Exchange rates and monetary spillovers*

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Abstract

When does the combination of flexible exchange rates and inflation-targeting monetary policy guarantee insulation from global financial conditions? We examine a dynamic global game model of international investment flows where, for some combination of parameters, the unique equilibrium exhibits the observed empirical feature of prolonged episodes of capital inflows and appreciation of the domestic currency, followed by abrupt reversals where capital outflows go hand-in-hand with currency depreciation, a domestic bond market crash, and inflationary pressure.

Keywords: Financial crises, global games

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1 Introduction

The flexible exchange rate version of the Mundell-Fleming model (Fleming (1962), Mundell (1963)) lays out the case for how flexible exchange rates allow monetary authorities to pursue domestic macroeconomic objectives in a world of free capital flows. Post-crisis discussions of monetary spillovers have revisited this classic proposition. The BIS report on global liquidity (BIS 2011) is a recent exposition of cross-border monetary spillovers and it has been followed by an active literature which has examined the extent to which floating exchange rates fail to insulate monetary policy from external developments (see, for instance, Agrippino and Rey (2015), Rey, (2013, 2015), Bruno and Shin (2015a, 2015b)).

The broad picture emerging from this literature is that of a significant global comovement in leverage and asset prices that is related to US monetary policy. This global factor is associated with surge and sudden stops in capital flows, and with large exchange rate fluctuations that deviate from uncovered interest parity (UIP).

This paper offers a theoretical model in which the differential between the interest rate on an international funding currency and that of a small open economy generates excessive fluctuations in leverage, bond prices, and inflation in the small open economy. This instability is generated by the capital flows of global investors seeking to reap self-justified rents from carry trades. The rents are self-justified in the sense that large capital inflows (outflows) generate positive (negative) abnormal returns on carry trades that justify the flows in the first place.

We contribute to the theoretical literature on self-fulfilling international crises pioneered by Obstfeld (1996) along two dimensions. First, we write down a fully dynamic coordination game among global investors in infinite horizon in which investors' beliefs about each others' future positions are uniquely determined along the equilibrium path. Shocks to the interest-rate differential serve as their coordination device and affect their collective beliefs in a highly non-linear fashion. This way, we show that coordination games are not only useful to model snapshot crisis episodes, but can also help understand the protracted build-up of financial fragility that precedes them. Second, we embed this coordination game in a simple but standard monetary model of a small open economy. This enables us to identify the set of primitive parameters of the domestic economy under which it lends itself to such destabilizing speculation.

We proceeds in two steps. We first couch the novel economic mechanism through which global investors' portfolio choice generates monetary spillovers in a small open economy in the simplest possible environment: a perfect-foresight model. Monetary spillovers stem from two ingredients. First, the central bank in the small open economy uses an interest-rate rule that responds to global investors' inflows only insofar as they affect the price level, but that does not track their direct impact on asset prices (and thus on the real rate). As a result inflows (outflows) are deflationary (inflationary). The second ingredient is the assumption (borne out by the data) that the non tradable goods of the small economy have more rigid prices than the tradable ones. This implies that the inflationary impact of capital flows must operate through the prices of tradable goods, and thus leads to large fluctuations in the nominal exchange rate. We show that for reasonable parameters of the model, there are two stable steady-state solutions — one associated with capital inflows and the other with capital outflows. The steady state with capital inflows is associated with an appreciation of the domestic currency and a failure of uncovered interest parity (UIP) yielding an abnormal positive return on carry trades, and the steady state with capital outflows is its mirror image.

In the second step of our analysis, we build a stochastic version of this perfect-foresight model. We introduce exogenous shocks to the dollar interest rate and use the global-game techniques of Burdzy, Frankel, and Pauzner (2000, 2001) to refine the outcome of the model to a unique solution. We show that the state space can be partitioned into two regions— a region where all global investors pile into the local currency bond, and one in which they short these bonds to be long US dollar-denominated assets. The transition between the two regions can be triggered by small fluctuations in the US dollar interest rate and the endogenous changes in domestic financial conditions. The unique equilibrium features dynamics that are reminiscent of boom-bust cycles. An easing of US monetary conditions typically creates a prolonged episode of capital inflows, benign domestic financial conditions, and appreciation of the currency for the small economy. Subsequent small increases in the US rate do not immediately reverse the up-phase of the cycle but will abruptly reverse it when a "tantrum" boundary is reached. Hitting the "tantrum" boundary triggers a large currency depreciation, capital outflows, a crash in the domestic bond market, and inflationary pressure. These features are reminiscent of that experienced by a number of emerging economies during the 2013 "taper tantrum" episode that followed the announcement of a possible tapering of the highly accommodative US monetary policy.

Related Literature

Our approach is most closely related to models of financial instability which involve coordination problems and self-fulfilling speculative episodes. In a similar spirit, Farhi and Tirole (2012) and Schneider and Tornell (2004) offer related models of "collective moral hazard" in which the government bails out speculators if their aggregate losses are sufficiently large, thereby inducing a coordination motive among speculators. We formalise the dynamic coordination game among investors using the dynamic extension of global-game methods developed by Frankel and Pauzner (2000) and Burdzy, Frankel, and Pauzner (2001) to obtain a unique equilibrium outcome. We show that these global-game tools can be adapted to the situation where coordination motives coexist with congestion effects. This is important because most financial models with coordination motives also feature congestion effects. In a model of bank run, Goldstein and Pauzner (2004) adapt static global-game techniques to the case in which strategic complementarities similarly fail to hold everywhere. In a model of sovereign-debt refinancing, He, Krishnamurthy, and Milbradt (2015) also apply global-game techniques in a context in which a large debt size comes at the benefit of smaller congestion effects but at the cost of a higher rollover risk.

Most closely related to our work, He and Xiong (2012) apply the equilibrium selection techniques developed by Burdzy, Frankel, and Pauzner (2001) in a dynamic financial context — the roll-over of short-term debt.

We also relate to the theoretical literature that seeks to model both crises and the build up of fragility that precedes them. Lorenzoni (2008) builds a model in which commitment problems on both lending and borrowing sides lead to excessive borrowing ex-ante and excessive volatility ex-post. Sannikov (2014) endogenizes the build up of fragility by assuming that two assets are available, an illiquid one that generates more with experts and less with households than a liquid one. A key driver is that the illiquidity of an asset (the value it generates when operated by households) has no impact on the steady-state target leverage of experts, so that endogenous illiquidity risk taking by experts in quiet times alone can lead to large crises even absent large fundamental risk.

Our paper also relates to the literature on portfolio choice in incomplete markets. In a recent contribution, Gabaix and Maggiori (2015) introduce financial intermediaries that operate in incomplete global financial markets by intermediating gains from trade between countries. We also model global investors as financial institutions operating in incomplete markets.[1] Garleanu, Panageas and Yu (2015) present a model in which investors face costs to extend their participation in markets located on a circle for diversification purposes. Our result that the profitability of investment increases in the weight of others' participation bears similarities with their finding that participation and leverage reinforce each other, possibly leading to multiple equilibria.

The relationship between exchange rates and leverage is our point of contact with the literature on the determinants of vulnerability to financial crises. Gourinchas and Obstfeld (2012) conduct an empirical study using data from 1973 to 2010 and find that two factors emerge consistently as the most robust and significant predictors of financial crises, namely a rapid increase in leverage and a sharp real appreciation of the currency. The build up of leverage and currency appreciation also lead to a higher probability of future sharp deleveraging and important capital outflows in our model. Schularick and Taylor (2012) similarly highlight the role of leverage in financial vulnerability, especially that associated with the banking sector.

Finally, our results complement the recent work on the *risk-taking channel of currency appreciation*, introduced by Bruno and Shin (2015a, 2015b) in the context of cross-border banking, whereby currency mismatches on borrowers' balance sheets lead to credit supply effects of exchange rate fluctuations. The risk-taking channel relies on Value-at-Risk (VaR)-induced behaviour that is sensitive to tail risks of credit portfolios. Hofmann, Shim and Shin

[1] We discuss interesting differences between their conclusions and ours below in Section 2.4.

(2015) apply the risk-taking channel to domestic currency sovereign yields through shifts in the tail risk of diversified local currency sovereign bond portfolios. Whereas existing models of the risk-taking channel are static, our global game model solves for the dynamic path of the key macro variables.

2 A simple perfect-foresight model

Time is discrete and is indexed by t. There are two types of agents, households populating a small open economy and global investors. There is a single tradable good that has a fixed unit price in US dollars.

2.1 Households

The households live in a small open economy. They use a domestic currency that trades at S_t dollars per unit at date t, where the exchange rate S_t will be determined in equilibrium.

At each date, a unit mass of households are born. Households live for two dates, consume when young and old, and work when old. Each household receives an initial endowment at birth with nominal value $P_t W \geq 0$, where P_t is the domestic price level.[2] The cohort that is born at date t has quasi-linear preferences over bundles of consumption and labor (C_t, C_{t+1}, N_{t+1})

$$U\left(C_t, C_{t+1}, N_{t+1}\right) = \ln C_t + \frac{C_{t+1} - N_{t+1}^{1+\eta}}{R}, \tag{1}$$

where $\eta > 0$ and $R > 1$ is the subjective discount rate.

