HOW CAN IT WORK? ON THE IMPACT OF QUANTITATIVE EASING IN THE EUROZONE

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Abstract
How can quantitative easing (QE) work in the Eurozone (EZ)? We model the EZ as the aggregate of two countries characterised by New Keynesian output and inflation equations with a Tobinian money market equation that determines each country's interest rate as a spread above the common policy rate. High spreads determine negative output gaps and deflationary pressure. With the ECB policy rate at the zero lower bound, QE expands money supply throughout the EZ. We show that QE, if large enough, can indeed be effective by reducing country spreads and the ensuing output gaps. However, zero output and deflation gaps can be obtained for the EZ on average, but not for all single countries unless fully symmetric conditions are met. Therefore fiscal accommodation at the country level should also intervene, and we conclude that the coordination of fiscal and monetary policies is of paramount importance.

Keywords: Monetary Policy, ECB, Deflation, Zero-Lower-Bound, Fiscal Policy
JEL Codes: E3, E4, E5

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1. Introduction

The decision of the European Central Bank (ECB) to launch a quantitative easing (QE) programme testifies of both the persistence of poor macroeconomic conditions in the Eurozone (EZ) and a major change in the stance of the ECB with respect to its conventional approach to monetary policy. The ECB justifies its QE programme (e.g. Draghi 2014a, b) with the widening gap between both the actual and the expected inflation rates vis-à-vis the official target "not exceeding but close to 2%" per year, in a context where the policy rate is at the zero lower bound (ZLB) and conventional monetary weapons have proved ineffective owing to deep "segmentation" and "nationalisation" of financial markets. The programme will be carried on at least until March 2017, and in any case until the inflation expectations will be on target.

Though supported by academic research (e.g. Bernanke and Reinhart 2004, Eggertsson and Woodford 2004, Orphanides 2014, De Grauwe and Ji 2015) and welcomed by most EZ governments, international partners and official institutions, QE is still surrounded by some scepticism. The assessment of QE forerunners' experience (United States, United Kingdom, Japan) is mixed.\(^1\) Japan has not yet escaped from its long lasting stagnation, and while US and UK have been doing better than the EZ over the last five years, the impact of their large QE programmes is unclear. Quite reasonably, other concomitant factors, not least the fiscal stance of governments, also mattered. Exactly one year after the start of the ECB programme, the aggressive package of additional measures adopted on March 10, 2016, testifies that the expected results are yet to materialise. Hence two questions are particularly relevant as far as the EZ is concerned. First, how QE is expected to work where conventional monetary policy has failed. Second, what the fiscal stance of governments will be vis-à-vis QE and the EZ rules still in place. Clearly, these questions are interconnected, and here we seek to provide a simple, though sufficiently detailed, macro-policy framework to address both of them.

In section 2 we begin with a preliminary discussion of QE. Therein we clarify the rationale for QE put forward by the ECB, and how QE can be viewed as a reincarnation of monetary policy as a means to increase the amount of money available in the economy. We highlight that the critical situation in the EZ is that, whereas the policy rate is at the ZLB, the market nominal and real interest rates are in some countries well above zero. These high interest rates concur to determine persistent negative output gaps and deflationary expectations throughout the EZ, and they are therefore eligible as the intermediate target for the success of QE.

In order to analyse the implementation and impact of QE, in section 3 we introduce a model of the EZ which is meant to serve three main purposes. First, take into account the specific features of QE in the EZ, that we represent in a two-country system sharing a central monetary policy. Second, characterise QE appropriately as an expansion of money supply, i.e. ECB purchases of country-denominated risky assets, aimed at closing a persistent deflationary gap relative to the inflation target (Borio and Dysiatat 2010, Altavilla et al. 2015, Driffil 2016). Third, provide a stylised representation of the main channels of macroeconomic and monetary adjustment, both domestically and across countries (e.g. in't Veld, 2013). Research on the "theory" of QE is in progress. One can mostly find partial models of monetary policy (e.g. Altavilla et al. 2015) or DSGE models with New Keynesian foundations and "financial frictions" (e.g. Gertler and Karadi 2011, Curdia and Woodford 2011, Schabert 2014). In a view to balancing micro-detail and macro-parsimony, we model the EZ as the aggregate of two countries characterised by New Keynesian output and inflation equations and a Tobinian LM money market equation (e.g. Tobin 1980, 1982) that determines each country's interest rate as a spread above the common policy rate.2

This latter feature makes it explicit the financial dimension of the economies in the form of their representative risky assets and their degree of substitutability in portfolios within and across countries. At the same time, it allows to characterise the "portfolio channel" of QE: with the policy

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2 The insertion of the money market in the New Keynesian model is discussed in detail by Woodford (2008). Recent reappraisals of the Tobinian approach include Duca and Muellbauer (2013), Blanchard et al. (2015), Caballero et al. (2016).
rate at the zero lower bound, QE expands money supply throughout the EZ in exchange for risky assets. This approach has its own limits in that it may neglect important stumbling blocks in the transmission mechanism. However, it neatly focuses on one of the critical points of QE: namely the extent to which, for a given configuration of risks, exchanges of assets for liquidity with the central bank are transmitted to the interest rates.

The model supports the ECB official view that the QE is targeted to the reversal of deflationary expectations. It also highlights that instrumental to this goal is the elimination of persistent output gaps, both at the EZ and at the country level, and hence the reduction of country-specific interest-rate spreads. We show that QE, if large enough, can succeed for the EZ as a whole. Section 4 nevertheless shows that the ECB cannot also close individual countries’ output gaps, unless specific and unrealistic conditions are met. In this case fiscal accommodation at the country level should also intervene. We show that QE can enhance the effectiveness of fiscal policy, and therefore conclude that the coordination of fiscal and monetary policies is of paramount importance. Conclusions and policy implications are summarised in section 5.

