Endogenous growth and global divergence in a multi-country agent-based model

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SCIENCES PO OFCE WORKING PAPER n° 33, 2018/01/04
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WORKING PAPER CITATION

This Working Paper:
Downloaded from URL : www.ofce.sciences-po.fr/pdf/dtravail/WP2018-01.pdf
DOI - ISSN

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ABSTRACT

In this paper we present a multi-country, multi-industry agent-based model investigating the different growth patterns of interdependent economies. Each country features a Schumpeterian engine of endogenous technical change which interacts with Keynesian/Kaldorian demand generation mechanisms. National growth trajectories are driven by firms’ accumulation of technological knowledge, which in turn also leads to emergent specialization patterns in different industries. Interactions among economies occur via trade flows, stemming from the competition of firms in international markets. Simulation results show the emergence of persistent income divergence among countries leading to polarization and club formation. Moreover, each country experiences a structural transformation of its productive structure during the development process. Such dynamics results from firm-level virtuous (or vicious) cycles between knowledge accumulation, trade performances, and growth dynamics. The model accounts for a rich ensemble of empirical regularities at macro, meso and micro levels of aggregation.

KEY WORDS

Endogenous growth, structural change, technology-gaps, global divergence, absolute advantages, agent-based models.

JEL
F41, F43, O4, O3.

ABOUT OFCE

The Paris-based Observatoire français des conjonctures économiques (OFCE), or French Economic Observatory is an independent and publicly-funded centre whose activities focus on economic research, forecasting and the evaluation of public policy.
1 Introduction

Since its foundations, one of the major challenges in economics has concerned the drivers of the wealth of nations and their disparities across countries. This has been the concern of both classical economists, in primis Adam Smith and Marx, as well as their critics (such as List, see the reconstruction in Reinert 2009), and it has been addressed by a whole generation of economic historians including Landes (1969), Cipolla (1994) and Allen (2001). For long time, international disparities have been taken for granted as a self-evident empirical regularity. Only rather recently, one has begun to statistically document these phenomena. A while after the seminal contributions by Kuznets (1966) and Bairoch (1981), a burgeoning stream of research has uncovered new empirical regularities concerning the evolution of countries’ income distribution over time (see also Durlauf and Quah 1999; Durlauf et al. 2005).

On the interpretative side, the classics and a good deal of historians have agreed that technological change is a sort of primis inter pares driver of both country-specific growth and inter-country differences thereof. However, the theory has been much slower in acknowledging it. Modern growth theory in its inception reveals the importance of technical change, essentially by default, via the famous Solow’s residual typically labelled as “total factor productivity” or, as Abramovitz would put it, as the measure of the economists’ ignorance. Nevertheless, long after the lonely voice of Schumpeter such an acknowledgment has lead to the emergence of new models in the evolutionary perspective (Nelson and Winter 1982; Dosi et al. 1994, 2010; Silverberg and Verspagen 1995; Llerena and Lorentz 2004, see e.g.) and in the Neoclassical one (see e.g. Romer 1990; Aghion and Howitt 1992, 1997). However, in both paradigm there is a “lack of attention both to multi-sector growth models and to multi-country models with trade and capital flows” (Solow 2005). Moreover, the complex feedback between demand and supply at medium and long run frequencies are usually overlooked (Solow 2005).

In this work, we try to answer Solow’s pleas by building an agent-based evolutionary model featuring many countries and sectors. We aim to study how different endogenous growth trajectories and processes of structural change can lead to patterns of divergence (or convergence) among different economies. The multi-country framework enable us to study the dynamics of the whole cross-sectional distribution of aggregate incomes, entailing the possibility of phenomena such as polarization, convergence clubs and growth persistence (Quah 1996). Together, the multi-sectoral setting sheds a light also on the dynamics of structural change. Indeed, the growth process is also “qualitative” as it typically involves the transformation of the economic structure, and those countries that manage to build up the conditions for such transformation are also able to fill their technology gap and escape from poverty (McMillan et al. 2014; Lavopa and Szirmai 2014; Freeman and Soete 1997; Landes 1969; Reinert 2009). Finally, the open economy framework allows to study jointly the dynamics of international competition, trade and growth wherein international technology-gaps and absolute advantages/disadvantages bear long term effects on growth patterns.

The creative-destruction processes of innovations and the radical transformations involved with structural change cannot be accounted by equilibrium models studying growth along a steady-state path. This is even more so in a multi-country setting where heterogeneous firms compete in different international markets and industries. For these reasons, we employ an agent-based model (ABM). ABMs consider the economy as a complex evolving system (Arthur et al. 1997; Kirman 2010; Dosi et al. 2013). On the modeling side, see e.g. Cimoli (1988), Dosi et al. (1990), Los and Verspagen (2006), and Cimoli and Porcile (2013). For an introduction to the methodology see Tesfatsion and Judd (2006), LeBaron and Tesfatsion (2008), and Farmer and Foley (2009). Recent surveys of macroeconomic agent-based models are provided by Fagiolo and Roventini (2012, 2017).
2012), where macroeconomic empirical regularities emerge from out-of-equilibrium interactions of heterogeneous adaptive agents.

Building on Dosi et al. (1994) and on the “Keynes meets Schumpeter” framework (Dosi et al. 2010, 2013, 2015, 2017b; Lamperti et al., 2017), the model is populated by heterogeneous firms which belong to different countries and industries and compete in international markets. Firms strive to innovate and imitate their competitors in order to increase their productivity and, as a consequence, their market shares. Thus, the model features a fully micro-founded Schumpeterian engine of endogenous technical change. At the same time, well in tune with a Keynes-Kaldor perspective, changes in domestic and international demand conditions affect both economic fluctuations and growth trajectories. Firms exporting activities shape international trade flows and the evolution of current accounts and exchange rates between countries.

Simulation results show the emergence of endogenous growth cum fluctuations in countries’ development paths. However, countries exhibit different growth trajectories leading to global divergence, polarization, and clubs formation. These dynamics result from the heterogeneous processes of structural change taking place in every economy, interacting with sectoral specializations and trade performances. Indeed, leading countries possess absolute advantages with respect to the lagging ones, in line with evolutionary trade theories (Dosi et al., 1990). The interactions between Schumpeterian competition and Kaldorian aggregate demand feedback are responsible for such emergent dynamics. Indeed, at the microeconomic level, the innovative and imitative activities of firms determine their competitiveness and market shares in world markets, boosting their sales and providing the necessary resources for their R&D investments. In turn, the performances of firms shape the structural transformation in their domestic countries. Such emerging properties are all captured by the model which simultaneously accounts for a large set of macro, meso and micro empirical regularities.

The rest of this work is organized as follows. Section 2 introduces some stylized facts at different levels of aggregation which ought to be reproduced by endogenous growth models. In Section 3, the model is presented. Simulation results are showed in Section 4 and discussed in Section 5. Finally, Section 6 concludes.

2 Innovation and International Growth Patterns: Some Multi-Scale Evidence

Let us start by briefly reviewing the empirical evidence on economic growth in a multi-country perspective (see Durlauf et al., 2005; Jones, 2016, for macroeconomic surveys). Together, we will consider also empirical regularities at the micro and meso levels. The stylized facts (SF) presented in this section (cf. Table 7) will be the test-bed for evaluating the explanatory power of our model.

2.1 Macroeconomic growth and fluctuations

Many historical accounts have documented an exceptional rise in living standards over the past two centuries (Landes, 1969; Bordo et al., 2007; Maddison, 2010). Nevertheless, such a take off has taken place in a relatively small set of Western nations while, only in the post-WWII period, their club was joined by Japan and by a group of East Asian economies. Such catching up episodes have been rather

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3 For other endogenous growth agent-based models, see e.g. Silverberg and Verspagen (1995), Llerena and Lorentz (2004), Saviotti and Pyka (2008), Caiani et al. (2017). In particular, the interactions between structural change and economic growth is explored in Ciarli et al. (2010), Lorentz et al. (2016), Ciarli et al. (2017).
rare as have been phenomena of forging ahead or falling behind (Abramovitz, 1986). More generally, the era of self-sustained economic growth is undoubtedly associated to “the great divergence” (Allen, 2001): starting from similar pre-industrial conditions, countries are nowadays extremely differentiated in terms of several indicators including productivity levels and wealth per capita.