Domestic consumption services C_t are produced combining the tradable good C_t^T and two nontradable goods $C_t^{N_1}$ and $C_t^{N_2}$ according to the technology

$$C_t = \frac{\left(C_t^T\right)^\alpha \left(C_t^{N_1}\right)^\beta \left(C_t^{N_2}\right)^\gamma}{\alpha^\alpha \beta^\beta \gamma^\gamma}, \tag{2}$$

where $\alpha, \beta, \gamma \in (0,1)$ and $\alpha+\beta+\gamma = 1$. Domestic firms set by old households use labor input to produce. Due to quasi-linear preferences, our results do not depend on the specification

[2] If the endowment of young households is zero, then the global investors introduced below cannot have an aggregate short position in domestic bonds. A strictly positive endowment plays no other role than allowing such short positions.

of the firms' production functions. All that is needed is that both nontradable goods are produced in finite, non zero quantities at each date. Households collect labor income and the profits from their firms when old.

Nominal rigidities. An important ingredient of the model is that the prices of the nontradable goods are less flexible than that of the tradable good.[3] We formalize this as follows. First, we posit that the tradable good has a flexible price P_t^T in the domestic currency, and that the law of one price holds ("PPP at the docks"). Second, the first nontradable good N_1 also has a fully flexible price $P_t^{N_1}$. A linear technology enables the transformation of each date-t unit of N_1 into F units of the tradable good, where $F > 0$. The second nontradable good N_2 has a fixed price that we normalize to 1 without loss of generality. Denoting P_t^T, $P_t^{N_1}$, and $P_t^{N_2}$ the respective prices of these three goods, we have:

$$P_t^T S_t = 1, \tag{3}$$

$$P_t^{N_1} = F P_t^T, \tag{4}$$

$$P_t^{N_2} = 1. \tag{5}$$

Equation (3) is the statement of the law of one price. Equation (4) states that domestic households are indifferent between purchasing the tradable good or producing it out of N_1. Finally, (5) is the statement of the rigidity of N_2's price.

The introduction of the nontradable good with flexible price N_1 allows us to decouple the "openness" of the economy as measured by $\alpha = 1 - \beta - \gamma$ from the flexibility of the overall price index to changes in the tradable goods price, as measured by $1 - \gamma$. As is well-known, optimal spending across goods implies that prices must satisfy

$$\begin{aligned} P_t &= (P_t^T)^\alpha (P_t^{N_1})^\beta (P_t^{N_2})^\gamma \\ &= \left(P_t^T\right)^{1-\gamma} F^\beta, \end{aligned} \tag{6}$$

where (6) follows from (4) and (5).

Households have access to the domestic bond market, in which risk-free one-period bonds denominated in the domestic currency are available in zero net supply. The nominal interest

[3]This is consistent with evidence documented by Burstein, Eichenbaum, and Rebelo (2005).

rate on these bonds is set by the domestic central bank according to a rule to be described below.

2.2 Global investors

A unit mass of global investors have access to both the local-currency bond market and to US dollar-denominated one-period bonds. The exogenous nominal return on US dollar bonds is denoted by $I^* > 0$. Global investors consume outside the local economy, and their utility is increasing in the consumption of the tradable good.[4]

In forming their financial portfolios, global investors face limits on the size of their exposures in domestic bonds, reflecting leverage constraints or exposure caps imposed by internal risk limits.[5] We assume that the position in domestic bonds of any investor must lie in the interval $[P_t L^-, P_t L^+]$, where these limits are denominated in the domestic currency and

$$L^- > -W,$$

which ensures that households always consume positively.[6]

The return to a global investor from investing in the local currency bond market relative to the return on dollar bonds is given by

$$\Theta_{t+1} = \frac{S_{t+1}}{S_t} \frac{I_{t+1}}{I^*} \tag{7}$$

We may interpret Θ_{t+1} as the return to a carry-trade position in which the investor borrows dollars at rate I^* and then invests the proceeds in the local currency bond yielding I_{t+1}. Uncovered interest parity (UIP) holds when $\Theta_{t+1} = 1$.

We denote $L_t \in [L^-, L^+]$ the real net aggregate borrowing by young households from global investors at date t (possibly negative). Since the economy is deterministic, optimal

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[4]Whether they also derive utility from consuming other goods, and the curvature of their utility function are immaterial. This is only true because the economy is deterministic, and will no longer be so in Section 3.

[5]We do not consider the microfoundations of these limits. As is well-known, they could result from example from agency problems within globally investing firms such as, for example, a cash-flow diversion problem.

[6]Setting lending limits in real terms simplifies the exposition but is not crucial. Nominal rigidities in trading limits would actually amplify our results.

portfolio choice by global investors implies that L_t must satisfy:

$$L_t \begin{cases} = L^+ & \text{if } \Theta_{t+1} > 1, \\ = L^- & \text{if } \Theta_{t+1} < 1, \\ \in (L^-, L^+) & \text{if } \Theta_{t+1} = 1. \end{cases} \tag{8}$$

In words, global investors choose corner portfolios unless they are indifferent between investing in US-dollar denominated assets or in domestic bonds.

2.3 Monetary policy rule

We suppose that the domestic monetary authority sets the nominal interest rate between t and $t+1$, I_{t+1}, following the interest-rate feedback rule:

$$I_{t+1} = R \left(\frac{P_t}{P_{t-1}} \right)^{1+\Phi} \tag{9}$$

where

$$\Phi > 0 \tag{10}$$

The interest rate rule (9) follows the Taylor principle from (10) in that the nominal interest rate reacts more than one-for-one to the price level change. The Taylor principle is necessary for a determinate solution in many classes of monetary models (Taylor, 1993; Woodford, 2001), and our model also shares this feature as we see below. Setting the target inflation rate to zero is only a normalization.

Given our quasi-linear preferences, households' Euler equation can be written as

$$I_{t+1} = \frac{P_{t+1}}{P_t} \frac{R}{(L_t + W)} \tag{11}$$

2.4 Steady-state solution

We are now equipped to solve for the perfect-foresight steady-states of this economy. A steady state must be such that the domestic economy is in equilibrium and global investors form optimal portfolios at each date. Formally, a perfect-foresight steady-state is a solution to (3), (6), (8), (9), and (11).

We introduce the following notation.

$$\begin{aligned}
r &= \ln R, \\
\delta &= \ln\left(\frac{R}{I^*}\right), \\
\theta_t &= \ln \Theta_t, \\
i_t &= \ln I_t, \\
s_t &= \ln S_t, \\
l_t &= \ln(L_t + W), \\
\pi_{t+1} &= \ln\left(\frac{P_{t+1}}{P_t}\right).
\end{aligned}$$

The Euler equation (11) and the interest-rate rule (9) can be gathered as follows:

$$i_{t+1} = r - l_t + \pi_{t+1}, \tag{12}$$

$$i_{t+1} = r + (1+\Phi)\,\pi_t. \tag{13}$$

Together, they define a linear-difference equation for the path of inflation:

$$\pi_t = \frac{\pi_{t+1} - l_t}{1+\Phi} \tag{14}$$

which has a unique non-exploding solution:

$$\pi_t = -\sum_{k\geq 0} \frac{l_{t+k}}{(1+\Phi)^{k+1}}. \tag{15}$$

Equation (15) shows that current inflation is affected by current and future capital inflows $(l_{t+k})_{k\geq 0}$. The Taylor principle ensures that π_t is well-defined, since l_{t+k} is bounded and $\Phi > 0$.

This expression for π_t highlights that our model shares the generic feature of standard interest-rule based monetary models that inflation reflects anticipated future "shocks." In our context, the "shocks" are not the usual exogenously assumed policy shocks, but rather are the consequence of optimal portfolio choice by global investors.

Using (6) and (3), we have:

$$\pi_{t+1} = -(1-\gamma)\left(s_{t+1} - s_t\right). \tag{16}$$

Equation (16) is our exchange rate pass-through equation that expresses inflation in terms of exchange rate depreciation.

Plugging (16) and (11) in (7), we have

$$\begin{aligned} \theta_{t+1} &= s_{t+1} - s_t + i_{t+1} - \ln I^*, \\ &= -\frac{1}{1-\gamma}\pi_{t+1} + \pi_{t+1} - l_t + \delta, && (17) \\ &= \frac{\gamma}{1-\gamma}\sum_{k\geq 0}\frac{l_{t+k+1}}{(1+\Phi)^{k+1}} - l_t + \delta. && (18) \end{aligned}$$

where (17) follows from (16) and the Euler equation, and (18) follows from the solution for π_{t+1} given by (15).

We now determine the steady states in which the debt level l is constant over time. We introduce

$$\underline{l} \equiv \ln(W + L^-), \quad (19)$$

$$\bar{l} \equiv \ln(W + L^+). \quad (20)$$

For brevity the remainder fo the paper focusses on the case in which

$$\underline{l} < 0 < \bar{l}.$$

Proposition 1 ***(Multiplicity of steady-states)*** *Suppose there exists $l^* \in (\underline{l}, \bar{l})$ such that*

$$\frac{\gamma - \Phi(1-\gamma)}{(1-\gamma)\Phi} l^* + \delta = 0.$$

Then $l = l^*$ is a steady state in which uncovered interest parity (UIP) holds. If $\Phi(1-\gamma) > \gamma$, there is no other steady-state solution. However, if $\Phi(1-\gamma) < \gamma$, there are two further steady-state solutions; there is a steady state with maximum capital inflows ($l = \bar{l}$), and there is a steady state with maximum capital outflows ($l = \underline{l}$).