2. A preliminary note on quantitative easing

Operationally, QE is a catch-all that covers a number of different interventions of the central bank in the money market (e.g. Bernanke and Reinhart 2004; Borio and Disyatat 2010). However, these interventions do have one common feature in that they inject additional base money into the system (hence the qualification "quantitative") which is reflected into an equal expansion of the central bank’s assets (QE is often presented, as for example by the ECB, in terms of a target on the latter). Looking at this

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3 One is the role of bank intermediaries, particularly in the EZ (Angeloni et al. 2003, Creel et al. 2013, Gertler and Karadi 2011, Schabert 2014). Another is factors (more likely to arise in connection with banks) that may weaken, or totally impair, the transmission mechanism, such as a "liquidity trap" – the boundless absorption of money supply into portfolios– or the fact that expenditures of "financially constrained" households and firms are insensitive to the interest rate. By ignoring these "frictions" we obtain a more "friendly" transmission mechanism, whereas in reality shocks may have greater impact, and QE less impact, than implied by the model.
common feature, QE is also dubbed as "unconventional" monetary policy with respect to the direct management of interest rates, with limited role of central bank's trading of assets, epitomised by the Taylor Rule. However, one may say that QE is nothing but a reincarnation of the traditional textbook treatment of monetary policy, the "LM model" for short, whereby the central bank controls "the quantity of money" (or, more precisely, the monetary base of which total money supply is a multiple).

In this perspective, QE also takes on a Tobinian flavour, and it comes to overlap with other "non conventional" measures that not only do involve the dimension but also the composition of asset portfolios of both the central bank and its counterparties (Bernanke and Reinhart 2004; Borio and Dysiatat 2010). The well-known Tobinian mechanism is that, by exchanging some classes of assets with money in the counterparties' portfolios, the spread on such assets falls so that they become cheaper vehicles for financing expenditure. Altavilla et al. (2015) and Driffil (2016) provide evidence that the ECB is achieving this goal. Yet, as will be seen, this may not be sufficient for complete success of QE.

The case for QE, as generally explained and communicated, is that the central bank wishes to achieve a policy goal that it can no longer achieve by means of the "conventional" policy owing to the policy rate being at the "zero lower bound". Central banks that have so far engaged in QE have also communicated somewhat different policy goals: foster the recovery of economic activity, prevent a deflationary spiral, raise inflationary expectations, spur credit supply. In the EZ, according to the ECB, the main QE rationale is to stop a deflationary drift and realign inflation expectations with the 2% target (e.g. Draghi 2014a,b). This communication strategy is clearly in tune with the single mandate of the ECB for price stability.

The New Keynesian workhorse model provides an oft-heard narrative. The monetary stance is considered to be restrictive when in the so-called IS function we have the following inequality:

\[ \text{interest rate} - \text{expected inflation} > \text{equilibrium real rate}, \]

which at the ZLB is rewritten as

\[ - \text{expected inflation} > \text{equilibrium real rate} \]

This inequality yields a negative output gap (aggregate demand below potential supply). It may occur from various combination of factors such as very low or negative equilibrium real rate, too low inflation target of the
central bank, expected inflation below target or negative (e.g. Krugman 1998). The negative output gap, via the so-called Phillips Curve feeds back onto negative inflation gaps, which in turn triggers low or negative expected inflation in a vicious circle.

In fact, De Grauwe and Ji (2015) and Orphanides (2014) show that in the EZ, while the ECB policy rate has been dwelling at the ZLB since the end of 2012, clear symptoms of monetary restriction have developed as witnessed by the falling growth of base money, broad monetary aggregates and credit, and by, on average, high real interest rates in various countries (albeit with important differences) due to deflationary expectations.

Since it is widely agreed that in the EZ bank credit is the primary source of private expenditure (e.g. Angeloni et al. 2003, Creel et al. 2013), Figure 1 shows the interest rate on medium-term bank loans to non-financial corporations in all EZ countries in 2015, both in nominal and real terms for the concomitant one-year inflation rate. Recalling that the key reference rate for bank loans, the 3-month Euribor, remained around zero, one may note that: 1) nominal and real interest rates (spreads) were well above zero in all countries; 2) in some countries deflation made the real rate higher than the nominal one; 3) heterogeneity across countries was large.

Figure 1. Distribution of country nominal and real interest rates of bank loans to non-financial firms in 2015 (average monthly values)

![Figure 1: Distribution of country nominal and real interest rates of bank loans to non-financial firms in 2015 (average monthly values)](source)

Here we meet a crucial issue, specific to the EZ, which will be pivotal in our subsequent treatment. Obviously, EZ output and inflation gaps are nothing but the result of the gaps in each country. The cause of negative gaps at the country level should lie in its own IS inequality, i.e. the country’s real interest rate exceeding the equilibrium real rate. This should be the result of a specific spread over the policy rate charged by lenders, net of expected inflation:

\[(\text{policy rate} + \text{country spread}) - \text{(country) expected inflation} > \text{(country) equilibrium real rate}\]

Unless financial markets are perfectly integrated and arbitrated, and all countries share the same fundamentals, each country's interest rate may well be different from any other and from the policy rate. In fact, it is now well documented that the EZ financial markets have undergone a substantial "segmentation" in the aftermath of the crisis\(^4\). One reason, in terms of portfolio theory, is that asset substitutability has fallen both across classes of assets and, more importantly, across country denomination. These considerations put the QE operation in the EZ in the right perspective. First, for a given distribution of country spreads and inflation rates, at the ZLB the ECB is unable to further lower the common floor of nominal interest rates, with the consequence that the real interest rate in some countries may remain too high, generating negative output gaps and deflationary pressure that reverberate at the EZ level. At this point the question is: how can QE achieve what the direct control of the policy rate cannot?