Not surprisingly, the empirical growth literature has largely rejected the convergence hypothesis on the grounds of different econometric techniques. Indeed, there is no empirical support for the so-called \( \sigma \)-convergence (i.e. decreasing income dispersion among countries - Sala-i Martin, 1996), and \( \beta \)-convergence occurs only in subsamples of economies characterized by similar initial conditions and common characteristics (Durlauf and Johnson, 1995). Considering the dynamics of the whole cross-sectional distribution of country incomes, a series of works has shown instead a strong shift over time towards bimodality and polarization (Quah, 1996; Bianchi et al., 1997; Henderson et al., 2008; Castaldi and Dosi, 2009), and slow mobility across income “clubs” (Quah, 1993, 1997). Relative rankings among countries tend to be sticky and only few economies successfully completed the transition from low-income to high-income clubs.

Contrary to what implied by equilibrium models, steady growth trajectories are hardly found in real data. Across-period correlation in growth rates of individual countries are rather weak suggesting that development paths are unstable (Easterly et al., 1993; Pritchett et al., 2000) with alternating phases of acceleration and deceleration (Rodrik, 1999; Hausmann et al., 2005; Lamperti et al., 2016).

Concerning the statistical properties of growth rates distributions, Castaldi and Dosi (2009) find evidence of fat tails in the empirical density obtained by pooling together growth rates from different countries and years. Symmetrically, data display a negative scaling law between income levels and growth rates variability (Canning et al., 1998; Castaldi and Dosi, 2009). Loosely speaking, laggard countries tend to experience more severe aggregate fluctuations.

Let us sum up the first set of stylized facts (SF) concerning international growth patterns:

**SF 1** In the last two centuries per capita incomes have grown exponentially in all countries affected by the process of industrialization.

**SF 2** There have been a few historical episodes of catching up, forging ahead and falling behind.

**SF 3** Aggregate income dispersion has increased over time with no \( \sigma \)-convergence.

**SF 4** \( \beta \)-convergence does not appear unless under some form of ex-ante selection bias.

**SF 5** The cross-sectional income distribution reveals a tendency towards bimodality and polarization.

**SF 6** There is a general lack of mobility across income clubs. Relative rankings are rather sticky.

**SF 7** Growth rates are weakly correlated across periods. Growth trajectories are relatively unstable.

**SF 8** The distribution of international growth rates displays a Laplacian shape with fat tails.

**SF 9** The volatility of growth rates is negatively associated to income levels.

We now consider the short-run behavior of economies at the business cycle frequencies. First, there is clear evidence that mild recessions coexist with deep crises (Stiglitz, 2011, 2015). Consistently, Fagiolo et al. (2008) investigate the time-series distribution of output growth rates and find that fat tails robustly emerge. Moreover, since the seminal work of Burns et al. (1946), there are also robust stylized facts concerning co-movements and relative volatility between output, consumption and investment (see e.g. Watson and Stock, 1999; Napoletano et al., 2006). Total investment expenditure is more volatile than GDP which, in turn, fluctuates less than consumption. Finally, investment and consumption co-move with GDP and are coincident and procyclical variables. We can then add other stylized facts to the list:
Output grows exponentially displaying large endogenous fluctuations.

Mild recessions coexist with deep downturns.

Investment is more volatile than output while consumption is less volatile.

Investment and consumption are both procyclical and coincident variables.

### 2.2 Industrial dynamics

The process of development involves a structural transformation of the economy (Kuznets 1966). Structural change continuously shapes growth trajectories as resources migrate from traditional agricultural activities to manufacturing and, possibly, nowadays, to information-intensive sectors (Lavopa and Szirmai 2014). In turn, as emphasized by the structuralist approach to development, heterogeneity in productive structures is a fundamental source of income disparities (see e.g. Prebisch et al. 1950). Indeed, fast-growing economies usually manage to specialize and to develop absolute advantages in dynamic sectors characterized by wide learning opportunities and high income elasticities of demand (see Dosi et al. 1990 among many others). More recently, similar patterns have been re-discovered introducing the notions of “complexity” and “product space” (Hausmann et al. 2007; Hidalgo and Hausmann 2009; Tacchella et al. 2012; 2013; Cristelli et al. 2015), where a complexity measure is associated to each commodity using product-level data. In this way, one can relate the performance of a country to the overall complexity of its export basket. Results along these lines have largely confirmed old findings concerning the importance of capabilities accumulation in the production of sophisticated goods. Development is therefore conceived as a process of learning, diversification and self-discovery (Hausmann and Rodrik 2003; Cimoli et al. 2009).

As industries emerge and decline, the characteristics of such a process ought to be studied. Castaldi and Sapio (2008) analyze the distributional properties of industry growth rates finding evidence supporting fat-tailed densities in line with what observed at the country level.

The empirical regularities at the meso level can be summarized as follows:

- **Endogenous structural change accompanies the whole development and growth trajectories.**
- **Heterogeneous productive structures in terms of sectors and products are associated with different revealed performances.**
- **The distribution of industry growth rates are fat-tailed, too.**

### 2.3 Firm-level empirical regularities

Firms are major *loci* where innovation and technical change occurs. As a consequence, they are one of the primary engines of the dynamics observed at the industry and country level. Let us present some microeconomic stylized facts concerning firm dynamics (see Dosi 2007 for a recent survey on the topic).

All available data suggest strong and persistent heterogeneity among firms. Firms differ profoundly in their capabilities and organizational forms, they master different technologies and follow idiosyncratic learning trajectories (Nelson and Winter 1982; Dosi et al. 2001). This maps in firm productivity data which always reveal a large dispersion persisting over time (Bartelsman and Doms 2000). In turn, heterogeneous efficiency levels translate into different profitabilities and performances (Geroski et al. 1993). Partly as a result, the empirical evidence about firm size distribution robustly shows a departure from the Gaussian benchmark and the presence of right-skewness, i.e. few large firms coexist with many small units (Dosi 2007). Micro growth rates distributions are well approximated by fat-tailed
Laplace density (Bottazzi and Secchi, 2003, 2006). As argued in Dosi (2007), the presence of fat tails can be directly related to some underlying lumpiness in the growth process of firms as well as to the correlation structure stemming from the very process of competition (see also Dosi et al., 2016b). Note that growth-rate distributions observed at the firm, industry, and country level suggest that such lumpy process survives aggregation and possibly point at a universal scaling conjecture (Fagiolo et al., 2008).

Given the multi-country perspective of the model which follows, let us also consider firm performances in international markets. First, exporting businesses are only a little fraction of the total firm population (Bernard and Jensen, 1999; Bernard et al., 2012). Then a natural question arises: do exporters display any specific characteristics? Empirical evidence robustly shows that exporting firms are generally larger, more productive, have higher capital-intensity, employ more skilled workers and pay higher wages than non-exporting competitors (Bernard and Jensen, 1999; Bernard et al., 2012).

The foregoing firm-level empirical regularities conclude our list of multi-scale stylized facts:

**SF 17** There are large and persistent productivity differentials across firms within the same sector and country, at all the levels of aggregation, and even more so across countries.

**SF 18** The distribution of firm size departs from normality and is right skewed.

**SF 19** The distribution of firm growth rates exhibit fat tails.

**SF 20** Only few firms are exporters.

**SF 21** Exporters are larger and more productive than non-exporters.

### 3 The Model

The tall ambition of our model is indeed to account jointly for all the stylized facts listed above, or at least for a large part of them. The model features $N$ economies (indexed by $i$). Each country includes $M$ consumption-good industries (indexed by $h$) and a capital-good sector. Each consumption-good sector is populated by $S$ firms (indexed by $j$). Technologies of production are heterogeneous across firms and endogenously evolve via a stochastic process of innovation and imitation. For simplicity, we assume that search and innovation occur only in the consumption-good sector and take the form of labour productivity increases, i.e. technical progress is Harrod neutral. Finally, again for simplicity, countries are endowed with an infinite supply of labor. The proximate ancestors of the model are the multi-country ABM in Dosi et al. (1994) and the Keynes+Schumpeter family of models (Dosi et al., 2010, 2013, 2015, 2017b).

#### 3.1 Timeline of the events

In each each time step $t$ events proceed as follows:

1. Firms in the consumption-good industries perform R&D in order to discover new techniques and to imitate competitors closer to the technology frontier. If and when innovation or imitation are successful, firms can improve their labor productivity.

2. Production, investment and employment decisions take place. Given their expected demand, consumption-good firms set their desired production, hire workers accordingly and, if necessary, expand their productive capacity.