Proof of Proposition 1. First, note that for l fixed, the relative return to investing in the local currency bond given by (18) can be written as

$$\theta = \frac{\gamma - \Phi(1-\gamma)}{(1-\gamma)\Phi} l + \delta \quad (21)$$

Let $l^* \in (\underline{l}, \bar{l})$ be such that $\theta = 0$. For such an l^* investors are indifferent between investing in the local currency bond or the dollar bond. Hence, l^* is a steady-state solution of our model. If $\Phi(1-\gamma) > \gamma$, this is the only steady-state solution, since θ is decreasing in l: More (less) foreign lending makes foreign lending unprofitable (profitable). This case corresponds in particular to the fully flexible benchmark (γ=0).

Now consider the case where $\gamma > \Phi(1-\gamma)$. In this case, we have two further steady-state solutions corresponding to the corner solutions $l = \bar{l}$ and $l = \underline{l}$.

First, suppose that all investors choose to invest in the local currency bond. Then $l = \bar{l}$, so that $\theta > 0$, implying that investing in the local currency bond is strictly better than investing in the dollar bond. Hence, all investors invest in the local currency bond, thereby sustaining maximum inflows $l = \bar{l}$ as a steady-state solution. Conversely, suppose that all investors choose to invest in the dollar bond. Then $l = \underline{l}$, so that $\theta < 0$, implying that investing in the dollar bond is strictly better. Hence all investors invest in the dollar bond, sustaining $l = \underline{l}$ as a steady state. ■

Proposition 1 highlights the possibility of both self-fulfilling capital flow surges and outflows as extremal steady-state solutions of our model. These steady-states correspond to binding risk limits for global investors, failure of UIP ($\theta \neq 1$), and off-target inflation.

The intuition behind the multiplicity of steady states is as follows. First, as is transparent from relation (15), capital inflows push local bond prices up and this compression in yields leads inflation to be below target. Second, since the prices of nontradable goods are relatively stickier than that of the tradable good, this deflationary impact must operate relatively more through the prices of tradable goods, and thus through a large appreciation of the nominal exchange rate, as formalized by (16). If these two effects are sufficiently important, then capital inflows generate a sufficiently large exchange rate appreciation that this more than compensates global investors for holding expensive local bonds. This yields arbitrage profits, and the anticipation of future large capital inflows in the small open economy is self-fulfilling. The model is essentially symmetric, so everything also works the mirror-image way in the steady-state with extremal outflows.

It is interesting to contrast this mechanism with that in Gabaix and Maggiori (2015).

In their setup, tighter financial constraints lead to larger excess returns on carry trades because it forces global investors to leave carry-trade profits on the table. Here, conversely, if one interprets tighter financial constraints as narrower trading limits, then such tighter constraints lead to smaller excess returns on carry trades. This is because these excess returns are generated by the destabilizing impact of foreign capital flows in or out of the small economy. This impact increases in the size of the carry trade.

The condition $\gamma > \Phi(1-\gamma)$ is satisfied when nominal rigidities are sufficiently important (γ sufficiently large) and monetary policy sufficiently passive (Φ sufficiently small). Both conditions imply that small changes in inflation expectations are consistent with large swings in the nominal exchange rate.[7] Otherwise stated, the domestic monetary authority could eliminate extremal steady-states by committing to a sufficiently large Φ, which means committing to a sufficiently large reduction in the policy rate in the presence of large capital inflows and/or large appreciation of the exchange rate.

Remark on the monetary rule. The reader may have noticed that the interest-rate rule (10) does not fully track the impact of foreign flows on domestic bond prices and thus on the real rate. By doing so, a rule of the form

$$i_{t+1} = r - l_t + (1+\Phi)\,\pi_t \tag{22}$$

would eliminate the effect of inflows on the domestic CPI and thus on carry-trades returns. We find it realistic, however, to assume that the flows of "hot money" are more nimble than the domestic monetary authority. In addition, we believe that the extremal steady states could still arise under a rule such as (22) in a richer model of nominal rigidities.[8]

The purpose of this perfect-foresight analysis is to present our novel mechanism for self-justified destabilizing capital flows in the simplest and most transparent environment. Yet this perfect-foresight analysis raises two obvious isses:

[7]The standard assumption $\Phi = 0.5$ implies that extremal steady-states can occur as soon as $\gamma > 1/3$, corresponding to a fairly low level of nominal rigidities.

[8]Suppose for example that the consumption boom due to inflows results in inflation in the price of nontradable goods through overheating. If rule (22) maintains the overall price index constant, then this inflationary pressure must be compensated with deflation in the price of the tradable good. Such a deflation, if sufficiently large, will again create room for self-fulfilling excess returns on carry trades through the associated increase in the nominal exchange rate.

- The multiplicity of steady-states leaves unclear how agents can coordinate on any equilibrium behavior at all.
- If carry traders hold the same position forever, then the prices of non tradables and the real-rate target of the central bank should eventually adjust.

We now turn to a stochastic version of our benchmark model and employ global-game techniques to tie down a unique dynamic solution that solves both issues. The goal of the analysis is to provide the theoretical foundations of the dynamics of an open economy in which surges of capital inflows can be explained alongside the sudden reversals that happen in practice.

3 Stochastic model

We will proceed to develop a stochastic version of our model, and then solve for the uniquely determined time paths by using perturbation methods that resemble global-game methods, but which are better suited for dynamic contexts.

Begin by assuming that time is continuous. The fixed integer dates of the previous section are replaced by the arrival times of a Poisson process with intensity $\mu > 0$. Namely, at each arrival time T_n, a new cohort of households are born, and die at the next arrival time T_{n+1}. They value consumption and leisure only at these two dates, with preferences that are the same as that in our benchmark set-up:[9]

$$\ln C_{T_n} + \frac{1}{R} E_{T_n} \left[C_{T_{n+1}} - N_{T_{n+1}}^{1+\eta} \right].$$

At each arrival date T_n, the central bank sets a nominal rate $I_{T_{n+1}}$ between T_n and T_{n+1} according to the rule:

$$I_{T_{n+1}} = R \left(\frac{P_{T_n}}{P_{T_{n-1}}} \right)^{1+\Phi} \tag{23}$$

[9]The unconventional assumption that households discount consumption at a random future date at a fixed discount factor R greatly simplifies the algebra but plays no other role. Accordingly, we will define the interest rate on US and local bonds as a fixed coupon between two arrival dates.

Replacing integer dates with dates that arrive at a constant rate is not essential, and our set-up is designed for tractability. It is designed to ensure that the global investors' portfolio choice problem described below is time homogeneous.

We now are more specific than in the previous section about global investors' preferences in order to characterise their portfolio choice. We follow Gabaix and Maggiori (2015) and model global investors as a unit-mass of financial institutions that can form zero-cost portfolios in bonds denominated in either currency at each date T_n with size within $[P_{T_n}L^-, P_{T_n}L^+]$ (in units of the domestic currency). The date-T_n trade is unwound at the next date T_{n+1} and the realised profit or loss is paid to the old households at this date. Each firm maximises the expected value of future consumption paid to all future households discounted at the households' subjective discount rate R between two arrival dates. We still suppose that

$$\ln(L^- + W) \equiv \underline{l} < 0 < \ln(L^+ + W) \equiv \bar{l}. \tag{24}$$

The two following modifications to the benchmark model are key to generate equilibrium uniqueness.

Shocks to the US dollar rate. First, we assume that the interest rate on US dollar-denominated bonds between two arrival dates T_n and T_{n+1} is given by

$$I^*_{T_{n+1}} = R\left(1 - w_{T_n}\right), \tag{25}$$

where w_t is a Wiener process with volatility σ and no drift.[10]

Imperfect liquidity. Second, we assume that the capital market is imperfectly liquid in the following sense. Each investor can revise his investment strategy only at switching dates that are generated by a Poisson process with intensity $\lambda > 0$. These switching dates are independent across investors. In between two switching dates, each global investor commits to a strategy and thus to lend to (or borrow from) households a committed amount within $[P_{T_n}L^-, P_{T_n}L^+]$ at each arrival date T_n (if any).

This form of execution uncertainty has a key property that will yield equilibrium uniqueness: Every investor knows that some other investors will revise their trading strategy almost surely between his current switching date and the next one.

[10] We will discuss more general stochastic processes below. Proofs are simpler in the case of a standard Wiener process.

Remark. ***(Arbitrage versus good deal and the role of trading limits)*** It is important to stress that the exogenous trading limits $[P_{T_n}L^-, P_{T_n}L^+]$ fulfill a very different role from that played in Section 2. In the previous section, it was necessary to impose such limits regardless of global investors' preferences because carry trades were (possibly) textbook arbitrage opportunities given the deterministic environment. As is well-known, any agent with increasing utility over consumption has an infinite demand for an arbitrage opportunity absent any financial constraint. In this stochastic environment, we will see that carry-trade portfolios generate losses with a non-zero probability in equilibrium. Thus, any risk-averse agent would form finite portfolios. Trading limits here only play the role of a very tractable substitute for risk aversion that is commonplace in models in which agents' attitude towards risk is not the main focus.[11] An interesting extension consists in studying risk limits that vary with exchange rate movements, as is the case in practice (see Hofmann, Shim and Shin (2015)).

