in the literature (5) - Extensive and intensive margin elasticities

- An individual who works attains a level of utility:
  \[ U[w(L_0 - L) + R - F, L] \]

- The optimal duration of leisure, \( L(w) \) verifies the FOC, \( U_L - wU_C = 0 \)

- The gain from adjusting hours when the wage goes from \( w \) to \( w_1 \) is:
  \[ G_L = U \left[ w_1(L_0 - L(w_1)) + R - F, L(w_1) \right] \]
  \[ - U \left[ w_1(L_0 - L(w)) + R - F, L(w) \right] \]
Main results in the literature (6) - Extensive and intensive margin elasticities

- The approximation of this gain by a first-order Taylor expansion around point \( w_1(L_0 - L(w_1)) + R - F, L(w_1) \) gives:

\[
G_I = [L(w_1) - L(w)] \\
\times \left[ U_L \left[ w_1(L_0 - L(w_1)) + R - F, L(w_1) \right] \\
- w_1 U_C \left[ w_1(L_0 - L(w_1)) + R - F, L(w_1) \right] \right]
\]

- Considering this approximation, Figure 1.17 shows that a wage increase from level \( w \) to \( w_1 \) induces a shift from point \( A \) to point \( B \) if the duration of work remains unchanged at its initial level.

- The utility increases and the gain corresponds to a shift from the point \( B \) to \( C \).
Main results in the literature (7) - Extensive and intensive margin elasticities

**Figure 1.17**
The consequence of a wage increase from $w$ to $w_1$ on the intensive-margin labor supply.
Main results in the literature (8) - Extensive and intensive margin elasticities

- In contrast to the adjustment at the intensive margin, the gains from adjusting hours at the extensive margin are not null.

- Assuming that the individual has an interest in not working at wage \( w \) but has an interest in working at wage \( w_1 > w \), the gain from the shift from non-work to work is given by:

\[
G_E = U \left[ w_1(L_0 - L(w_1)) + R - F, L(w_1) \right] - U(R, L_0)
\]

- A first-order Taylor expansion around point \([w_1(L_0 - L(w_1)) + R - F, L(w_1)]\) gives:

\[
G_E \approx FU_{C} \left[ w_1(L_0 - L(w_1)) + R - F, L(w_1) \right]
\]

- Considering this approximation, Figure 1.18 shows that
  - The gain from shifting from initial point \( A \) to \( C \) may be greater than the second-order gain obtained by shifting from \( B \) to \( C \).
Main results in the literature (9) - Extensive and intensive margin elasticities

Figure 1.18
The consequence of a wage increase from \( w \) to \( w_1 \) on the extensive-margin labor supply.
Main results in the literature (10) - Micro and macro elasticities

- Macro elasticity measures the elasticity of aggregate labor supply at the country level. The “aggregate hours elasticity” is equal to the sum of the extensive and intensive elasticities.

- Chetty et al. (2011b) have summarized the micro and macro evidence on the extensive and intensive margins in the following table 4.

- It shows that the structural micro and macro estimates of Hicksian elasticities match on both margins. Thus, they are consistent with the observed differences in aggregate hours across countries with different tax systems.

- But Frisch elasticities do not: estimates are small when based on micro evidence but large when based on macro studies.
Main results in the literature (11) - Micro and macro elasticities

<table>
<thead>
<tr>
<th></th>
<th>Intensive Margin</th>
<th>Extensive Margin</th>
<th>Aggregate Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State (Hicksian)</td>
<td>micro 0.33</td>
<td>0.26</td>
<td>0.59</td>
</tr>
<tr>
<td>Steady State (Hicksian)</td>
<td>macro 0.33</td>
<td>0.17</td>
<td>0.50</td>
</tr>
<tr>
<td>Intertemporal</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Substitution (Frisch)</td>
<td>micro 0.54</td>
<td>0.28</td>
<td>0.82</td>
</tr>
<tr>
<td>Substitution (Frisch)</td>
<td>macro [0.54]</td>
<td>[2.30]</td>
<td>2.84</td>
</tr>
</tbody>
</table>