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*As the aim of the model is to study the emergence of technology-driven endogenous growth and convergent/divergeney patterns in a large cross-section of countries, we do not model the financial sector. In this respect, the model is directly comparable with most of the works in the “New Growth” literature.*
3. The capital-good sector in each country receives orders from firms in the consumption-good industries, hire workers, and start the production.

4. Monetary wages and exchange rates are set at the national level.

5. International imperfectly competitive consumption-good markets opens. Workers spend their income on both domestic and imported goods. Firms’ market shares evolve according to their price competitiveness.

6. Entry and exit occur. Firms with quasi-zero market share exit the market and are replaced by new ones.

7. Machines ordered at the beginning of the period are delivered and become part of the capital stock for the following one.

At the end of each time step, the aggregate variables (e.g. GDP, investments, consumption, exports, imports, etc.) are computed by summing the corresponding microeconomic variables. The next sections will provide a detailed description of the model.

3.2 The consumption-good sector

The consumption-good sector in each country is composed by $M$ industries and $S$ firms per industry. Firms are the key drivers of technical change. They invest in R&D ($RD$) a fixed proportion of their past sales ($SS$):

$$RD_{j,h}^i(t) = \rho SS_{j,h}^i(t-1), \quad (1)$$

with $\rho \in (0,1]$. Total R&D expenditures are then split between innovative ($IN$) and imitative ($IM$) efforts:

$$IN_{j,h}^i(t) = \lambda RD_{j,h}^i(t) \quad (2)$$

$$IM_{j,h}^i(t) = (1 - \lambda) RD_{j,h}^i(t), \quad (3)$$

with $0 \leq \lambda \leq 1$.

Innovation and imitation are modelled as a two-step stochastic process. In the first step, a draw from a Bernulli distribution ($\theta$) determines whether firms succeed in their search activities. Probabilities of success ($\theta_{in}$, $\theta_{im}$) are an increasing function of R&D expenditures and of firms’ search capabilities ($\xi_{1,2}>0$):

$$\theta_{in}^i_{j,h}(t) = \min \left\{ \theta_{max}; 1 - e^{-\xi_1 IN_{j,h}^i(t)} \right\} \quad (4)$$

$$\theta_{im}^i_{j,h}(t) = \min \left\{ \theta_{max}; 1 - e^{-\xi_2 IM_{j,h}^i(t)} \right\} \quad (5)$$

Firms succeeding in innovation discover a new production technique associated with a labour productivity coefficient $A_{in}$:

$$A_{in}^i_{j,h}(t) = A_{j,h}^i(t-1)(1 + x_{j,h}^i(t)) \quad \text{where:} \quad x \sim Beta(\alpha_1, \beta_1) \quad (6)$$

5. As common in other evolutionary models (Chiaromonte and Dosi, 1993; Dosi et al., 1994, 2010), R&D strategies are assumed to be entirely routinized and time-invariant. Notice that the assumption of fixed R&D expenditure coefficients is quite in tune with firms actual behaviours (Nelson and Winter, 1982; Dosi, 1988; Dosi and Egidi, 1991).

6. We impose an upper bound $\theta_{max} < 1$ to account for the fact that there is always a minimum degree of uncertainty involved in search activities.
The multiplicative increase \((x)\) is drawn from a Beta distribution with parameters \((\alpha_1, \beta_1)\) and support \([x_1, \bar{x}_1]\), with \(x_1 \in [-1, 0]\) and \(\bar{x}_1 \in [0, 1]\). The shape and support of the Beta distribution captures technological opportunities. Given the high degree of uncertainty characterizing the innovation process, the newly discovered techniques may well be less productive than the ones currently mastered by firms. Technological opportunities and firms’ search capabilities define the characteristics of the technological regime (Dosi 1988; Dosi and Nelson 2010).

Firms able to successfully imitate their competitors will copy randomly a technique \((\text{Aim})\) from the latter. The probability to imitate a specific firm is inversely proportional to the technological distance, measured by Euclidean metric. In tune with the technology-gap literature, we assume that foreign techniques are more difficult to imitate than domestic ones (on the point see Abramovitz 1986; Dosi et al. 1990; Fagerberg et al. 2005). Therefore, if firms are based in different countries, the distance between their technical coefficients is augmented by a multiplicative parameter \(\epsilon > 1\). As we shall see in Section 5 the ease of imitation of foreign technologies plays a crucial role in driving catching-up among countries.

Finally, once both the innovation and imitation processes are completed, each firm selects the most efficient production technique among those that it can master, i.e. the one entailing the higher labor productivity:

\[
A_{j,h}^i(t) = \max \left\{ A_{j,h}^i(t-1); \text{Ain}_{j,h}^i(t); \text{Aim}_{j,h}^i(t) \right\}
\]  

(7)

Given the nominal wage level \((W)\) fixed at the country level (see Equation 23 below), firms set price \((p)\) as a mark-up \((m)\) on the unit cost of production:

\[
p_{j,h}^i(t) = (1 + m_{j,h}^i(t)) \frac{W_{j,h}^i(t)}{A_{j,h}^i(t)}
\]

(8)

The mark-up ratio evolves according the dynamics of past market shares \((f)\):

\[
m_{j,h}^i(t) = m_{j,h}^i(t-1)(1 + \frac{f_{j,h}^i(t-1) - f_{j,h}^i(t-2)}{f_{j,h}^i(t-2)})
\]

(9)

with \(\nu > 0\).

Consumption-good firms produce their output using both labour and capital. While labor productivity grows over time as result of technical change, the capital-output ratio \((B)\) remains constant.\(^7\) Firms set desired production \((Qd)\) according to adaptive demand expectations \((D)\):\(^8\)

\[
Qd_{j,h}^i(t) = f(D_{j,h}^i(t-1), D_{j,h}^i(t-2), \ldots, D_{j,h}^i(t-k)).
\]

(10)

Desired production is constrained by productive capacity. Thus, actual production \((Q)\) is computed as:

\[
Q_{j,h}^i(t) = \min \left\{ Qd_{j,h}^i(t), \frac{K_{j,h}^i(t)}{B} \right\}
\]

(11)

where \(K\) is the stock of capital.

Capacity constrained firms invest to expand their capital stock. More specifically, expansion invest-
ments ($Ie$) occur whenever the desired capital stock ($Kd$) exceeds the actual one.

$$Ie_{j,h}^i(t) = Kd_{j,h}^i(t) - K_{j,h}^i(t),$$  \text{(12)}

with $Kd_{j,h}^i(t) = BQd_{j,h}^i(t)$. Firms invest also to cover (constant) capital depreciation ($\delta$). Hence, replacement investments ($Ir$) are simply:

$$Ir_{j,h}^i(t) = \delta K_{j,h}^i(t),$$  \text{(13)}

with $\delta \in (0, 1)$. The law of motion of capital stocks is then equal to:

$$K_{j,h}^i(t + 1) = K_{j,h}^i(t) + Ie_{j,h}^i(t).$$  \text{(14)}

3.3 The capital-good sector

In each country, domestic firms acquire their machines from an aggregate (i.e. unmodeled ‘single firm’) capital-good sector. Total production ($Q_k$) equals the sum of the orders from domestic firms ($I^i$):

$$Q_k^i(t) = I^i(t).$$  \text{(15)}

The labor productivity in capital-good sectors is assumed to track the average country level $A^i(t)$. In turn, employment is equal to:

$$L_k^i(t) = \frac{Q_k^i(t)}{A^i(t)}.$$  \text{(16)}

Finally, prices tracks the unit cost of production.