Local risk-neutrality implies that global investors choose corner portfolios. We deem "long" a global investor who committed to maximum lending L^+ at his last switching date, and "short" one who committed to the maximum borrowing L^-. We let x_t denote the fraction of long global investors at date t. Note that the paths of the process $(x_t)_{t\in\mathbb{R}}$ must be Lipschitz continuous, with a Lipschitz constant smaller than λ. The aggregate real net lending L_{T_n} taking place at an arrival date T_n is then equal to

$$L_{T_n} = x_{T_n}L^+ + (1 - x_{T_n})L^-. \tag{26}$$

It corresponds to stopping the process of the (real) aggregate committed amount by global investors L_t at the arrival dates T_n at which this amount is actually lent (or borrowed).

Suppose that a global investor has a chance to revise his position at a date t such that

$$T_{n-1} < t < T_n. \tag{27}$$

Denoting T_λ his next switching date, the expected unit return from the carry trade — the

[11] In recent work, Albagli, Hellwig, and Tsyvinski (2015) use a similar assumption to generate a high tractability and thus new insights in a standard noisy REE asset-pricing model. The binary portfolio choice of risk-neutral carry traders is particularly suited to our iterated-dominance solution.

expected value from committing to lend one additional real unit to each future cohort until T_λ — is

$$\Pi_t = E_t \left[\sum_{m \geq 0} \frac{1_{\{T_\lambda > T_{n+m}\}}}{R^m} \frac{P_{T_{n+m}} S_{T_{n+m}}}{P_{T_{n+m+1}} S_{T_{n+m+1}}} \left(\frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{S_{T_{n+m}} R} - 1 + w_{T_{n+m}} \right) \right]. \tag{28}$$

Expression (28) states that the global investor earns the carry-trade return associated with each arrival date until he gets a chance to revise his position.[12]

The evolution of the economy is then fully described by two state variables, the exogenous state variable w_t and the endogenous state variable x_t. The exogenous state variable w_t directly affects only the expected return on carry trade Π_t while the endogenous one x_t directly affects both the carry trade return and the equilibrium variables $(L_{T_n}, I_{T_n}, P_{T_n}, S_{T_n})$ of the domestic economy. We are now equipped to define an equilibrium.

An equilibrium is characterized by a process x_t that is adapted to the filtration of w_t and has Lipschitz-continuous paths such that:

[12]To arrive at (28), note that investing one real unit at arrival date T_j costs USD $P_{T_j} S_{T_j}$. The net rate of return $(S_{T_{j+1}}/S_{T_j}) I_{T_{j+1}} - I^*_{T_{j+1}}$ applies to this dollar amount. The resulting consumption for old households at date T_{j+1} is then this USD profit divided by $P_{T_{j+1}} S_{T_{j+1}}$. Re-arranging and discounting these terms yields (28).

$$I_{T_{n+1}} = R\left(\frac{P_{T_n}}{P_{T_{n-1}}}\right)^{1+\Phi}, \tag{29}$$

$$E_{T_n}\left[\frac{I_{T_{n+1}}P_{T_n}}{P_{T_{n+1}}}\right] = \frac{R}{L_{T_n}+W}, \tag{30}$$

$$\frac{S_{T_{n+1}}}{S_{T_n}} = \left(\frac{P_{T_{n+1}}}{P_{T_n}}\right)^{\frac{-1}{1-\gamma}}, \tag{31}$$

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \text{if } \Pi_t < 0, \\ \lambda(1-x_t) & \text{if } \Pi_t > 0, \end{cases} \tag{32}$$

where

$$\Pi_t = E_t\left[\sum_{m\geq 0}\frac{1_{\{T_\lambda > T_{n+m}\}}}{R^m}\frac{P_{T_{n+m}}S_{T_{n+m}}}{P_{T_{n+m+1}}S_{T_{n+m+1}}}\left(\frac{S_{T_{n+m+1}}I_{T_{n+m+1}}}{S_{T_{n+m}}R} - 1 + w_{T_{n+m}}\right)\right].$$

Exactly as in the perfect-foresight case, equilibrium in the domestic economy is characterized by the Taylor rule (29), households' Euler equation (30), and the pass-through equation (31). Equation (32) states that global investors make optimal portfolio choices. They become long at switching dates at which the expected return on the carry trade is positive (or remain long if this was their previous positions), and short if this is negative (or remain short if this was their previous positions).

Note that relations (29) and (31) are identical to their counterparts in the perfect-foresight case except for the re-labelling of dates. They are in particular log-linear. Conversely, the Euler equation (30) now features an expectation over the inverse of inflation given the stochastic environment. As a result, the system of equations defining the equilibrium is no longer log-linear in $L_t + W$. For the remainder of the paper, we will solve for a linearized version of these equilibrium equations:

Linear approximation. We solve for an equilibrium process x_t that satisfies the first-order expansions of (29), (30), (31), and (32) in $\underline{l}$ and $\bar{l}$.

This boils down to normalizing $W = 1$[13] and assuming that L^+ and L^- are sufficiently small that global investors have a small impact on the domestic real rate, the rate of inflation

[13]so that the real rate is R when $x_t = 0$.

and the appreciation/depreciation of the nominal exchange rate between two arrival dates. Importantly, this does not rule out that the domestic price level P_{T_j} and nominal exchange rate level S_{T_j} possibly reach large or small values because the cumulative impact of inflows can be large if global investors trade in the same direction for a long time. Nor does it imply restrictions on the size of the interest rate differential since w_t is unbounded. All that it imposes is that the domestic real rate and nominal variables do not fluctuate too much between two arrival dates.

Up to this approximation, we have:

Proposition 2 ***(Unique equilibrium)*** *Suppose that*

$$\gamma > \Phi(1-\gamma). \tag{33}$$

For μ/λ sufficiently large, there exists a unique equilibrium defined by a decreasing Lipschitz function f such that

$$dx_t = \lambda\left(\mathbb{1}_{\{w_t > f(x_t)\}} - x_t\right) dt, \tag{34}$$

where $\mathbb{1}_{\{.\}}$ denotes the indicator function.

Stochastic bifurcations. Equation (34) states that the equilibrium aggregate position of global investors x_t obeys an ordinary differential equation controlled by the stochastic process w_t. Such processes are known as *stochastic bifurcation models,* and are extensively studied in Bass and Burdzy (1999) and Burdzy et al. (1998). These mathematics papers establish in particular that for almost every sample path of w_t, there exists a unique Lipschitz solution x_t to the differential equation (34) defining the price dynamics for f Lipschitz decreasing. We will repeatedly use this result throughout the proof of Proposition 2.[14]

Equilibrium dynamics. The frontier f divides the (w, x)-space into two regions. Proposition 2 states that in the unique equilibrium, any investor decides to be long when the system is to the right of the frontier f at his switching date, and short when it is on the left of the frontier. Thus, net lending (and therefore the nominal exchange rate) will tend to rise

[14]These papers also establish convergence results ensuring that the process x_t that we obtain when linearizing in $\underline{l}$ and $\overline{l}$ converges to the exact equilibrium process as $\underline{l}, \overline{l} \to 0$.

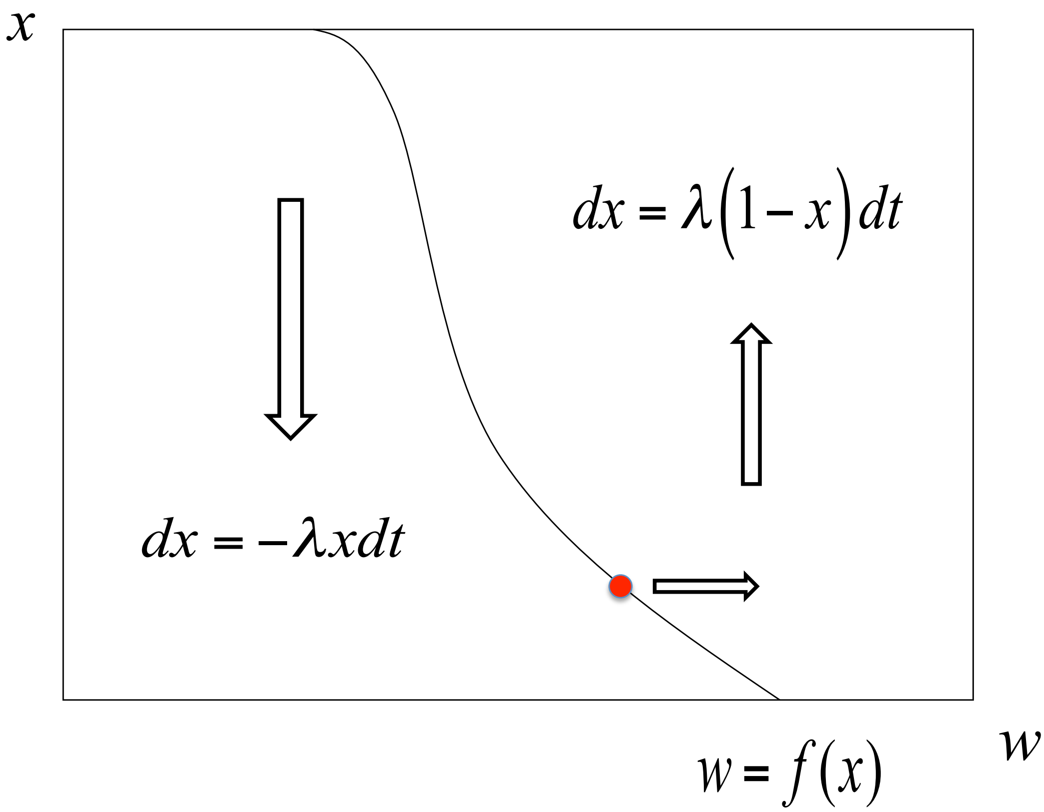

Figure 1: **Equilibrium dynamics.** The frontier *f* divides the *(w,x)*-space into two regions. In the unique equilibrium, lending increases in the right-hand region and declines in the left.

in the right-hand region, and tend to fall in the left-hand region, as indicated by the arrows in Figure 1.