3. The model

We model the EZ as the aggregate of two open economies with independent governments, a common currency and a single central bank (ECB). In a view to balancing micro-detail and macro-parsimony, for each economy we re-elaborate, in a simple and manageable way, the standard New Keynesian macro-policy framework consisting of three equations: one for the goods market, one for the inflation rate, and one for the interest rate.

The latter results from a Tobinian money market that determines the interest rate as a spread above the common policy rate.

The structures of the two economies (the parameters of the model and the latent general equilibrium values – such as potential output, the natural interest rate, inflation target) are alike and remain constant. They however differ in the riskiness of their representative domestic assets underlying the determination of the spread.

The EZ variables observed by the ECB are the average of the two countries’ variables. The ECB operates at the ZLB, and QE consists of an expansion of money supply, i.e. purchases of country-denominated risky assets. Since our focus is on policy responses to shocks, the short-run macroeconomic adjustments are assumed to occur instantly, and we do not develop the dynamic behaviour of the system. The key point is the effect of the shock on the system, when there is no built-in mechanism of self-adjustment except a policy action. Consequently, we run a comparative analysis of different policy actions. Unless otherwise stated, all variables are log-deviations from trend or equilibrium values, except interest rates which are expressed as spreads above the (zero) common policy rate.

3.1. Output and inflation

For each country $i = 1, 2, j = \text{not}-i$, and each time unit (the time subscript is omitted), we first posit the following output market-clearing condition (or IS equation) in terms of deviation of realized output from potential output:

$$y_i = \alpha_p(y_{pi} + u_{pi}) + \alpha_g d_i + \alpha_x x_i$$

where $y_i$ is the output gap, which consists of three components, private domestic demand $y_{pi}$, the net contribution of the public sector to domestic demand $d_i$, and net exports $x_i$. The coefficients $\alpha_p, \alpha_g, \alpha_x$ must sum to one:

$$\alpha_p + \alpha_g + \alpha_x = 1$$

This is a simplifying assumption, common to this class of models that focus on macroeconomic shocks. However we recognise that differences in economic structures are important in the EZ, especially after the crisis, which also had structural effects, as is testified by the ongoing downward revisions of potential output in all countries. We shall informally discuss the consequences of structural differences or changes where necessary.

6 In President Draghi’s words “This orientation [of our monetary policy] implies that there are types of shocks that are relevant for our price stability assessment, and those that are not. The relevant type of shocks are those that are likely to persist into the medium-term and affect medium-term inflation expectations” (2014b, p.3).
demand, i.e. the government’s primary deficit $d_i$, and the net contribution of the foreign sector, i.e. the foreign trade balance $x_i$, each weighed by the respective GDP share in steady state ($\alpha_p, \alpha_g, \alpha_x$). In addition, an exogenous shock to domestic private demand $u_{pi}$ may arrive randomly.

Domestic private demand is expressed as a negative function of the deviation of the real interest rate from the "natural rate" at which potential output is realised and the economy remains in steady state. In the standard formulation, the real interest rate is represented by the nominal policy rate set by the central bank net of expected inflation. As a characteristic feature of the EZ, we instead allow the real interest rate to differ across countries, by the extent of a spread $s_i$ of the local nominal rate relative to the common policy rate, net of the local expected inflation gap $\pi_i$. The latter is likewise the deviation of expected inflation from the EZ target; we call $\pi_i < 0$ expected deflation. Therefore,

$$y_{pi} = -\sigma(s_i - \pi_i)$$

Therefore, the country real interest rate may rise "too high" $(s_i - \pi_i) > 0$ relative to the given natural rate as a result of a positive spread and/or expected deflation.\(^7\)

As to the public sector, the public primary deficit $d_i$ is the deviation from its steady-state value, which is set to zero. This is the net contribution of the public sector being the difference between additional demand created by expenditure for goods and services and its subtraction due to taxation. Following the EZ policy framework (e.g. EU Commission 2013), we distinguish between a discretionary and a cyclical component of the primary deficit. For simplicity we attribute the cyclical component to the sole tax revenue (i.e. we ignore automatic cyclical expenditure such as unemployment benefits). Assuming that the government keeps a constant revenue/GDP rate $\tau$, the actual revenue deviates from its steady-state level

\(^7\) This formulation can be derived directly from households’ choice of their optimal consumption path, so that they shift consumption from the present to the future according to their intertemporal elasticity of substitution. As to investment, in this class of models (e.g. Casares and McCallum 2006), deviations of investment from steady state (pure capital replacement) are sensitive to the same variable as consumption, via Tobin’s $q$, net of the adjustment cost component. Hence equation (2) may be extended to include, at least in part, the investment component of private demand.
proportionally to the output gap, \( \tau y_i \). Then we treat the discretionary component as a change in public expenditure, or fiscal shock \( u_{gi} \).

\[ d_i = u_{gi} - \tau y_i \]

EZ governments also face a mandatory deficit constraint, but for the time being it is not necessary to model it explicitly (we shall return to this point in section 4.3).