3.4 Market dynamics

Market selection regulates the distribution of international demand for different consumption goods across firms. In each country, total consumption corresponds to the wage bill. For simplicity, we assume that workers spend an equal proportion $d_h = 1/M$ of their income in each consumption-good industry.\footnote{Such asymmetric behavior is driven by a positive bias towards optimism as well as by a general concern about the possibility to lose market shares. For a complete discussion of the topic see \cite{Kaldor1951}.}

Each firm is competing in $N$ national markets all characterized by imperfect information. As goods are homogeneous within each industry, firms’ competitiveness depends on the price they charge. Naturally, in foreign markets, firms’ prices are affected by the exchange rate and by trade costs \cite{AndersonVanWincoop2004}. More specifically, given a firm $j$, operating in industry $h$ and based in country $i$, its competitiveness in country $k$ is given by:

$$E_{j,h}^{i,k}(t) = \frac{1}{p_{j,h}^i(t)e^{i,k}(t)(1 + \tau)},$$  \text{(17)}

where $e^{i,k}$ stands for the nominal exchange rate between countries $i$ and $k$, and the parameter $\tau$ captures additional costs for competing in foreign markets (equal to zero if $i = k$ and strictly positive if $i \neq k$). The average competitiveness ($\bar{E}$) for industry $h$ in country $k$ is computed summing up firm

\text{\footnote{Such assumption implies that sectoral income elasticities of demand are constant and equal to 1. This is obviously a simplification: within the evolutionary tradition, the role of structural change driven by changes in patterns of consumption is extensively analyzed in \cite{Verspagen1992}, \cite{Montobbio2002}, \cite{Ciarli2010} and \cite{Lorentz2015}.}}
competitiveness over countries weighted by their market shares:

\[
\bar{E}_k^h(t) = \sum_{i=1}^{N} \sum_{j=1}^{S} E_{j,k}^{i,k}(t) f_{j,k}^i(t - 1).
\]  

(18)

Finally, market selection affects firms’ market shares \((f)\) by means of a quasi-replicator dynamics:  

\[
f_{j,k}^i(t) = f_{j,k}^i(t - 1)(1 + \chi \frac{E_{j,k}^{i,k}(t) - \bar{E}_k^h(t)}{\bar{E}_k^h(t)}),
\]  

(19)

with \(\chi > 0\). In a nutshell, the market shares of more competitive firms in each market will expand, while those of the less efficient ones will shrink. The parameter \(\chi\) accounts for the strength of competition in the market. The market share in the global market of firm \(j\) competing in industry \(h\) is computed as follows:

\[
f_{j,k}^i(t) = \sum_{k=1}^{N} f_{j,k}^i(t) / N.
\]  

(20)

Given the wage \((W)\) and aggregate national employment \((L)\), the domestic demand \((D_{int})\) of each firm corresponds to:

\[
D_{int}^i_{j,h}(t) = W^i(t)L^i(t) d_h f_{j,k}^i(t), \quad \text{with: } i = k
\]  

(21)

Symmetrically the demand for exports \((D_{exp})\) is:

\[
D_{exp}^i_{j,h}(t) = \sum_{k \neq i} W^k(t)L^k(t) e_{k,i}^i(t) d_h f_{j,k}^i(t)
\]  

(22)

International competition is also characterized by Schumpeterian exit and entry dynamics. At each time step, firms with quasi-zero market shares exit the market and are replaced by entrants. The number of firms is thus constant in each industry.  

\(\text{Empirical findings support indeed the idea that entrants are proportional to the number of incumbents (Geroski, 1995). More precisely, firms’ initial techniques are obtained applying to the domestic average productivity in the industry a multiplicative shock drawn from a Beta (} \alpha_2, \beta_2) \text{ with support } [x_2, \bar{x}_2] \text{ (where: } x_2 \in [-1, 0] \text{ and } \bar{x}_2 \in [0, 1]). \text{ Such assumption is consistent with recent theoretical and empirical appraisals pointing out the cumulativeness and the specificity of national learning patterns (Fagerberg, 1994; Cimoli and Dosi, 1995).}\)

3.5 The macroeconomic framework

In each country, the functioning of the labour market is regulated by institutional rules. The supply of labour is infinitely elastic to variations in demand (in line with Lewis, 1954; Cornwall, 1977). Hence, total employment is determined in the goods markets by the total labour demand of consumption- and

11The quasi-replicator dynamics differs from the canonical one since it allows for negative market shares. The standard replicator dynamics instead evolves on the unit simplex. Conversely, the “quasi-replicator” also determines firms death: through the entry and exit process, firms with negative market shares are replaced by a new entities. For a deeper discussion of the replicator dynamics model see Silverberg et al. (1988), Dosi et al. (1995) and Dosi et al. (2016b).

12Empirical findings support indeed the idea that entrants are proportional to the number of incumbents (Geroski, 1995).

13More precisely, firms’ initial techniques are obtained applying to the domestic average productivity in the industry a multiplicative shock drawn from a Beta (} \alpha_2, \beta_2) \text{ with support } [x_2, \bar{x}_2] \text{ (where: } x_2 \in [-1, 0] \text{ and } \bar{x}_2 \in [0, 1]). \text{ Such assumption is consistent with recent theoretical and empirical appraisals pointing out the cumulativeness and the specificity of national learning patterns (Fagerberg, 1994; Cimoli and Dosi, 1995; Fagerberg and Verspagen, 2002; Cimoli et al., 2009).}
Monetary wages are determined by institutional factors as in Dosi et al. (2010):

\[ W^i(t) = W^i(t-1)(1 + \psi_1 g_{prod}^i(t-1) + \psi_2 g_{empl}^i(t-1) + \psi_3 g_{cpi}^i(t-1)), \]  

(23)

with \( \psi_{1,2,3} \geq 0 \). That is, wages are affected by past growth rates of national productivity \( g_{prod} \), employment \( g_{empl} \), and consumption price index \( g_{cpi} \).

Concerning exchange rates \( e \), they evolve according to past current account conditions with a stochastic noise:

\[ e^i(t) = e^i(t-1)(1 + \gamma TB^i(t-1) \bar{Y}(t-1) + u^i(t)) \quad u_t \sim \mathcal{N}(0, \sigma_e), \]  

(24)

where \( TB \) stands for trade balance, \( \bar{Y} \) is world GDP, \( u \) is a white noise, and the parameter \( \gamma \) regulates the sensitivity of the adjustment defining the exchange rate regime. In line with the literature on BOP-constrained growth (see e.g. McCombie and Thirlwall 1994; Thirlwall 1979), such a formulation tries to capture in a parsimonious way the long-run tendency of exchange rates to move in order to balance current accounts among countries.

At the end of each time step, national aggregates are determined simply summing up the corresponding micro variables. Thus, national consumption \( (C) \), total exports \( (EXP) \) and imports \( (IMP) \) are computed as:

\[ C^i(t) = W^i(t)L^i(t); \]  

(25)

\[ EXP^i(t) = \sum_{h=1}^{M} \sum_{j=1}^{S} D_{exp}^{i,j,h}(t); \]  

(26)

\[ IMP^i(t) = C^i(t) - \sum_{h=1}^{M} \sum_{j=1}^{S} D_{int}^{i,j,h}(t). \]  

(27)

Naturally, the trade balance is \( TB^i(t) = EXP^i(t) - IMP^i(t) \). The GDP \( (Y) \) of country \( i \) is then equal to:

\[ Y^i(t) = C^i(t) + I^i(t) + EXP^i(t) - IMP^i(t) \]  

(28)

Of course, trade balances of all countries cancel out at the global level:

\[ \sum_{i=1}^{N} TB^i(t)e^i(t) = 0. \]

4 Simulation Results

How does the model fare in reproducing the empirical regularities presented in Section 2? The results generated by the model are analyzed by means of numerical simulations. We impose identical initial conditions and structural parameters across countries and firms. In this way, we can explore the

---

14 For macroeconomic agent-based models explicitly accounting for decentralized labor-market dynamics see e.g. Fagiolo et al. (2004); Dawid et al. (2012); Riccetti et al. (2015); Dosi et al. (2016a, 2017b,a) and the survey in Fagiolo and Roventini (2017).

15 The exchange rate between two countries \( i \) and \( j \) can be computed as: \( e^{i,j} = \frac{e^i}{e^j} \). Model properties are robust also to a scenario with fixed exchange rates \( (\gamma = 0; \sigma = 0) \). Results are available from the authors upon request.

16 Even if most of behavioural rules and interaction mechanisms are grounded on the empirical evidence, we did not explicitly perform any calibration exercises. However, simulations results robustly emerge for a large part of the parameter space. For recent developments in the fields of validation and calibration of ABMs see e.g. Lamperti (2017), Guerini and Moneta (2017) and Grazzini et al. (2017). A survey of the literature is provided by Fagiolo and Roventini (2017).
endogenous emergence of heterogeneity across firms and industries, and study whether it generates convergent/divergent international growth patterns.

Below we present the results of Monte Carlo simulations. Structural parameters are presented in Table 9 in the Appendix.

We first consider the growth trajectories emerging at the international level (Section 4.1). We then zoom in and study industrial and firm dynamics (Section 4.2).

### 4.1 Endogenous growth and divergent patterns

Let us start by considering the dynamics of per capita income of the sixty countries composing our world economy (cf. Figure 1). First, the model endogenously generates secular exponential growth in incomes per capita (SF 1) and divergent patterns across countries. Simulations (cf. Figure 2) also show emergent episodes of forging-ahead, catching-up and falling-behind (SF 2).