The main features of these dynamics can be seen from Figure 1. Starting from the red dot on the frontier, a positive shock on w will pull the system to the right of it. Unless the path of w_t is such that a larger negative shock brings it back on the frontier immediately, a more likely scenario is that lending grows for a while so that x_t becomes close to 1, in which case $\frac{dx_t}{dt}$ becomes close to 0. If cumulative negative shocks on w eventually lead the system back to the left of the frontier, then there are large outflows

$$\frac{dx_t}{dt} \simeq -\lambda.$$

These dynamics therefore correspond to prolonged episodes of appreciation of the domestic currency, large cumulated capital inflows, and benign domestic financial conditions following a negative shock on the US interest rate. Subsequent small increases in the US interest rate do not reverse these dynamics until a tipping point is reached. This point triggers a large currency depreciation, important capital outflows, a crash in the domestic bond market, domestic inflation, and a tightening of domestic monetary policy. These features of sudden stops correspond to that experienced by the "Fragile Five" (Brazil, Indonesia, India, South Africa and Turkey) following Bernanke's testimony about the possible "tapering" of the highly accommodative US monetary policy (see Aoki, Benigno, and Kiyotaki, 2015).

It is admittedly not surprising that the equilibrium displays periods of capital inflows or outflows given that investors' positions are assumed to be sticky and bounded. The interesting part of these dynamics lies in our view in the subtle nonlinear impact of the fundamental (the US interest rate) on investors' coordination. After they have reaped positive excess returns on carry trades for a long time, only a large accumulation of negative news can lead investors to switch beliefs about each others' future positions and thus about the profitability of carry trades. When the tipping point is reached, however, a small incremental negative news has a disproportionate impact on investors' position. We consider this to be a signature pattern of episodes of destabilizing speculation that stochastic-bifurcation models capture parsimoniously.

Expected return on carry trades. The expected return on the carry trade at date t,

Π_t, is zero if and only if $w_t = f(x_t)$. It is positive if (w_t, x_t) is on the right of the frontier f in the (w, x)-space and negative if it is on the left of f. Thus, carry trades exhibit abnormal expected returns that increase in the net open interest. They are risky trades though, as an investor can be stuck in a position that generates losses when other investors revert their trade before his trade unwinds.

Congestion effects. Condition (33) is already the one that generates multiple steady-states in the perfect-foresight case. Here, it is necessary in order to obtain a decreasing frontier f, and thus large bifurcations, for exactly the same economic reasons.[15] It is not sufficient, however. The additional condition that μ/λ be sufficiently large is also required in order to obtain a decreasing frontier. It means that capital must flow sufficiently smoothly into the domestic economy: The arrival rate of domestic trading counterparties for global investors must be sufficiently large relative to the frequency at which global investors can revise their committed position. We offer detailed explanations for this condition in the proof of Proposition 2 below. The broad intuition is that if this condition is not satisfied, then global investors create sufficiently large congestion externalities for each other that this acts as a stabilizing force that more than offsets the destabilizing ones presented in the perfect-foresight case. Otherwise stated, if the capital market is too congested, then capital inflows cannot destabilize the small open economy even when its monetary rule is sufficiently passive that (33) holds. This interplay between the stance of monetary policy and the ability of the domestic capital market to channel foreign flows is novel, to our knowledge.

Proof of Proposition 2

Our proof follows the same roadmap as that in Frankel and Pauzner (2000), with additional complexity induced by the congestion effects.

More precisely, the first step consists in using relations (29) to (31) to express the nominal exchange rate and interest rate as functions of the expected future paths of capital inflows L_t. This yields in turn a relatively simple expression for the expected return on the carry trade Π_t as a function of these expected capital inflows:

[15] Absent this condition, the frontier would be increasing, and small shocks on w_t would translate into small shocks on x_t in the same direction so that the system remains close to the frontier.

Lemma 3 ***(Linearized expected return)*** *At first-order w.r.t.* $\underline{l}, \bar{l}$*, the expected return on the carry trade is*

$$\Pi_t = \int_0^{+\infty} \xi(v) E_t \left[l_{t+\frac{v}{\mu}} \right] dv + \frac{w_t}{\frac{\lambda}{\mu} + \rho}, \tag{35}$$

where

$$\xi(v) = \left(\frac{\chi\omega}{\omega - \rho - \frac{\lambda}{\mu}} - 1 \right) e^{-(\rho + \frac{\lambda}{\mu})v} - \frac{\chi\omega}{\omega - \rho - \frac{\lambda}{\mu}} e^{-\omega v}, \tag{36}$$

$$l_t = x_t \bar{l} + (1 - x_t)\, \underline{l}, \tag{37}$$

$$\rho = 1 - \frac{1}{R}, \tag{38}$$

$$\omega = \frac{\Phi}{1 + \Phi}, \tag{39}$$

$$\chi = \frac{\gamma}{(1 - \gamma)\, \Phi}. \tag{40}$$

Proof. See the appendix. ■

The factor $\xi(v)$ that discounts future capital inflows in (35) is first negative, then positive as v spans $[0, +\infty)$.[16] Thus, an increase in the expected net inflow $E_t[l_{t+s}]$ raises the date-t expected return on the carry trade only for s sufficiently large, and reduces it otherwise. This is the key technical difference between our setup and the abstract games studied by Burdzy, Frankel, and Pauzner, in which the expected payoff at date-t increases in (their equivalent of) l_{t+s} for each s, t.

This property of $\xi(v)$ reflects congestion externalities. Whereas an investor benefits from the long positions of his successors after he has lent to the domestic economy, inflows occurring between a switching date and the next arrival date at which he will actually lend are conversely detrimental to him. They lead to a currency appreciation that reduces the upside from the carry trades executed once the arrival dates occur. With constant inflows (as was the case in the perfect-foresight economy), this congestion effect is not sufficient to stabilize the economy when $\chi > 1$.[17] In this stochastic version of the model, however, expecting higher future inflows does not always necessarily translate into higher expected profits if the

[16] Notice that this is so regardless of the sign of $\omega - \rho - \frac{\lambda}{\mu}$.

[17] To see this, note that $\int \xi = (\chi - 1)/(\lambda/\mu + \rho) > 0$.

market is too congested, as an investor would rather see outflows and dollar appreciation between his switching date and the date at which he starts his streak of carry trades. The following lemma is instrumental. It shows that this effect is not sufficient to stabilize capital flows when markets are not too congested—a situation that seems empirically plausible.

Lemma 4 ***(Uncongested capital market)*** *Suppose that $\chi > 1$. There exists M such that for all $\mu/\lambda > M$, the following is true. Suppose that two processes x_t^1 and x_t^2 satisfy*

$$\begin{aligned} &0 < x_0^1 \leq x_0^2 < 1, \\ &\textit{For } i = 1, 2,\ dx_t^i = \lambda \left(1_{\left\{ w_t > f^i\left(x_t^i\right) \right\}} - x_t^i \right) dt, \end{aligned}$$

where f^i is decreasing Lipschitz and $f^2 \leq f^1$. Then the expected profit at date 0 is smaller under x_t^1 than x_t^2, strictly so if $f^1 \neq f^2$ and/or $x_0^1 \neq x_0^2$.

Proof. See the appendix. ■

Lemma 4 states that if (33) holds and μ/λ is sufficiently large, then future inflows make current carry trades more attractive because the reinforcing effect overcomes the congestion effect. In the balance of the paper, we suppose that the conditions in Lemma 4 are satisfied. We now show that there is in this case a unique Lipschitz process x_t that satisfies the equilibrium conditions.[18]

First, the proof of Lemma 4 also shows that the case in which x_t obeys $\frac{dx_t}{dt} = -\lambda x_t$ for all $u \geq 0$ corresponds to a lower bound on the expected carry-trade return. When x_t obeys such dynamics, there exists a frontier f_0 such that

$$w_t = f_0(x_t) \implies \Pi_t = 0. \tag{41}$$

The frontier f_0 is decreasing from Lemma 4 (with $f^1 = f^2 = +\infty$) and is clearly affine and thus Lipschitz.[19] Thus an admissible equilibrium process must be such that investors who have a chance to switch when the system is on the right of f_0 become long.