The foreign trade balance \( x_i \) consists of two components, intra and extra-EZ, with respective shares of \( \theta \) and 1-\( \theta \). Intra-EZ trade depends on the change in the intra-EZ real exchange rate \( (\pi_j - \pi_i) \), given by the relative inflation gaps in the two countries, and on the business cycle in the two countries \( (y_j - y_i) \). As to extra-EZ trade, we assume that world prices and output remain on trend, so that the only relevant variable is the change in the extra-EZ real exchange rate, given by the rate of change of the euro exchange rate \( \varepsilon \) (\( \varepsilon > 0 \) denotes depreciation) \textit{vis-à-vis} the local inflation gap. We assume unit elasticity of all the trade components with the relevant variables. Therefore,

\[ x_i = \theta((\pi_j - \pi_i) + (y_j - y_i)) + (1 - \theta)((\varepsilon - \pi_i) - y_i) \]

The euro exchange rate is driven by (deviations from) uncovered interest parity with the rest of the world (ROW), i.e.

\[ \varepsilon = \phi(r_w - r + \varepsilon^e) \]

where \( r \) is (the change in) the EZ policy rate, \( r_w \) is the equivalent for the ROW and \( \varepsilon^e \) is the expected depreciation rate of the euro. Thus, the euro depreciates to the extent that \( r_w + \varepsilon^e > r \), where \( \phi \) measures the responsiveness of world capital movements to the interest-rate differential (for simplicity, we set \( \phi = 1 \)). As to \( \varepsilon^e \), we adopt the "PPP view" according to which exchange-rate expectations are driven by the inflation differential \( \pi - \pi_w \); this is zero when inflation is on trend in the EZ and the ROW. In a world at the ZLB, differences of level and change in the policy rates are negligible, \( r_w \approx r \approx 0 \). Assuming that world inflation remains on trend, \( \pi_w = 0 \), the euro exchange rate is fully driven by the inflation gap in the EZ, \( \varepsilon = \pi \), that is deflation \( \pi < 0 \) makes the euro appreciate, and vice versa.

The supply side of each economy is represented by the relationship between the output gap and the deviations of inflation from its expected value. According to the standard New Keynesian Phillips Curve (PC), that assumes monopolistic competition with sticky prices, the current inflation
gap is determined by its expected value and the current output gap, i.e. in our terms,
\[ \pi_i = \beta \pi_i + \eta y_i + u_{\pi} \]
where \( \beta \) is a discount factor, \( \eta \) is the elasticity of price changes to output gaps and \( u_{\pi} \) is a white-noise random shock. The rational expectation (RE) of the inflation gap is therefore the statistical expected value of (6):
\[ \pi_i = E(\pi_i) = \frac{\eta}{1-\beta} E(y_i) \]
where \( E(\cdot) \) denotes the unbiased statistical expected value. Clearly, in this setup, the RE of the inflation gap is uniquely conditioned by the statistical expected value of output gaps. The key implication is that \( \pi_i \) is zero only if the price setters can rationally expect the output gap to be zero. As shown by Woodford (2003, ch. 3), the standard Taylor rule ensures that the output gap is zero when inflation is on target. This provides the anchor for the expected inflation in general equilibrium, so that \( \pi_i \) can be gauged as a reversible fluctuation around the central bank's target, which supports the RE that \( \pi_i = E(\pi_i) = 0 \).

This RE equilibrium can be upset as the agents cease to have a rational basis to believe that \( E(y_i) = 0 \). To see this point in detail, let \( y_i^e \) denote a generic expectation of the output gap and substitute it for \( E(y_i) \) in equation (7). To the extent that \( y_i^e \neq 0 \), the actual inflation gap becomes
\[ \pi_i = \eta y_i + \eta \beta (1-\beta)^{-1} y_i^e + u_{\pi} \]
and if \( y_i^e < 0 \), the actual inflation gap takes a negative drift. Therefore, the concern for a persistent, expectation-driven, deflationary bias in the economy has little to do with exogenous shocks to the PC and much to do with the entrenchment of the belief that the output gap will remain negative. To pin down this phenomenon with observable data in a simple way, let the belief \( y_i^e \) be the expected value of the output gap persisting with probability \( p \) or reverting with probability \( 1-p \), i.e. \( y_i^e = p y_i \). As a result we can write
\[ \pi_i = \omega y_i \]
with \( \omega = p \eta (1-\beta)^{-1} \) measuring the weight of persistence expectations (PE).

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8 Hence \( 1-p \) can be interpreted as a measure of the confidence in the central bank's control over the business cycle.
9 In applied quantitative macro-models \( \beta \) is set close to 1. Note therefore that even a small persistence probability \( p \) may magnify \( \omega \) substantially.
The actual inflation gap thus becomes
\[(10) \quad \pi_i = \pi_y y_i + u_{\pi_i}\]
where \(\pi_y = \eta + \beta \omega\).

Note that, as \(y_{\pi_i} = E(y_i) = 0\) supports the zero-gaps RE equilibrium \(\pi_{\pi_i} = E(\pi_i) = 0\), so a persistent output gap, such that eventually \(y_{\pi_i} = E(y_i) = y_i\), with \(p \rightarrow 1\), supports another, nonzero-gaps, RE equilibrium where \(\pi_y = \eta/(1-\beta)\), and \(\pi_{\pi_i} = E(\pi_i) = \pi_y y_i\); So long as \(0 < p < 1\), the economy is not in RE equilibrium \(\pi_{\pi_i} \neq E(\pi_i)\); however, we shall allow for this possibility by using (9) as the equation of the expectation formation, so that we can examine two scenarios: the "normal" one, when \(\omega = 0\) and the ECB should prevent the formation of nonzero-gaps expectations by consistently realising \(E(y) = 0\), and the "persistence" one, where \(\omega > 0\) and the ECB should curb existing PE.

Substituting equations from (2) to (10) into (1) we obtain the following bilateral\(^{10}\) form of the IS equation:
\[(11) \quad y_i = \left[-\sigma' s_i + \alpha_p u_{p_i} + \alpha_g u_{g_i} - \alpha_x u_{\pi_i} + \alpha'_x (y_j + \pi_j) + \alpha''_x \epsilon_j]\right] \Omega_y\]
where \(\sigma' = \alpha_p \sigma\), \(\alpha'_x = \alpha_x \theta\), \(\alpha''_x = \alpha_x (1-\theta)\) and \(\Omega_y = [1 + \alpha_g \tau + \alpha_x (1 + \eta) - \omega(\sigma' - \beta \alpha_x)]^{-1}\)

We have thus a detailed account of (changes in) the various internal and external variables and shocks that may generate e.g. a negative output gap under the normal condition that \(\Omega_y > 0\). Given the respective parameters, the impact of these events is larger the smaller is the magnitude of the common denominator (or the larger the respective "multiplier" of each shock).