Simulated GDP data do not reveal any tendency to convergence. The first two moments of the

---

17Income and productivity variables are always expressed at constant prices and exchange rates.
Figure 3: Income distribution moments dynamics. Monte Carlo 5% confidence intervals in grey

income distribution increase over time (cf. Figure 3), thus rejecting the $\sigma$-convergence hypothesis as in real data (SF 3). Again, in tune with the empirical evidence, $\beta$-convergence does not occur (SF 4). More precisely, we sequentially regress the average growth rates ($g_y$) for the period $t, t + t^*$ versus the initial income levels ($y$): 

$$g_y(t, t + t^*) = \alpha + \beta y(t).$$

Results in Figure 4 show that negative and significant estimates of $\beta$ are ephemeral episodes. The unconditional convergence hypothesis thus fails unless introducing some specific selection bias. Such conclusion is reinforced by the Monte Carlo averages of $\beta$ estimates, which monotonically approach zero over time (see Figure 5). This suggests that, as the technological distance among countries increases, imitation and catching-up become more difficult. The foregoing results are corroborated by the different convergence tests proposed by Bernard and Durlauf (1991).

However, as the moments of the income distribution do not fully account for its time dynamics (Quah 1996), we show in Figure 6 the evolution of the whole empirical density of international incomes, which clearly moves from an unimodal shape towards a bimodal one at the end of the simulation (SF 5). In turn, the model endogenously generates two convergence clubs for poor and advanced countries, with the latter being relatively smaller than the former. Such results are corroborated by bimodality tests (cf. Table 1) commonly employed in the growth literature (Bianchi et al. 1997; Henderson et al. 2008). The Silverman tests rejects unimodality ($M = 1$) at the 10% level already at $t = 200$ while the bimodality hypothesis ($M = 2$) cannot be rejected. Consistently with the empirical findings of Henderson et al. (2008), the more conservative DIP test fails to reject unimodality during the simulation. However, the

---

18 We estimate a battery of augmented Dickey-Fuller equations and apply the Engel-Granger procedure for cointegration. The results, available on request, are in line with those of Bernard and Durlauf (1991).

19 Income (and productivity) data are normalized taking logs and subtracting the cross-country average to remove common trends: $y_{i,t} = \log Y_{i,t} - \bar{Y}_t$. Where $Y$ is the original variable and $\bar{Y}$ is an average across countries. As a result, the corresponding growth rate densities are centered on zero. The same normalization is performed for industry- and firm-level data, when studying distributional properties.
decreasing trend in the p-values clearly provides further evidence against unimodality.

Relatedly, the estimation of transition probability matrix for five different classes of country income (cf. Table 2) reveals a general lack of mobility within the distribution\(^\text{20}\). Indeed, the high probability values along the main diagonal suggest that relative country rankings are sticky (SF 6). Moreover, the associated ergodic distribution shows that the probability mass tend to (asymptotically) concentrate on the tails, pointing, once again, at an on-going process of polarization.

Let us now consider the scaling behavior of output growth rates. Consistently with Castaldi and Dosi (2009), we order pooled normalized per-capita income observations \((y)\) and we divide them in equally populated bins. Then, we regress the mean and the standard deviation of growth rates \((g)\) in

\[\hat{p}_{i,j} = \frac{n_{i,j}}{n_i}\]

where \(n_i\) is the number of observations in state \(i\) and \(n_{i,j}\) is the number of observed transition from \(i\) to \(j\). This corresponds to the maximum likelihood estimators of true probabilities (Norris, 1998).
each class $i$ versus the associated average income level ($\bar{y}$):

$$\mu(g)_i = \alpha + \beta \bar{y}_i + \epsilon_t$$

$$\log(\sigma(g)_i) = \alpha + \beta \bar{y}_i + \epsilon_t$$

A plot for a single realization is presented in Figure 7 while Table 3 provides Monte Carlo averaged estimated coefficients. We find that the volatility of $g$ (in logs) scales negatively with income levels (SF 9), suggesting that poor countries are subject to more severe aggregate fluctuations than advanced economies. The positive relationship found between growth rates and income levels instead points at the existence of dynamic increasing returns in production (Castaldi and Dosi, 2009).

Countries do not appear to follow a steady growth trajectory. In line with the empirical evidence, the average across-periods correlation of country growth rates (cf. Table 4) are rather weak, suggesting that growth experiences are relatively unstable (SF 7).

We then investigate the statistical properties of output growth rates distributions. More specifically, we fit the exponential-power family of densities over the empirical distribution of cross-country growth rates. In tune with the empirical evidence (SF 8), the estimated $b$ parameter is close to unity (cf. Table 5), i.e. a Laplacian shape with tails much fatter than the Gaussian benchmark provides a good fit.

Following Bottazzi and Secchi (2003), we fit a symmetric Subbotin function which has the form:

$$f(x) = \frac{1}{2ab^{1/4}\Gamma(1+1/b)} e^{-\frac{1}{b}|\frac{x-m}{a}|^b},$$

where $m$ is the location parameter, $a$ accounts for the scale and $b$ governs the fatness of the tails. For $b = 2$ the distribution converges to a normal whereas for $b = 1$ it describes the Laplace distribution. Estimates of the Subbotin are performed also with industry- and firm-level data and are reported in Table 5.
<table>
<thead>
<tr>
<th>Time Step</th>
<th>M=1</th>
<th>M=2</th>
<th>M=3</th>
<th>DIP test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2482</td>
<td>0.3975</td>
<td>0.4030</td>
<td>0.8410</td>
</tr>
<tr>
<td></td>
<td>(0.0380)</td>
<td>(0.0371)</td>
<td>(0.0344)</td>
<td>(0.0238)</td>
</tr>
<tr>
<td>100</td>
<td>0.1516</td>
<td>0.2982</td>
<td>0.3890</td>
<td>0.4762</td>
</tr>
<tr>
<td></td>
<td>(0.0243)</td>
<td>(0.0336)</td>
<td>(0.0378)</td>
<td>(0.0480)</td>
</tr>
<tr>
<td>200</td>
<td>0.0684</td>
<td>0.4526</td>
<td>0.4582</td>
<td>0.2803</td>
</tr>
<tr>
<td></td>
<td>(0.0204)</td>
<td>(0.0390)</td>
<td>(0.0351)</td>
<td>(0.0444)</td>
</tr>
<tr>
<td>300</td>
<td>0.0620</td>
<td>0.4603</td>
<td>0.5191</td>
<td>0.1795</td>
</tr>
<tr>
<td></td>
<td>(0.0255)</td>
<td>(0.0416)</td>
<td>(0.0337)</td>
<td>(0.0402)</td>
</tr>
<tr>
<td>400</td>
<td>0.0373</td>
<td>0.4155</td>
<td>0.4205</td>
<td>0.1627</td>
</tr>
<tr>
<td></td>
<td>(0.0181)</td>
<td>(0.0356)</td>
<td>(0.0333)</td>
<td>(0.0332)</td>
</tr>
<tr>
<td>500</td>
<td>0.0336</td>
<td>0.4642</td>
<td>0.4577</td>
<td>0.1537</td>
</tr>
<tr>
<td></td>
<td>(0.0160)</td>
<td>(0.0345)</td>
<td>(0.0305)</td>
<td>(0.0342)</td>
</tr>
</tbody>
</table>

Table 1: *p*-values from multimodality tests at different time steps; *variable*: per capita GDP. Monte Carlo standard errors are in brackets.