[18]One can see by inspection fo the proof of Lemma 4 that M does not depend on the volatility σ of w_t, a useful property when we will let $\sigma \to 0$ in the following.

[19]The frontier simply obtains from writing $E_t\left[l_{t+\frac{v}{\mu}}\right] = \underline{l} + \left(\bar{l} - \underline{l}\right) x_t e^{-\frac{\lambda v}{\mu}}$ in (35).

Define now f_1 such that

$$w_t = f_1(x_t) \implies \Pi_t = 0 \tag{42}$$

if for all $u \geq 0$,

$$\frac{dx_{t+u}}{du} = \begin{cases} -\lambda x_{t+u} & \text{if } w_{t+u} < f_0(x_{t+u}), \\ \lambda(1 - x_{t+u}) & \text{if } w_{t+u} > f_0(x_{t+u}). \end{cases} \tag{43}$$

That is, f_1 is such that an investor is indifferent between being long or short when the system is on f_1 at his switching date if he believes that other investors become long if and only if they are on the right of f_0. This function f_1 must be decreasing. Suppose otherwise that two points (w, x) and (w', x') on f_1 satisfy

$$x' > x,$$
$$w' \geq w.$$

Then applying Lemma 4 with $f^2 = f_0, f^1 = f_0 + w' - w$ contradicts that both points generate the same expected carry-trade return. We also show in the appendix that f_1 is Lipschitz, with a Lipschitz constant smaller than that of f_0.

By iterating this process, we obtain a limit f_∞ of the sequence of frontiers $(f_n)_{n \geq 0}$ that is decreasing Lipschitz as a limit of decreasing Lipschitz functions with decreasing Lipschitz constants. The process

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \text{if } w_t < f_\infty(x_t), \\ \lambda(1 - x_t) & \text{if } w_t > f_\infty(x_t) \end{cases} \tag{44}$$

is an admissible equilibrium since by construction, if all investors switch to being short to the left of f_∞ and to being long to the right, the indifference point for an investor also lies on f_∞. We now show that this is the only equilibrium process.

Consider a translation to the left of the graph of f_∞ in (w, x) so that the whole of the curve lies in a region where w_t is sufficiently small that being short is dominant regardless of the dynamics of x_t. Call this translation f_0'. To the left of f_0', going short is dominant. Then construct f_1' as the rightmost translation of f_0' such that an investor must choose to be short to the left of f_1' if he believes that other investors will play according to f_0'. By iterating this

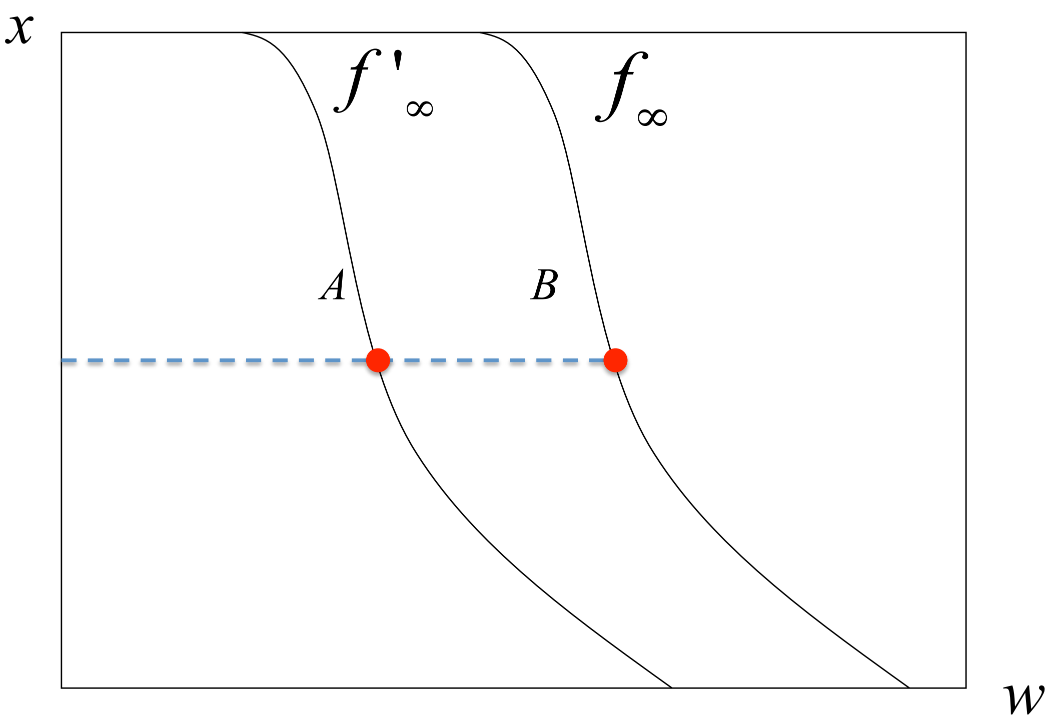

Figure 2: **Uniqueness of the limiting boundary.** This figure illustrates the argument for the uniqueness of the equilibrium boundary separating the two regions. The boundary f'_{∞} coincides with f_{∞}.

process, we obtain a sequence of translations to the right of f_0'. Denote by f_∞' the limit of the sequence. Refer to Figure 2.

The boundary f_∞' does not necessarily define an equilibrium strategy, since it was merely constructed as a translation of f_0'. However, we know that if all others were to play according to the boundary f_∞', then there is at least one point A on f_∞' where the investor is indifferent. If there were no such point as A, this would imply that f_∞' is not the rightmost translation, as required in the definition.

We claim that f_∞' and f_∞ coincide exactly. The argument is by contradiction. Suppose that we have a gap between f_∞' and f_∞. Then, choose point B on f_∞ such that A and B have the same height - i.e. correspond to the same x. But then, since the shape of the boundaries of f_∞' and f_∞ and the values of x are identical, the paths starting from A must have the same distribution as the paths starting from B up to the constant difference in the initial values of w. This contradicts the hypothesis that an investor is indifferent between the two actions both at A and at B. If he were indifferent at A, he would strictly prefer being long at B, and if he is indifferent at B, he would strictly prefer being short when in A. But we constructed A and B so that investors are indifferent in both A and B. Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no "gap", and we must have $f_\infty' = f_\infty$. ■

Proposition 2 shows that adding exogenous shocks w_t to the carry return eliminates the indeterminacy of the perfect-foresight case. More precisely, equilibrium uniqueness stems from the interplay of these shocks with the fact that each investor, when he receives a switching opportunity, needs to form beliefs about the decisions of the investors that will have an opportunity to switch between now and his next switching date. Suppose that (w_t, x_t) is close to a dominance region in which investors would prefer a course of action for sure, but just outside it. If w_t was fixed, it may be possible to construct an equilibrium for both actions, but when w_t moves around stochastically, it will wander into the dominance region between now and the next opportunity that the trader gets to switch with some probability. This gives the investor some reason to hedge his bets and take one course of action for sure. But then, this shifts out the dominance region, and a new round of reasoning takes place given the new boundary, and so on.

Remark 1. (Bounded shocks to the US rate) We model the interest-rate differential as a Brownian motion for expositional simplicity. It is easy to see that we could write it as $d(w_t)$, where w_t is a standard Brownian motion, and d a Lipschitz increasing function, possibly bounded as long as there are still dominant actions for w_t sufficiently large or small.

Remark 2. (Transitory shocks to the US rate) While a strong persistence in shocks to the US rate is undoubtedly realistic, extensions of this framework can also accommodate for various forms of mean-reversion (Burdzy, Frankel, and Pauzner, 2001, or Frankel and Burdzy, 2005).

The case of small shocks

The limiting case in which the volatility σ of the interest-rate differential tends to zero yields useful insights. It is possible to characterize the shape of the frontier f in this case.

In this section we denote the frontier f_σ to emphasize its dependence on σ. Suppose the economy is in the state $(f_\sigma(x_t), x_t)$ at date t. That is, it is on the equilibrium frontier. For some arbitrarily small $\varepsilon > 0$, introduce the stopping times

$$\begin{aligned} T_1 &= \inf_{u \geq 0} \left\{ x_{t+u} \notin (\varepsilon, 1-\varepsilon) \right\}, \\ T_0 &= \sup_{0 \leq u < T_1} \left\{ w_{t+u} \neq f_\sigma(x_{t+u}) \right\}. \end{aligned}$$

In words, T_1 is the first date at which x_t gets close to 0 or 1, and T_0 is the last date at which x_t crosses the frontier before T_1. If T_0 is small in distribution, it means that the economy is prone to bifurcations. That is, it never stays around the frontier for long. Upon hitting it, it quickly heads towards extreme values of x. The next proposition shows that this is actually the most likely scenario when σ is small. This, in turn, yields a simple explicit determination of the frontier.