### 3.2. The money market and the interest-rate spread

In normal times, the policy rate set by the ECB affects the demand side of each country via the real interest rate, while the inflation target provides the anchor for the expected inflation. However, here we have to deal with "special times" in which the policy rate is at the ZLB, and the central bank deliberately turns the Taylor Rule off and shifts to QE with the unconditional objective of closing the inflation gap. We should model this new monetary policy stance from the point of view of each country in the EZ.

In the first place, we need to introduce the money market of each country.

\(^{10}\) That is, a quasi-reduced form of \(y_i\) where \(y_j\) appears explicitly.
A variety of microfoundations are available. For the reasons discussed in section 2, we find it suitable a Tobinian foundation on portfolio theory (e.g. Tobin 1980, 1982; see Appendix A1). Key to this approach is the degree of substitutability between money and assets and across different assets depending on outstanding stocks, their riskiness, and risk preferences. What configuration is most accurate is a thorny empirical issue, but as discussed in section 2, it is now largely believed that, whereas the pre-crisis regime of the EZ would approach "perfect substitutability", after the crisis the EZ capital markets have become significantly "segmented", which we translate into the assumptions that (i) there is a segmentation between within-country and cross-country substitution, and (ii) assets are imperfect substitutes across countries. Like Blanchard et al. (2015), we assume that money demand in each country is expressed by domestic agents who seek to optimise their money holdings vis-à-vis interest-bearing domestic assets in view of their non-financial transactions. Besides there are EZ "global investors" who seek pure financial returns by optimising their portfolios of assets from different countries, which gives rise to intra-EZ capital movements.11 

Therefore, we first have a money demand equation for each country such that the rate of change in money demand results from

\[ m_{di} = \pi_i + m_y y_i - m_s s_i + u_{mi} \]

The inflation and output gaps, \( \pi_i \) and \( y_i \), trigger excess demand for transaction balances with positive elasticity. The interest rate on domestic assets (i.e. the country spreads on the common policy rate, \( s_i \)) is the opportunity cost of money, and it triggers substitution between money and the domestic assets according to the semi-elasticity \(-m_s\). Finally, money demand can be shifted by exogenous shock \( u_{mi} \). Portfolio theory shows that money-asset substitutability is poorer (\( s_i \) is smaller) when risk and/or risk aversion are higher. In turn, these conditions are more likely when the underlying asset stock is high (see Appendix A1).

Money supply in each country has two sources (e.g. Goodhart 1989; Tamborini 2001). The first is direct injection from the union's money market, \( \mu_i \). The second is the share of the union's money stock that is circulated by way of intra-union payment imbalances \( b_i \), so that surplus

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11 For simplicity we exclude non-EZ assets.
countries gain money to the expenses of deficit countries. These imbalances are the result of the current account (which for simplicity we identify with the trade component only) and capital movements i.e.

\[ b_i = \theta((\pi_j - \pi_i) + (y_j - y_i)) + b_s(s_i - s_j) \]

The trade account has already been defined above, whereas capital movements are driven towards one country or the other depending on the interest-rate differential (i.e. the respective spread over the common policy rate) given the degree of cross-country asset substitution measured by the parameter \( b_s \). If they are perfect substitutes, we are in the case of "perfect capital mobility" in the Mundell-Fleming tradition. Instead, given risk aversion, assets from different countries are imperfect substitutes depending on differences in relative riskiness and outstanding stocks which may lower the magnitude of \( b_s \) (see Appendix A1). Notice that here the capital market operates in a "normal" situation in which a higher spread signals a higher risk premium, but it attracts capital inflows and does not trigger a capital flight to safety. Therefore, the money supply flowing into in each country can be specified as follows:

\[ m_{si} = b_i + \mu_i \]

whereas in the aggregate \( \Sigma b_i = 0 \) and \( \Sigma \mu_i = \mu \), where \( \mu \) is the rate of money creation in the EZ.14

At this point, we are in a position to examine how the money market conditions affect the country spreads. Let us compute the value of \( s_i \) that satisfies the money market equilibrium \( m_{d_i} = m_{si} \). The result is

\[ s_i = [(m_y + \theta)(y_i + (1+\theta)\pi_i - \theta(y_j + \pi_j) + b_s s_j + (u_{mi} - \mu_i)]\Omega_s \]

12 As to extra-union imbalances, in the EZ they are pooled together by the ECB and affect its stock of official reserves and (possibly) money supply. In practice, being on a free float, this component is almost negligible.

13 In addition, cross-country substitututability may be further impaired to the extent that international investors discounts specific cross-border risks or displays "home bias" (i.e. risk aversion is higher or lower for assets issued in specific countries)

14 \( \mu_i \) is also a simple and straightforward way to capture the role of the banking system. Note, then, that \( \Sigma \mu_i = \mu \) can be read bi-directionally. From the right to the left it indicates how the money creation by the ECB is allocated to the single countries. This reading is appropriate to QE and we shall use it subsequently. However, in "conventional times", when the ECB sets the terms of borrowing in the money market and stands by, we can read from left to right the total amount of money creation due to each country's banking system borrowing in the union's money market, or the extent of endogenous money creation.
where $\Omega_s = [b_s + m_s]^{-1}$

Each country spread depends on the output and inflation gaps of both countries, the spread of the other country, and its own money demand and supply shocks. The quantitative impact of these variables on the spread mainly depends on two parameters, $m_s$ and $b_s$, which respectively measure asset-substitutability within and across countries. The change in the spread is greater the smaller they are, i.e. with poorer substitutability. All this supports the concern that high-risk and high-debt (private and/or public) countries have a spread strongly sensitive to shocks.