<table>
<thead>
<tr>
<th>N. obs.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>5751.28</td>
<td>0.9325</td>
<td>0.0663</td>
<td>0.0011</td>
<td>0.0001</td>
<td>0</td>
</tr>
<tr>
<td>(213.2393)</td>
<td>(0.0033)</td>
<td>(0.0032)</td>
<td>(0.0001)</td>
<td>(0.0000)</td>
<td></td>
</tr>
<tr>
<td>6601.52</td>
<td>0.0682</td>
<td>0.8528</td>
<td>0.0777</td>
<td>0.0013</td>
<td>0</td>
</tr>
<tr>
<td>(160.1320)</td>
<td>(0.0013)</td>
<td>(0.0022)</td>
<td>(0.0018)</td>
<td>(0.0001)</td>
<td></td>
</tr>
<tr>
<td>4979.2</td>
<td>0.0008</td>
<td>0.1222</td>
<td>0.7763</td>
<td>0.0988</td>
<td>0.0019</td>
</tr>
<tr>
<td>(132.6577)</td>
<td>(0.0001)</td>
<td>(0.0019)</td>
<td>(0.0020)</td>
<td>(0.0016)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>4161.98</td>
<td>0</td>
<td>0.0022</td>
<td>0.1252</td>
<td>0.7689</td>
<td>0.1037</td>
</tr>
<tr>
<td>(124.6708)</td>
<td>(0.0002)</td>
<td>(0.0024)</td>
<td>(0.0029)</td>
<td>(0.0017)</td>
<td></td>
</tr>
<tr>
<td>8326.02</td>
<td>0</td>
<td>0.0001</td>
<td>0.0014</td>
<td>0.0458</td>
<td>0.9528</td>
</tr>
<tr>
<td>(83.2227)</td>
<td>(0.0000)</td>
<td>(0.0001)</td>
<td>(0.0015)</td>
<td>(0.0014)</td>
<td></td>
</tr>
<tr>
<td>Ergodic</td>
<td>0.2495</td>
<td>0.2281</td>
<td>0.1468</td>
<td>0.1166</td>
<td>0.2591</td>
</tr>
<tr>
<td></td>
<td>(0.0099)</td>
<td>(0.0057)</td>
<td>(0.0044)</td>
<td>(0.0041)</td>
<td>(0.0028)</td>
</tr>
</tbody>
</table>

Table 2: 3-step transition probability matrix and implied ergodic distribution; *variable*: per capita GDP. Monte-Carlo standard errors are in brackets.

<table>
<thead>
<tr>
<th>Std. Dev.</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binned OLS</td>
<td>-0.2077</td>
</tr>
<tr>
<td></td>
<td>(0.0090)</td>
</tr>
</tbody>
</table>

Table 3: Scaling relations (Binned OLS); *variable*: per capita GDP. Monte-Carlo standard errors are in brackets.
Figure 7: Scaling laws (single realization); \textit{left}: Growth rates standard deviation vs. income levels; \textit{right}: Growth rates mean vs. income levels

<table>
<thead>
<tr>
<th>Period length</th>
<th>5</th>
<th>8</th>
<th>10</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple</td>
<td>0.2755</td>
<td>0.2171</td>
<td>0.1647</td>
<td>0.0293</td>
</tr>
<tr>
<td></td>
<td>(0.0043)</td>
<td>(0.0046)</td>
<td>(0.0052)</td>
<td>(0.0050)</td>
</tr>
<tr>
<td>Rank</td>
<td>0.2261</td>
<td>0.1680</td>
<td>0.1266</td>
<td>0.0263</td>
</tr>
<tr>
<td></td>
<td>(0.0031)</td>
<td>(0.0036)</td>
<td>(0.0043)</td>
<td>(0.0042)</td>
</tr>
</tbody>
</table>

Table 4: Average across-periods correlation in growth rates; \textit{variable}: per capita GDP. Monte-Carlo standard errors are in brackets.

<table>
<thead>
<tr>
<th></th>
<th>b</th>
<th>a</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Per capita income</td>
<td>1.0171</td>
<td>0.0248</td>
<td>-0.0028</td>
</tr>
<tr>
<td></td>
<td>(0.0056)</td>
<td>(0.0002)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>Output</td>
<td>0.9776</td>
<td>0.0476</td>
<td>-0.0015</td>
</tr>
<tr>
<td></td>
<td>(0.0061)</td>
<td>(0.0005)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>Per capita income (time series)</td>
<td>1.1423</td>
<td>0.0252</td>
<td>0.0234</td>
</tr>
<tr>
<td></td>
<td>(0.0378)</td>
<td>(0.0007)</td>
<td>(0.0004)</td>
</tr>
<tr>
<td>Output (time series)</td>
<td>1.1102</td>
<td>0.0513</td>
<td>0.0276</td>
</tr>
<tr>
<td></td>
<td>(0.0307)</td>
<td>(0.0011)</td>
<td>(0.0007)</td>
</tr>
<tr>
<td>Industry output</td>
<td>0.5791</td>
<td>0.0135</td>
<td>-0.0073</td>
</tr>
<tr>
<td></td>
<td>(0.0026)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
</tr>
<tr>
<td>Firms output (country pooling)</td>
<td>1.1435</td>
<td>0.0926</td>
<td>-0.0105</td>
</tr>
<tr>
<td></td>
<td>(0.0151)</td>
<td>(0.0013)</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>Firms output (single industry)</td>
<td>1.1495</td>
<td>0.0926</td>
<td>-0.0105</td>
</tr>
<tr>
<td></td>
<td>(0.0153)</td>
<td>(0.0012)</td>
<td>(0.0005)</td>
</tr>
</tbody>
</table>

Table 5: Exponential power parameters estimation at different levels of aggregation. Monte-Carlo standard errors are in brackets.
Figure 8: Macro growth rates distributions (empirical density vs. normal fit); left panels: cross-sectional pooling; right panels: time series growth rates for a single economy.

Similar results are also found when one considers the time-series distribution of output growth rates for a given country (cf. right panels in Figure 8). This in turn implies that the growth process of country is characterized by endogenous fluctuations and (rarer) deep crises (SF 11, cf. Fagiolo et al., 2008). Finally, we consider the business-cycle properties of macroeconomic time series. In line with the empirical evidence (Watson and Stock, 1999), the detrended series of aggregate investment is more volatile than GDP, while the latter fluctuate less than aggregate consumption (SF 12). Moreover, cross-correlations among macro variables at the business cycle frequencies suggest that consumption, investment, employment and productivity are procyclical as they track GDP fluctuations (SF 13).

4.2 Emergent structural change and firm heterogeneity

The foregoing macroeconomic patterns result from a rich dynamics at the industry level shaped by the innovative activities of firms and by processes of market selection. First, the evolution of industry output shares for four randomly selected countries (cf. Figure 9) reveals that the model is able to generate endogenous structural change (SF 14). Note that at the beginning of the simulation, economies are equal also in terms of specialization. However, the relative weights of industries evolve over time, leading to heterogeneous productive structures across countries (SF 15). Interestingly, in some economies, sectors appear to emerge and decline rapidly while others seem to experience more stable dynamics. This, in turn, implies that the patterns of structural change also differ across countries (McMillan et al., 2014). Moreover, disparate specialization trajectories are found to drive different growth performances. To highlight this point, we report in Figure 10 the productivity gap (at the end of the simulation) disaggregated by industries between the subset of countries in the top income decile vis-à-vis those in the bottom one. The gap appears to be significant in almost all sectors. In line with evolutionary intuitions (Dosi et al., 1990) and with empirical findings.

\footnote{Due to space reasons, we do not report the results in the paper. They are available from the authors upon request.}
Figure 9: Industry output shares evolution (4 randomly selected countries)

Figure 10: Productivity-gaps by industry between leaders and laggards. Monte Carlo 5% confidence intervals are given by black bands. Leaders and laggards are selected as respectively the top and the bottom 10% countries in terms of average income ranking during the last 100 steps.
Figure 11: Industry and firm growth rates distributions (empirical density vs. normal fit)

Figure 12: Firms productivity standard deviation (4 randomly selected industries)
The heterogeneity across sectors is also revealed by the distribution of within-country growth rates for industry output. Once again, there is a strong departure from normality with emerging fat tails (cf. Subbotin estimates in Table 5 and the left panel in Figure 11): different industries experience large growth episodes and sharp contractions (SF 16).

The stylized facts of industrial dynamics result from the interactions of heterogeneous innovating firms. In tune with microeconomic evidence (SF 17), there is persistent productivity heterogeneity across firms (cf. Figure 12). The productivity differentials map into different market shares, profits levels and eventually size. The distribution of firm size is indeed right-skewed (SF 18), suggesting the co-existence of few successful large entities with many small businesses (cf. Figure 13). Firms growth rate distributions exhibit a fat-tailed “tent” shape (SF 19), alike those found at the industry and country levels (cf. Subbotin estimates in Table 5 central and right panels in Figure 11). It seems that lumpy growth processes at the micro level are not washed away by aggregation, suggesting a possible “universal” mechanism of growth for firms, industries and countries.