Proposition 5 ***(Small shocks)***

1. *As $\sigma \to 0$, T_0 converges to 0 in distribution, and the probability that $\frac{dx_t}{dt} > 0$ (respectively $\frac{dx_t}{dt} < 0$) over $[T_0, T_1]$ converges to $1 - x_t$ (x_t respectively).*

2. *As $\sigma \to 0$, the frontier f_σ tends to an affine function. For μ/λ sufficiently large, the slope of this function is increasing in Φ and decreasing in γ.*

Proof. See the appendix. ■

First, Proposition 5 clears the concern that in equilibrium, x would only exhibit small fluctuations around a fixed value because Brownian paths cross the frontier too often. As σ becomes smaller, the system exhibits more frequent bifurcations towards extremal values of x. When the system reaches the frontier, it is all the more likely to bifurcate towards capital outflows when cumulative inflows have been large (x large). Thus the model does generate "destabilizing carry trades," whereby global investors generate self-justified excess returns on the carry trade that persist beyond the exogenous trigger until a large reversal occurs.

The second point in Proposition 5 relates the slope of the frontier f_σ to the monetary parameters of the model Φ and γ in this case of small shocks. The slope of the frontier affects the dynamics of capital inflows and in turn the exchange-rate dynamics. If the graph of the frontier is closer to being horizontal in the (w, x) plane, then the system should cross the frontier less often, and thus do so only for more extreme values of x. Carry-trade returns should in this case exhibit more serial correlation and fatter tails. Point 2 states that, at least for μ/λ sufficiently large, the frontier is flatter when Φ is smaller, and γ larger. In other words, if monetary policy does not respond much to capital inflows, then carry trade returns should exhibit more skewness.

4 Empirical content

Our model generates a rich set of qualitative features, of which many have empirical implications. As a by-product of our analysis, we provide in particular a novel explanation for the seemingly high Sharpe ratio generated by carry-trade strategies.

Profitability of FX carry trades

The equilibrium expected return on the carry trade increases with respect to w and x, it is positive on the right of the frontier f in the (w, x) plane and negative on the left. On the other hand, the interest-rate differential increases in w and decreases in x. We have indeed:

Lemma 6 ***(Interest-rate differential)*** *At first-order, the interest-rate differential at a given arrival date T_n is given by*

$$R\left(w_{T_n} - l_{T_n} - \frac{1}{1+\Phi}\int_0^{+\infty} e^{-\omega s} E_{T_n}\left[l_{T_n+\frac{s}{\mu}}\right] ds\right). \tag{45}$$

Proof. See the appendix. ■

The interest-rate differential increases w.r.t. w but decreases w.r.t. l (and thus x) because the current domestic real rate is lower and future deflation more likely when l is large. Thus the expected return on the carry trade is not unambiguously increasing in the interest-rate differential. Yet, when the interest-rate differential is sufficiently large in absolute terms, it must be that the system is on the right (left) of the frontier when the differential is positive (negative). Thus, we have:

A positive (negative) interest-rate differential predicts a positive (negative) return on the carry-trade for sufficiently large absolute differentials.

In particular, for $\underline{l}$, $\bar{l}$ sufficiently small, most of the interest-rate differential is due to the exogenous component w. The threshold above which a positive (negative) interest-rate differential is associated with a positive (negative) excess return on the carry trade is arbitrarily small. This rationalization of carry-trade returns as self-fulfilling genuine excess returns contrasts with existing theories that seek to explain the return on carry trades as a compensation for (possibly mismeasured) risk. Farhi and Gabaix (2015) thorougly survey this existing literature. We do not deny that a significant fraction of carry-trade returns may reflect risk premia, and view our theory as a complement to such risk-based considerations rather than a competing alternative.

Peso problem

A large literature argues that the return on the carry trade partly reflects a risk premium for rare and extreme events that may not show in finite samples (see, e.g., Farhi and Gabaix, 2015, or Lewis, 2007, and the references herein). We closely connect to this literature as follows. Fix $\epsilon > 0$ small. The expected return on the carry trade is 0 starting both from $(f(\epsilon), \epsilon)$ and $(f(1-\epsilon), 1-\epsilon)$ in the (w, x) plane. Yet from Proposition 5, as σ becomes small, most paths starting from $(f(\epsilon), \epsilon)$ will exhibit long periods of appreciation of the domestic currency ended with rare (and large) depreciations, while paths starting from $(f(1-\epsilon), 1-\epsilon)$ will feature a symmetric prolonged depreciation. The interest-rate differential is positive in the former case and negative in the latter. Thus, due to rare reversals, finite samples should yield that a positive interest-rate differential predicts a positive excess return on the carry trade even when the true expected return is zero. More generally, rare reversals imply that finite samples should lead to an overestimation of the absolute magnitude of abnormal returns on the carry trade.

Profitability of FX momentum strategies

Proposition 5 shows that as $\sigma \to 0$, the system often bifurcates in one direction. This implies that, at least at a sufficiently short horizon, returns are positively autocorrelated, so that momentum strategies in FX markets should generate a positive excess return. The key economic force behind this profitability of momentum strategies is that once carry traders coordinate on a course of action, they stick to it until a sufficiently large reversal of the interest-rate differential leads them to switch to a different strategy. Such a rationalization of momentum returns with coordination motives is novel to our knowledge.

Monetary policy and carry-trade returns

In addition to relating to the above existing empirical findings, the model also generates a new range of predictions on the relationship between the stance of monetary policy and the distribution of the returns on carry-trade strategies. Proposition 5 suggests that the frontier is flatter when Φ is smaller and γ larger. Otherwise stated, if an economy is such that the CPI is not too sensitive to the exchange rate, and/or the central bank not too aggressive, then this economy should be more prone to large fluctuations in carry-trade activity because it will experience more prolonged bifurcations. Thus the returns on carry-trade and momentum strategies should have fatter tails.

5 Concluding remarks

The independence of monetary policy under liberalised capital flows and floating exchange rates has been a benchmark principle in international finance. In our paper, we have explored a parsimonious model of global investors facing each other in a dynamic global game and found that under plausible conditions, the model generates boom bust cycles associated with coordinated capital inflows and outflows. In such a setting, monetary conditions depend on the coordination outcome of investors who have access to the domestic bond market, as well as on the economic fundamentals. Thus, we qualify the proposition that a floating exchange rate guarantees monetary autonomy by showing that as capital flows more smoothly into a small open economy, then a commitment to a more aggressive monetary response to capital flows is required in order to discourage destabilizing carry trades.

Assuming that households are risk-neutral over late consumption dramatically simplifies the analysis. With strictly concave preferences, the current real rate would depend on consumption growth, so that we could no longer abstract from the impact of foreign lending on quantities and thus production in the domestic economy as we are able to do here. We find it useful to derive our novel mechanism for self-fulfilling profitable carry trades in a highly tractable framework that describes the interplays of the ingredients at work in a fully transparent fashion. An interesting avenue for future research, which would be more simulation-based, is the study of the impact of such carry trades on quantities under more standard preferences.

A Appendix

A.1 Proof of Lemma 3

The first-order expansion of the Euler equation (30):

$$\ln I_{T_{n+1}} + \ln E_t \left[\frac{P_{T_n}}{P_{T_{n+1}}} \right] = \ln R - \ln(L_{T_n} + W) \tag{46}$$

in $\underline{l}$, $\bar{l}$ yields:

$$\ln I_{T_{n+1}} - E_t \left[\ln \frac{P_{T_{n+1}}}{P_{T_n}} \right] = \ln R - l_{T_n} \tag{47}$$

where

$$l_{T_n} = \underline{l}(1 - x_{T_n}) + \bar{l} x_{T_n}. \tag{48}$$

Combined with the Taylor rule (29), this yields domestic inflation as a function of future expected inflows as in the perfect-foresight case:

$$\ln \frac{P_{T_n}}{P_{T_{n-1}}} = - \sum_{k \geq 0} \frac{E_{T_n} \left[l_{T_{n+k}} \right]}{(1 + \Phi)^{k+1}}, \tag{49}$$

As in the perfect-foresight case, (31) yields in turn:

$$E_{T_n} \left[\ln \frac{S_{T_{n+1}} I_{T_{n+1}}}{R S_{T_n}} \right] = \frac{\gamma}{1-\gamma} \sum_{k \geq 0} \frac{E_{T_n} \left[l_{T_{n+k+1}} \right]}{(1 + \Phi)^{k+1}} - l_{T_n}. \tag{50}$$

One can write (28) as

$$\Pi_t = E_t \left[\sum_{m \geq 0} \frac{1_{\{T_\lambda > T_{n+m}\}}}{R^m} E_{T_{n+m}} \left[\frac{P_{T_{n+m}} S_{T_{n+m}}}{P_{T_{n+m+1}} S_{T_{n+m+1}}} \left(\frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{R S_{T_{n+m}}} - 1 + w_{T_{n+m}} \right) \right] \right]. \tag{51}$$

At first-order w.r.t. $\underline{l}$, $\bar{l}$,

$$E_{T_{n+m}} \left[\frac{P_{T_{n+m}} S_{T_{n+m}}}{P_{T_{n+m+1}} S_{T_{n+m+1}}} \left(\frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{R S_{T_{n+m}}} - 1 \right) \right] = E_{T_{n+m}} \left[\ln \frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{R S_{T_{n+m}}} \right] \tag{52}$$

$$= \frac{\gamma}{1-\gamma} \sum_{k \geq 0} \frac{E_{T_{n+m}} \left[l_{T_{n+m+k+1}} \right]}{(1 + \Phi)^{k+1}} - l_{T_{n+m}}. \tag{53}$$