In this type of model money demand shocks may be an important source of macroeconomic instability, and they may capture phenomena that have played a role during the development of the financial crisis, such as a sudden surge in liquidity preference $u_{mi} > 0$ that pushes the spread up. However, money-security substitution also implies that the money market equilibrium should be consistent with the security market equilibrium, and, by Walras Law, excess demand in the money market should be equal to excess supply in the security market, and vice versa (Tobin, 1969). For our purposes, we focus on public bonds as representative of the security market. Therefore, excess supply of public bonds, due to a government deficit, should be matched by excess demand for money, or $d_i = u_{mi}$. We can thus see that the country spread is increasing in the its public deficit, a fundamental assumption at the roots of the EZ design. To the extent that the spreads are correlated ($b_s > 0$), the spread in one country is also increasing in the other's public deficit, another concern that has shaped the EZ design.

Money supply shocks are particularly important as they are the vehicle of the transmission of central operations to the countries. We see that these shocks have the same and symmetric effect as the demand ones in both countries. This fact has two interesting consequences. The first is that an increase in money supply is more effective on the spread precisely when and where it is needed, i.e. when $s_i$ is large because $m_s$ is small (domestic asset substitutability is low) and issuing new liabilities to finance expenditure is more costly. The second is the case in which $\mu_i = d_i$, i.e. money financing of the public deficit with null effect on the spread. Note that the cross-border transmission channel now operates for the good of the other country too.

3.3 The three equations at the country level
Each country is characterised by three equations determining \((y_i, \pi_i, s_i)\) for any shock \((u_{pi}, u_{gi}, u_{\pi i}, u_j, \varepsilon)\). These equations are crucial since they provide the foundation of the whole QE policy, as we shall see in detail.

In the first place, in principle for any exogenous shock there exists an optimal decentralised policy response, either monetary or fiscal, i.e. a pair \(\mu^*_i\) and/or \(u^*_gi\) such that \(\pi_i = 0.15\). Therefore, the centralised policy response by the ECB should rest on the premise that the decentralised one is unfeasible. On the monetary side, the national banking systems may be unable, or unwilling, to borrow from the ECB at the given policy rate so as to expand domestic money supply as much as necessary. On the fiscal side, governments may be inhibited, or unwilling, to activate the necessary fiscal stimuli.

For the sake of concreteness, we have sought to provide a tentative quantification of the parameters of the three equations based on available direct or indirect sources (see Appendix A2). This is done just for illustrative purposes, with no claim of rigorous measurement. The results are reported below, for the base case in which there are no expectations of persistent output gap, and the case of a 5% probability of persistence in parentheses underneath.

\[
\begin{align*}
y_i &= -0.127s_i + 0.636u_{pi} + 0.523u_{gi} - 0.035u_{\pi i} + 0.023(y_j + \pi_j) + 0.012\varepsilon \\
\pi_i &= 0.086y_i + u_{\pi i} \\
s_i &= 1.245(y_i + \pi_i) - 0.495(y_j + \pi_j) + 0.4s_j + 0.75(u_{gi} - \mu_i)
\end{align*}
\]

To begin with the IS function, the parameters share the common multiplier \(\Omega_y\) smaller than 1 (0.872 in the base case) indicating the shock-absorbing capacity of the system. As to fiscal shocks, the parameter is in line with the pre-crisis consensus that set fiscal multipliers in the range between 0.5 and 1.\(^{16}\) The PC function has quite a small output elasticity, as is standard in estimated or calibrated New Keynesian models.

\(^{15}\) Note that decentralized policy responses should be coordinated owing to their reciprocal spillovers.

\(^{16}\) As is well known, the pre-crisis consensus on fiscal multipliers has been challenged by a number of empirical studies pointing to a large upgrading of estimates well above 1 or even 2. Here the difference with the traditional
The LM function shows that the spread is highly sensitive to all shocks. This matches a well-known empirical regularity about the volatility of interest rates during the crisis. Our model (see above 3.2 and Appendix A1) captures one main reason for high sensitivity, namely poor asset substitutability, reflected in the relative small magnitude of the parameters of the LM function found in the crisis period (see Appendix A2) such that the common multiplier $\Omega_s$ is barely below unity (0.75), i.e. portfolio adjustments provide little shock absorption. The other side of the coin is that the cross-country transmission of spreads is of limited extent (0.4). Combined with poor asset substitutability, the quantitative importance of the intra-EZ BP channel of the spread is confirmed. A large part of the sizeable increase of the spread during a domestic boom is due to the deterioration of the BP (contraction of domestic money supply vis-à-vis increase in the demand for transaction balances) whereas the entire decrease of the spread triggered by a foreign boom is due to the improvement of the BP (expansion of domestic money supply). Exogenous shocks to domestic money supply, too, have an important impact on the country spread.

It can be seen that the introduction of even a small probability assigned to persistence of output/inflation gaps makes a nontrivial difference for the IS and PC functions. The IS function is affected, becoming more sensitive to all shocks, because its common multiplier is amplified (from 0.872 to 0.908), while the PC output elasticity rises substantially. For a given recessionary shock, the negative inflation gap grows much larger.