Finally, the model also replicates some pieces of empirical evidence on firm-dynamics and international trade. In Table 6, we report some Monte Carlo statistics on exporters shares and premia. Market selection mechanisms allow only a small fraction of total domestic firms (around 6.5%) to penetrate in foreign markets (SF 20). As observed in real data, there are premia associated to the export status (SF 21): exporters are more productive, they employ more workers and display higher sales than non-exporting competitors. The second row in Table 6 shows that such features persist also when controlling for industry-specific characteristics.
## Exporters premia

<table>
<thead>
<tr>
<th></th>
<th>Exp. Share (%)</th>
<th>Productivity (%)</th>
<th>Employment (%)</th>
<th>Tot. sales (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Country level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.5975</td>
<td>74.3132</td>
<td>34.5098</td>
<td>202.2531</td>
</tr>
<tr>
<td></td>
<td>(0.0775)</td>
<td>(0.5667)</td>
<td>(3.8768)</td>
<td>(3.2267)</td>
</tr>
<tr>
<td><strong>Industry level</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16.8310</td>
<td>124.2774</td>
<td>136.0739</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0847)</td>
<td>(2.6970)</td>
<td>(2.4416)</td>
<td></td>
</tr>
</tbody>
</table>

*Notes: A firm is considered exporter at t if \( f_{i,t} > f_{min} \cdot 1.05 \) in at least one country. Where: \( f_{min} = \frac{1}{(N\times S)} \). Export premia are computed as: \([\log(X_{EXP}) - \log(X_{NEXP})] \cdot 100\). Where: \( X_{EXP} \) and \( X_{NEXP} \) are averages respectively for exporters and non-exporters.*

Table 6: Exporters shares and premia. Monte-Carlo standard errors are in brackets.

## 5 General discussion

A summary of all the stylized facts is provided in Table 7. Simulation results have shown that a parsimonious multi-country agent-based model can account for endogenous growth and a rich ensemble of empirical regularities at different levels of aggregation. Such results can be achieved with only two basic drivers. On the supply side, an endogenous engine of technical change is grounded on firm-specific innovative and imitative activities. On the demand side, Keynesian/Kaldorian mechanisms endogenously determine aggregate demand and its distribution across countries (via technological gaps/leads and foreign trade multipliers).

Note that commonly found explanatory variables such as the education level or the degree of political stability are not even modelled here, while other variables such as R&D propensities are assumed identical across-countries and time-invariant. Therefore, they cannot be at the root of the ubiquitous emergent dynamics of differentiation and divergence.

Indeed such “secular” phenomena may be robustly accounted for by the interaction between idiosyncratic learning, trade competitiveness and demand generation under conditions of dynamic increasing returns. In line with Myrdal (1957) and Kaldor et al. (1967), virtuous and vicious cycles of cumulative events at the firm level survive aggregation and affect overall trade balances and GDP growth. At the micro level, a virtuous cycle in our model is typically given by:

i. An idiosyncratic productivity increase either via innovation or imitation.

ii. If such productivity gain is not compensated by increases in wages or by an appreciation of the exchange rate (both system-level variables in our model), the firm will be able to increase its price competitiveness in both national and foreign markets, boosting its sales, exports, and output.

iii. In the following period, higher sales entail higher R&D expenditures which, in turn, increase the probability of achieving a new productivity increase, etc.

The asymmetric accumulation and propagation of (endogenous) productivity and demand shocks at the firm level is then responsible for the emerging macro divergence.

The foregoing sequence of cumulative feedback propagates to the macroeconomic level both on the supply and on the demand side. Concerning the former, start by noticing that when the firm size distribution is skewed (as in our model, cf. Figure 13), the macrodynamics will be affected by the evolution of few large firms. This is in line with the granular hypothesis by Gabaix (2011), although it has little to do with propagation. More important for our purposes here is the possibility of (easier)
Table 7: Summary of stylized facts

<table>
<thead>
<tr>
<th>Stylized facts</th>
<th>Related literature</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Macroeconomic level</strong></td>
<td></td>
</tr>
<tr>
<td>SF 1       Secular increase in per-capita incomes</td>
<td>Maddison (2010)</td>
</tr>
<tr>
<td>SF 2       Endogenous catching up, forging ahead and falling behind episodes</td>
<td>Abramovitz (1995)</td>
</tr>
<tr>
<td>SF 3       Rising income dispersion</td>
<td>Sala-i Martin (1996), Lee et al. (1997)</td>
</tr>
<tr>
<td>SF 5       Bimodality in the cross-sectional distribution of income</td>
<td>Bianchi et al. (1997), Henderson et al. (2008)</td>
</tr>
<tr>
<td>SF 6       Lack of mobility across income classes</td>
<td>Quah (1993), Easterly et al. (1993), Pritchett et al. (2000)</td>
</tr>
<tr>
<td>SF 7       Low across-period correlation of growth rates</td>
<td>Easterly et al. (1993), Pritchett et al. (2000)</td>
</tr>
<tr>
<td>SF 8       Fat-tailed distribution of international growth rates</td>
<td>Castaldi and Dosi (2009)</td>
</tr>
<tr>
<td>SF 9       Growth-rate standard deviation scales negatively with income levels</td>
<td>Canning et al. (1998), Castaldi and Dosi (2009)</td>
</tr>
<tr>
<td>SF 10      Self-sustained growth in GDP with endogenous fluctuations</td>
<td>Maddison (2010)</td>
</tr>
<tr>
<td>SF 11      Mild recessions coexist with deep downturns</td>
<td>Stiglitz (2015), Fagiolo et al. (2008)</td>
</tr>
<tr>
<td>SF 12      Relative volatility of GDP, consumption and investment</td>
<td>Baxter and King (1999)</td>
</tr>
<tr>
<td>SF 13      Cross-correlation of macro variables</td>
<td>Watson and Stock (1999), Napoli et al. (2006)</td>
</tr>
<tr>
<td><strong>Industry level</strong></td>
<td></td>
</tr>
<tr>
<td>SF 14      Endogenous structural change</td>
<td>Kuznets (1966)</td>
</tr>
<tr>
<td>SF 15      Heterogeneity in productive structures</td>
<td>Dosi et al. (1990), McMillan et al. (2014)</td>
</tr>
<tr>
<td>SF 16      Fat-tailed distribution of industry output growth rates</td>
<td>Castaldi and Sapi (2008)</td>
</tr>
<tr>
<td><strong>Firm level</strong></td>
<td></td>
</tr>
<tr>
<td>SF 17      Persistent across-firm heterogeneity in productivity</td>
<td>Bartelsman and Doms (2000), Dosi (2007)</td>
</tr>
<tr>
<td>SF 18      Skewed firm size distribution</td>
<td>Bottazzi and Secchi (2003), Dosi (2007)</td>
</tr>
<tr>
<td>SF 20      Not all firms export</td>
<td>Bernard and Jensen (1999), Bernard et al. (2012)</td>
</tr>
<tr>
<td>SF 21      Exporters are more productive and larger than non-exporters</td>
<td>Bernard and Jensen (1999), Bernard et al. (2012)</td>
</tr>
</tbody>
</table>

domestic imitation which implies some sort of “dynamic spillovers”. These interdependencies tend to generate co-movements between unit at the micro level which will not be averaged out when increasing the scale of observation.\(^{24}\) In the model, national productivity interdependences are enhanced by the process of firm entry as entrants’ initial productivity is linked to the average one in the country. On the demand side, exports translates into demand impulses for the domestic economy, whereby an external demand shock amplifies via more output, more employment, more wages, yet more demand etc. (that is, the foreign trade multiplier).\(^{25}\) As a result of these transmission mechanisms, self-reinforcing divergence in productivity and income levels will also be found in aggregate data. Moreover, in line with Kaldorian development theory, high productivity growth will be associated also with positive export performances and trade surpluses.

To investigate these links, Table 8 presents some statistics for leader and laggard countries. We start by pooling together industry-level data on productivity for the two groups. Advanced economies display both higher average productivity levels (at the end of the simulation) and faster productivity growth. Trade balances tend to be (on average) positive for rich countries and negative for laggards. Relatedly, leaders also exhibit a larger share of firms that are able to penetrate in foreign markets. These results indeed suggest that evolutionary microfoundations can robustly yield Kaldorian cycles of cumulative causation.\(^{26}\)

Exchange rates adjustments, international barriers to competition, as well as foreign imitation oppor-
<table>
<thead>
<tr>
<th></th>
<th>Leaders</th>
<th>Laggards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industry productivity levels</td>
<td>9.4593</td>
<td>9.0032</td>
</tr>
<tr>
<td></td>
<td>(0.0567)</td>
<td>(0.0613)</td>
</tr>
<tr>
<td>Industry productivity growth</td>
<td>0.0157</td>
<td>0.0149</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>Normalized trade balance</td>
<td>0.0017</td>
<td>−0.0011</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Industry exporters share</td>
<td>0.0794</td>
<td>0.0398</td>
</tr>
<tr>
<td></td>
<td>(0.0016)</td>
<td>(0.0011)</td>
</tr>
</tbody>
</table>

Table 8: Leaders vs. Laggards characteristics. Monte-Carlo standard errors are in brackets. Leaders and laggards countries are selected as respectively the top and the bottom 10% countries in terms of average income ranking during the last 100 simulation steps.