Thus,

$$\Pi_t = E_t\left[\int_0^{+\infty}\sum_{m\geq 1}\mu^m\left(\frac{s}{R}\right)^{m-1}\frac{e^{-(\lambda+\mu)s}}{(m-1)!}\left[\begin{array}{c}\int_0^{+\infty}\frac{\gamma}{1-\gamma}\sum_{k\geq 1}\mu^k\frac{u^{k-1}}{(k-1)!}\frac{e^{-\mu u}}{(1+\Phi)^k}l_{t+s+u}du\\ -l_{t+s}+w_t\end{array}\right]ds\right], \tag{54}$$

$$= E_t\left[\int_0^{+\infty}e^{-\left(\frac{\lambda}{\mu}+\rho\right)s}\left(\int_0^{+\infty}\chi\omega e^{-\omega u}l_{t+\frac{s+u}{\mu}}du - l_{t+\frac{s}{\mu}} + w_t\right)ds\right], \tag{55}$$

$$= \int_0^{+\infty}e^{-(\frac{\lambda}{\mu}+\rho)v}\left(\chi\omega\int_0^{v}e^{-(\omega-\frac{\lambda}{\mu}-\rho)u}du - 1\right)E_t\left[l_{t+\frac{v}{\mu}}\right]dv + \frac{w_t}{\frac{\lambda}{\mu}+\rho}, \tag{56}$$

and integrating yields the result. (Note that we ignored here the terms in $w_t l_t$ because we use this approximation of Π_t only for initial values w_t of the same order of magnitude as l_t.)

A.2 Proof of Lemma 4

Suppose $\chi > 1$. Consider two processes x_t^1 and x_t^2 that satisfy the conditions stated in Lemma 4 with $x_0^1 < x_0^2$. Lemma 2 in Burdzy, Frankel and Pauzner (1998) states that almost surely,

$$x_t^2 \geq x_t^1 \text{ for all } t \geq 0. \tag{57}$$

This implies in particular that whenever investors switch to being long along a sample path of (w_t, x_t^1), so do they along the sample path of (w_t, x_t^2) that corresponds to the same sample path of w_t. This is because it must be that (w_t, x_t^2) is on the right of the frontier f^2 whenever (w_t, x_t^1) is on the right of the frontier f^1. Thus, the process

$$y_t = x_t^2 - x_t^1 \tag{58}$$

satisfies

$$0 < y_0 < 1, \tag{59}$$

$$\frac{dy_t}{dt} = \lambda(\epsilon_t - y_t), \tag{60}$$

In order to prove the Lemma, we only need to find M such that for all $\mu/\lambda > M$,

$$\Delta = \int_0^{+\infty}\left(\left(\frac{\chi\omega}{\omega-\rho-\frac{\lambda}{\mu}} - 1\right)e^{-\left(\frac{\lambda}{\mu}+\rho\right)v} - \frac{\chi\omega}{\omega-\rho-\frac{\lambda}{\mu}}e^{-\omega v}\right)y_{\frac{v}{\mu}}dv \geq 0. \tag{61}$$

for all deterministic process y_t that obeys (59) and (60). The result then obtains from taking expectations over all paths of w_t.

To prove (61), we introduce the function ζ that satisfies

$$\begin{cases} \frac{d\zeta(v)}{dv} = -\left(\left(\frac{\chi\omega}{\omega-\rho-\frac{\lambda}{\mu}} - 1\right) e^{-\left(\frac{\lambda}{\mu}+\rho\right)v} - \frac{\chi\omega}{\omega-\rho-\frac{\lambda}{\mu}} e^{-\omega v}\right), \\ \lim_{+\infty} \zeta = 0. \end{cases}$$

Integrating by parts, we have

$$\Delta = \zeta(0)y_0 + \frac{1}{\mu}\int_0^{+\infty} \zeta(v)\frac{dy_{\frac{v}{\mu}}}{dv}dv, \tag{62}$$

$$= \zeta(0)y_0 + \frac{\lambda}{\mu}\int_0^{+\infty} \zeta(v)(\epsilon_{\frac{v}{\mu}} - y_{\frac{v}{\mu}})dv. \tag{63}$$

Further,

$$y_{\frac{v}{\mu}} = y_0 e^{-\frac{\lambda v}{\mu}} + \frac{\lambda}{\mu}\int_0^{v} e^{-\frac{\lambda}{\mu}(v-u)}\epsilon_{\frac{u}{\mu}}du, \tag{64}$$

and thus

$$\Delta = y_0\left(\zeta(0) - \frac{\lambda}{\mu}\int_0^{+\infty} \zeta(v)e^{-\frac{\lambda v}{\mu}}dv\right) \tag{65}$$

$$+ \frac{\lambda}{\mu}\left[\int_0^{+\infty} \epsilon_{\frac{v}{\mu}}\left(\zeta(v) - \frac{\lambda}{\mu}\int_v^{+\infty} \zeta(u)e^{-\frac{\lambda}{\mu}(u-v)}du\right)\right]. \tag{66}$$

We have

$$\lim_{\frac{\lambda}{\mu}\to 0} \zeta(0) = \frac{\chi-1}{\rho} > 0, \tag{67}$$

ζ is increasing then decreasing beyond a value that stays bounded as λ/μ tends to zero, and $\int_0^{+\infty}\zeta$ converges. Thus for λ/μ sufficiently small,

$$\zeta(v) - \frac{\lambda}{\mu}\int_v^{+\infty} \zeta(u)e^{-\frac{\lambda}{\mu}(u-v)}du \tag{68}$$

is positive for all $v \geq 0$, which yields that Δ is positive, and concludes the proof.

A.3 Complement to the proof of Proposition 2

We prove here that f_1 is Lipschitz with a constant that is smaller than that of f_0, that we denote K_0. Suppose by contradiction that two points (w_t, x_t) and (w_t', x_t') on f_1 satisfy

$$x' > x, \tag{69}$$

$$\frac{x_t' - x_t}{w_t - w_t'} < \frac{1}{K_0}. \tag{70}$$

We compare the paths x'_{t+u} and x_{t+u} corresponding to pairs of paths of w'_{t+u} and w_{t+u} that satisfy for all $u \geq 0$

$$w_{t+u} - w'_{t+u} = w_t - w'_t. \tag{71}$$

It must be that for such pairs of paths:

$$x'_{t+u} - x_{t+u} \leq (x'_t - x_t)e^{-\lambda u}. \tag{72}$$

Otherwise it would have to be the case that (w', x') can be on the right of f_0 when (w, x) is not. Suppose by contradiction that this can be. Let T denote the first time at which this occurs. It must be that

$$K_0 e^{-\lambda T}(x'_t - x_t) \geq w_{t+T} - w'_{t+T} = w_t - w'_t, \tag{73}$$

a contradiction with (70).

Thus along such paths of $w'_{t+u} - w_{t+u}$, $x'_{t+u} - x_{t+u}$ shrinks at least as fast as when investors switch to being short all the time. Together with (70), this implies that the expected return on the carry trade cannot be the same in (w_t, x_t) and (w'_t, x'_t), a contradiction.

A.4 Proof of Proposition 5

The first point is a particular case of Theorem 2 in Burdzy, Frankel, and Pauzner (1998). To prove the second point, notice that as $\sigma \to 0$, starting from a point on the frontier,

$$E_t\left[x_{t+v}\right] \simeq (1 - x_t)\left(1 - (1 - x_t)e^{-\lambda v}\right) + x_t^2 e^{-\lambda v} \tag{74}$$

because the system bifurcates upwards with probability $1 - x_t$ and downwards with probability x_t in the limit. Plugging this in (35) and writing that the expected return is zero yields a slope of the frontier equal to

$$-(\bar{l} - \underline{l})(\chi - 1)$$

as $\lambda/\mu \to 0$. This means that the absolute value of the slope of the frontier varies as χ w.r.t. γ, Φ for σ, λ/μ sufficiently small. This proves the proposition.

A.5 Proof of Lemma 6

We have

$$I_{T_n+1} - R(1 - w_{T_n}) = R\left(\left(\frac{P_{T_n}}{P_{T_{n-1}}}\right)^{1+\Phi} - 1 + w_{T_n}\right), \tag{75}$$

$$\simeq R\left(w_{T_n} - E_{T_n}\left[\sum_{k\geq 0} \frac{l_{T_n+k}}{(1+\Phi)^k}\right]\right), \tag{76}$$

$$= R\left(w_{T_n} - l_{T_n} - \int_0^{+\infty} \sum_{k\geq 1} \frac{\mu^k s^{k-1} e^{-\mu s}}{(k-1)!(1+\Phi)^k} E_{T_n}\left[l_{T_n+s}\right] ds\right), \tag{77}$$

$$= R\left(w_{T_n} - l_{T_n} - \frac{1}{1+\Phi}\int_0^{+\infty} e^{-\omega s} E_{T_n}\left[l_{T_n+\frac{s}{\mu}}\right] ds\right). \tag{78}$$

This proves the lemma.

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