Table 4: Micro vs. Macro Labor Supply Elasticities. Each cell shows a point estimate of the relevant elasticity based on meta analyses of existing micro and macro evidence. Micro estimates are identified from quasi-experimental studies; macro estimates are identified from cross-country variation in tax rates (steady state elasticities) and business cycle fluctuations (intertemporal substitution elasticities). The aggregate hours elasticity is the sum of the extensive and intensive elasticities. Macro studies do not always decompose intertemporal aggregate hours elasticities into extensive and intensive elasticities. Therefore, the estimates in brackets show the values implied by the macro aggregate hours elasticity if the intensive Frisch elasticity is chosen to match the micro estimate of 0.54.
Source: Chetty et al. (2011, Table 1, p. 2).
Main results in the literature (12) - Micro and macro elasticities

Theorists and empirical researchers are left with two possibilities: either the micro estimates are based on models that overlook important factors that could increase elasticities, or macroeconomists of the business cycle do not have the right model available, one that would describe economic fluctuations consistently with observed agents’ behavior.
Main results in the literature (13) - The elasticity of labor supply of men and women

- Labor supply elasticities for men are very small and insignificant, whereas that of women are somewhat larger.
- According to the theoretical models, these results indicate that, within the household, fiscal reforms affect principally the participation decisions of women, since they have lower wages than those of men and in all likelihood possess a comparative advantage when it comes to household production.
- The difference between the elasticities of the labor supplied to the market by men and by married women is explained by the fact that women’s labor is regarded as more substitutable for domestic work than that of men, especially when the woman is less qualified than her spouse.
Main results in the literature (14) - The cost of leisure and the productivity of home production

- In the basic models of labor supply, the cost of leisure is measured as an opportunity cost, that is as forgone wages.
- Gonzalez-Chapela (2007) obtains an intertemporal elasticity of labor supply of 0.16 to the price of leisure goods and 0.25 to wages for working-age men.
- This means, for a man working 2,000 hours per year, that a fall of 1% in the price of leisure goods would prompt a fall of 3.2 hours in the length of time worked annually.
- An exogenous source of variation, such as the weather, could also derive changes in the utility:
  - When the weather is good, the opportunity cost of working is greater than it is on days when it rains. For the US, Connolly (2008) finds that men reduce their investment in leisure time by 30 minutes on rainy days so that they can work longer.
Main results in the literature (15) - Micro and macro elasticities

- The impact of children on the working women is negative
- Bloom et al. (2009) estimate the effect of fertility on female labor force participation
  - They find that removing legal restrictions on abortion significantly reduces fertility and that a birth reduces a woman’s labor supply by almost 2 years
- The ability to control birth timing is also an important factor that probably bolstered female participation
- The availability of child care is another possible factor that may explain the female employment rate
Summary and conclusion (1)

- According to the neo-classical theory of labor supply, every individual trades off between consuming a good and consuming leisure.

- The supply of individual labor is positive if the current wage exceeds the reservation wage.
  - If labor supply is positive, the marginal rate of substitution between consumption and leisure is equal to the hourly wage.

- The relation between the individual supply of labor and the hourly wage is the result of combined substitution and income effects.

- The substitution effect implies an increasing relation between the wage and labor supply, while the income effect works in the opposite direction if leisure is a normal good.
Summary and conclusion (2)

- When an individual has the opportunity to devote a part of her time to household production, at the optimum, the hourly wage is equal to the marginal productivity of household work.

- As a general rule, the mechanism of substitution of leisure over time implies that the permanent component of the evolution of real wages has a smaller effect on labor supply than the transitory component.

- The elasticity of labor supply by women is, in general, greater than that of men, which is generally small, although this difference diminishes over time.