Divergent growth is also associated with different patterns of structural change and specialization trajectories. At the beginning of the simulation countries are symmetrically specialized, producing an equal share of output in each industry. As technical change takes place, some firms will become leaders in their respective industries increasing their market shares on international markets. At the country level, this will cause a rise in output shares in the industries where leaders are located and a relative fall in other activities. In other words, specializations patterns are triggered by absolute advantages with respect to international competitors, while standard Ricardian inter-industry cost differences play almost no role. As emphasized in the evolutionary trade theory (Dosi et al., 1990), output composition is shaped by the evolution of technological gaps and leads. In Section 4.2, we showed how absolute technological advantages emerge for leader countries along the simulation history (cf. Figure 10). Such results highlight the importance of developing absolute advantages (or reducing absolute disadvantages) in many activities as the primary source of economic success.

6 Concluding remarks

In this work we developed an agent-based multi-country model in order to investigate endogenous growth patterns of divergence/convergence among different economies. The model bridges theoretical insights from evolutionary theory with applied research in the technology-gap tradition.

Simulation results show indeed the generic emergence of divergent and complex growth dynamics exhibiting a strong tendency towards polarization and clubs formation. Furthermore, each country

27Evolutionary economists have long way argued in favour of industrial policies (Freeman, 1989; Cimoli et al., 2009). The importance of industrial policies is also confirmed by various historical studies (Amsden, 1989; Kim and Nelson, 2000; Wade, 1990; Nelson and Pack, 1999; Lee, 2013).

28Since wages are tied to average national productivity, industries which are more productive than the national average will show a reduction in unit costs. This may yield a correlation between output shares and unit labour costs which can be spuriously interpreted as providing support to comparative advantages-based theories. Nevertheless, this is only a consequence of the process of structural and technological transformation inducing industry cost-adjustments via nominal wages variations.

29Castellacci (2007) mentions this task as one of the most fascinating challenges in the field.
experiences an endogenous transformation of its productive structure during the development process. Both aspects are emergent outcomes of the co-evolution of Schumpeterian microfoundations and aggregate demand propagation mechanisms in tune with Kaldorian development theory. Indeed, at the microeconomic level, the innovative performances of firms lead to knowledge accumulation, increasing production and exports, which in turn trigger structural transformation and changed patterns of specialization. Overall, such dynamics leads to the emergence of virtuous and vicious development trajectories among countries. The robustness of the model is corroborated by the fact that it accounts for a rich ensemble of empirical regularities at macro, meso and micro levels of aggregation.

The model can be extended along several research avenues. First, one can introduce a more sophisticated characterization of trade interactions, accounting for finer evidence on international trade (Bernard et al. 2012). These would allow us to test different industrial and macroeconomic policies that laggard countries could implement to catching up with the technological frontier. Finally, financial markets and international movement of capitals should be modeled in order to develop a more sophisticated exchange rate dynamics and account for emerging financial crises that could freeze or stop the development of countries.

Acknowledgments
We thank Mauro Napoletano, Alberto Russo, Giorgio Fagiolo, Carolina Castaldi, Andre Lorentz, Alessandro Caiani, Mary Kaltenberg, Francesco Lamperti, Pietro Santoleri and Caterina Santi for providing useful comments and discussions. We are also grateful to participants of the EMAEE 2017 conference in Strasbourg, the WEHIA 2017 conference in Milan, the CEF 2017 conference in New York, the FinGro 2017 conference in Milan and the MEIDE 2017 conference in Montevideo. All usual disclaimers apply. The authors acknowledge the support by the European Union’s Horizon 2020 research and innovation program under grant agreement No. 649186 - ISIgrowth.

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Roventini, A., Dosi, G., Napoletano, M., Stiglitz, J. E., and Treibich, T. (2016). Expectation formation, fiscal policies and macroeconomic performance when agents are heterogeneous and the world is changing. Working paper series forthcoming, Laboratory of Economics and Management (LEM), Scuola Superiore Sant’Anna, Pisa, Italy.


Appendix. Parameter Values

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of countries</td>
<td>$N$</td>
<td>60</td>
</tr>
<tr>
<td>Number of industries</td>
<td>$M$</td>
<td>30</td>
</tr>
<tr>
<td>Number of firms (each industry)</td>
<td>$S$</td>
<td>20</td>
</tr>
<tr>
<td>Sectoral demand shares</td>
<td>$d_h$</td>
<td>$1/30$</td>
</tr>
<tr>
<td>Capital-output ratio</td>
<td>$B$</td>
<td>3</td>
</tr>
<tr>
<td>Mark-up adjustment parameter</td>
<td>$\nu$</td>
<td>0.04</td>
</tr>
<tr>
<td>R&amp;D investment propensity</td>
<td>$\rho$</td>
<td>0.04</td>
</tr>
<tr>
<td>R&amp;D allocation parameter</td>
<td>$\lambda$</td>
<td>0.5</td>
</tr>
<tr>
<td>Firms search capabilities</td>
<td>$\xi_{1,2}$</td>
<td>0.08</td>
</tr>
<tr>
<td>First stage probabilities upper bound</td>
<td>$\theta_{max}$</td>
<td>0.75</td>
</tr>
<tr>
<td>Beta distribution parameter</td>
<td>$(\alpha_1, \beta_1)$</td>
<td>(1,5)</td>
</tr>
<tr>
<td>Beta distribution support</td>
<td>$[\underline{x}_1, \overline{x}_1]$</td>
<td>[-0.05,0.25]</td>
</tr>
<tr>
<td>Beta distribution parameter (ent.)</td>
<td>$(\alpha_2, \beta_2)$</td>
<td>(1,5)</td>
</tr>
<tr>
<td>Beta distribution support (ent.)</td>
<td>$[\underline{x}_2, \overline{x}_2]$</td>
<td>[-0.03,0.15]</td>
</tr>
<tr>
<td>Foreign imitation penalty</td>
<td>$\epsilon$</td>
<td>5</td>
</tr>
<tr>
<td>Foreign competition penalty</td>
<td>$\tau$</td>
<td>0.05</td>
</tr>
<tr>
<td>Replicator dynamics parameter</td>
<td>$\chi$</td>
<td>1</td>
</tr>
<tr>
<td>Wage sensitivity parameters</td>
<td>$(\psi_1, \psi_2, \psi_3)$</td>
<td>(1, 0, 0)</td>
</tr>
<tr>
<td>Exchange rates flexibility</td>
<td>$\gamma$</td>
<td>0.1</td>
</tr>
<tr>
<td>Exchange rates shocks std. dev.</td>
<td>$\sigma_e$</td>
<td>0.002</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$</td>
<td>0.02</td>
</tr>
<tr>
<td>Monte-Carlo replications</td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>

Table 9: Benchmark Parametrization
Its 1981 founding charter established it as part of the French Fondation nationale des sciences politiques (Sciences Po), and gave it the mission is to “ensure that the fruits of scientific rigour and academic independence serve the public debate about the economy”. The OFCE fulfils this mission by conducting theoretical and empirical studies, taking part in international scientific networks, and assuring a regular presence in the media through close cooperation with the French and European public authorities. The work of the OFCE covers most fields of economic analysis, from macroeconomics, growth, social welfare programmes, taxation and employment policy to sustainable development, competition, innovation and regulatory affairs.

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Sciences Po is an institution of higher education and research in the humanities and social sciences. Its work in law, economics, history, political science and sociology is pursued through ten research units and several crosscutting programmes. Its research community includes over two hundred twenty members and three hundred fifty PhD candidates. Recognized internationally, their work covers a wide range of topics including education, democracies, urban development, globalization and public health. One of Sciences Po’s key objectives is to make a significant contribution to methodological, epistemological and theoretical advances in the humanities and social sciences. Sciences Po’s mission is also to share the results of its research with the international research community, students, and more broadly, society as a whole.

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