4. QE at work

4.1. The ECB and the EZ economy as a whole

In order to introduce the ECB’s behaviour we should now move to the EZ level. The correct road towards the EZ level starts from the three-equations systems of each country. In fact, the ECB should know and exploit the exact structure of the transmission mechanism of QE, by which we mean the Keynesian multiplier typically greater than 1, is the absence of components of private expenditure directly dependent on GDP discussed previously.
whole set of country equations and their parameters resulting in the EZ economy as a whole. Now, upon solving for the two countries' endogenous variables, and taking their average values, we obtain the following EZ system in matrix format (EZ variables are denoted by non-indexed symbols)

\[ (17) \quad [y, \pi, s]' = A[u_p, u_g, u_\pi, \mu]' \]

The three endogenous EZ variables result to be determined by the average shocks to aggregated demand, fiscal policy, inflation, and by the rate of money creation \( \mu \). As to the exchange rate, we have assumed that it is driven by the EZ inflation gap (see section 3.1), i.e. \( \varepsilon = \pi \), so that it, too, is endogenised. The coefficient matrix \( A \) conveys important information.

First, substantial structural uncertainty exists in that the signs of all coefficients are ambiguous. This is the consequence of two phenomena. One is that each shock has both a direct impact on the correspondent endogenous variable and an indirect effect via the concomitant adjustment of the other endogenous variables.\(^{17}\) The second is reciprocal spillovers that amplify the simple aggregation of country effects. To see this, start again from each country's IS and suppose that a slump occurs in country \( j \), \( y_j < 0 \), while nothing happens in country \( i \). Then the EZ output loss will not just be \( y_j/2 \) but larger, owing to the reciprocal spillovers between the two countries. The same occurs with shocks to spreads, as can be seen by means of the LM equation (15): an increase in the spread of country \( j \) also raises the spread of country \( i \) through the BP channel, so that the increase of the average spread is magnified.

Assuming that the indirect effects and the spillover effects are sufficiently small, the coefficient signs are those reported in system (18). In parentheses we also report the figures obtained with our empirical parameters, which are indeed consistent with this assumption. Since the expectations of persistent deflation play an important role in the ECB communication, system (18) reports figures for this case.

\(^{17}\) For instance, a negative demand shock \( u_p < 0 \) affects the output gap \( y \) directly and the spread \( s \) indirectly. The direct effect generates a negative output gap, which also reduces the spread. The final effect remains negative on both \( y \) and \( s \) if its direct impact on \( y \) is larger than the recovery of output due to the concomitant fall of \( s \).
$$\begin{bmatrix} y & \pi & s \end{bmatrix} = \begin{bmatrix} + & + & - & + \\ (.936) & (.413) & (-.356) & (.179) \\ + & + & + & + \\ (.482) & (.213) & (.817) & (.092) \\ + & + & + & - \\ \end{bmatrix} \begin{bmatrix} u_p & u_g & u_\pi & \mu \end{bmatrix}^\prime$$

4.2. The mechanics of QE and its policy implications

System (18) may be useful to understand why QE may be necessary in the first place. The EZ inflation gap $\pi$ can be negative after a private demand fall $u_p < 0$, a fiscal contraction $u_g < 0$, and/or because of a direct deflationary shock $u_\pi < 0$. Consider $u_p < 0$: it has a large impact on both $y$ and $\pi$, though $s$ is reduced in parallel. Commensurate (about twice the shock) coordinated fiscal stimuli $u_{gi} > 0$ would be the most effective response for correcting both recession and deflation; yet, if compatible with fiscal rules, their drawback is a substantial rise in $s$, which may be detrimental for countries with high debt. With fiscal stimuli tightly constrained, a policy of generalised "competitive" deflation across countries $u_\pi < 0$ would reduce $s$ but it might nonetheless put monetary policy under stress, giving limited support to $y$ (one third of the shock) via the extra-EZ trade\textsuperscript{18} while magnifying the fall of $\pi$. Indeed, QE $\mu > 0$, appears as the weapon of last resort. Notably, QE has a substantial effect on $s$ whereas the effect on $y$ is of lesser magnitude, and the final effect on $\pi$ is rather small (indeed, the transmission mechanism is $\mu \to s \to y \to \pi$). The ultimate reason is that the standard quantification of the slope of the PC is in the range of few centesimal points, although, as previously seen, the persistence expectations raise it substantially. The immediate policy implication is that QE should be activated on a large scale.

In this setup, the ECB has the single instrument $\mu$ for one (unconditional) official target: close the negative inflation gap $\pi$. In principle, this appears to be a problem with a well-defined solution. Let us work it out in detail with reference to the empirical coefficient values in system (18). For each of the shocks $u_p$, $u_g$, and $u_\pi$ we can compute the optimal $\mu^*$, i.e. the rate of money creation that sets the inflation gap to zero, and gauge its effect on the

\textsuperscript{18} Generalized deflation has a zero-sum effect on intra-EZ trade.
remaining endogenous variables (see Table 1). In all cases, a positive $\mu^*$ is warranted, with three qualifications. First, the quantitative responses are different, and the ECB should identify the type of shock originating the inflation gap. Second, the more the PC is flat, the larger $\mu^*$ should be; hence, it may be comforting to see that in the presence of deflationary expectations, which make the PC steeper, QE is both necessary and more efficient. Third, apart from quantitative differences of QE, the overall macroeconomic effects are also notably different for different shocks.

Table 1. Optimal $\mu^*$ and its effects on the other endogenous variables ($\rho=0.05$)

<table>
<thead>
<tr>
<th>$u_\pi&lt;0$</th>
<th>$u_\pi&lt;0$</th>
<th>$u_\pi&lt;0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu^*$</td>
<td>5.2</td>
<td>2.3</td>
</tr>
<tr>
<td>$y$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$s$</td>
<td>-7.5</td>
<td>-6.2</td>
</tr>
</tbody>
</table>

If QE reacts to $u_\pi<0$, $y$ is also closed and $s$ is reduced. This outcome is entirely consistent with the logic of targeting both the actual and the expected inflation gap. It is often argued that the relevant shock is $u_\pi<0$ due to the oil price fall; if this is the case, then QE should be larger, $s$ should fall more and $y$ should become positive as long as the shock is not reversed. This is essentially the "overshooting" policy strategy envisaged by Eggertsson and Woodford (2004).

Our conclusion is that a QE programme of the appropriate size can, in principle, succeed for the EZ as a whole. However, it should be considered that, as said at the beginning, the transmission mechanisms in our model can in reality be impaired by a number of factors that may prevent the relevant spreads from falling all the way (e.g. in the credit markets) or make private expenditure insensitive to the fall of spreads. These factors are not easily overcome by merely expanding the scale of QE. Here we shall focus on another stumbling block that may lie in the way of QE success.

4.3. Asymmetric shocks and the need for fiscal adjustment

Let QE be successful for the EZ as whole (i.e. on average). Yet remember that what surfaces at the EZ level is the result of what is going on at the country level, which crucially depends on:
• the degree of correlation of shocks across countries (we denote with $c_{ji} \in [-1, 1]$ the extent of a shock in country $j$ given a shock in country $i$)
• the distribution of money creation between the countries (we denote with $\phi_i$ the share of country $i$ in the rate of money creation)

Even in our (most favourable) case in which the countries are structurally equal, the EZ average outcome exactly reflects the country-level outcomes only if a new "divine coincidence" occurs$^{19}$:
• the relevant shock is symmetric, $c_{ji} = 1$
• each country receives the same share of aggregate money creation, $\phi_i = 0.5$

Let us consider as an example the case of $u_p < 0$, and compare the outcomes of the centralised QE in Table 1, with those shown in Table 2 for an asymmetric shock to country 1. In the first place, asymmetry entails that the average EZ shock is just one half of the shock to country 1. Then, the equal distribution of $\mu^*$ between the two countries implies that country 1 is underadjusted and country 2 is overadjusted, that is, QE is too little for country 1 and too much for country 2. In particular, note that the stabilisation of $\pi$ and $y$ at the EZ level hides residual negative output and inflation gaps in country 1 exactly matched by positive gaps in country 2.

<table>
<thead>
<tr>
<th>$\mu^*$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.0</td>
<td>$\pi_1$(-0.2)</td>
</tr>
<tr>
<td>$y$</td>
<td>0.0</td>
<td>$y_1$(-0.4)</td>
</tr>
<tr>
<td>$s$</td>
<td>$-3.8$</td>
<td>$s_1$(-4.4)</td>
</tr>
</tbody>
</table>

Analogous result occurs if we let the shock and QE be symmetric but the asymmetry is structural (e.g. QE is less effective in one country than in the other). Pragmatically, it may be argued that the centralised QE is better than nothing, for otherwise both countries would suffer from worse deflation and output gaps. However, in order to unbundle the average EZ variables

$^{19}$ "Divine coincidence" is a well-known term coined by Blanchard and Gali (2007) to denote the joint stabilisation of output and inflation in the New Keynesian models
and obtain the desired results at the country level, the ECB should know and exploit a huge amount of local information and should control the country distribution of the aggregate money creation. Both requirements may be hardly feasible in practice, but, more importantly, they may be objected as being in contrast with the ECB mandate that prevents ad hoc monetary policy actions for specific countries. In fact, the ECB has announced that it will control for the country distribution of QE in consideration of each country's share in the ECB capital. Yet this criterion is utterly unrelated to the optimal money creation at the country level\textsuperscript{20}. It may well be the case that more money creation will flow where it is needed the least. Therefore, non trivial problems of consistency arise for the correct design of the QE programme.

In the case of undesirable outcomes at the country level, the intervention of last resort is fiscal accommodation. This can consistently be obtained from the three-equation systems at the country level (16). With our parameters and a private demand shock, the fiscal response in each country that drives its own inflation and output gaps to zero, given the country distribution of QE, looks like the following

\begin{equation}
\begin{aligned}
u^*_{gi} &= -1.751u_{pi} - 0.15u_{pj} + 0.162u_{gj} - 0.435\mu_i - 0.289\mu_j
\end{aligned}
\end{equation}

The first important feature to note is that we are in the presence of a coordinated solution. The fiscal response of each country should also take into account the fiscal response, the demand shock, and the QE share of the other. Note the positive parameter of $u_{gj}$ in the equation: it draws attention to the fact that the fiscal stimulus in country $j$ may have a negative spillover onto country $i$, so that the latter's fiscal stimulus should be enlarged. In this model, the negative spillover is due to the rise of the spreads in both countries. Hence, the negative side of fiscal accommodation to QE may be that the spreads rise instead of falling. On the other hand, the negative parameters of $\mu_i$ and $\mu_j$ in equation (19) confirm that the concomitant QE, exerting some positive effect on inflation and output and restoring asset substitution, reduces both the extent of fiscal adjustments and their impact.

\textsuperscript{20} Notice in addition that the "country-specific" QE takes the form of national bond purchase, but nothing guarantees that the owners of those bonds are also residents of the country, so that money creation may not happen there.
on the spreads. Hence, QE may effectively relax the constraints that may prevent the decentralised fiscal solution.

As to the role of the ECB, it appears as the leader-player who chooses its own optimal QE for the EZ as a whole irrespective of the country responses. It is entitled to do so because the aggregate effect of the latter is neutral on the EZ target of zero inflation gap, though it is not neutral on the average spread. Hence, the combination of QE with fiscal accommodation does not necessarily entail a threat on monetary dominance. To have an idea of the magnitudes at stake compare Table 2 with Table 3, which is obtained from equation (19).

Table 3. Optimal centralised solution and fiscal accommodation at the country level (asymmetric shock $u_{p1} = -1\%$, $c_{21} = 0$, $\phi_1 = \phi_2 = 0.5$)

<table>
<thead>
<tr>
<th></th>
<th>$\mu^*$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$u^*_{g1}$</th>
<th>$u^*_{g2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$y$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$s$</td>
<td>-3.2</td>
<td>-3.6</td>
<td>-3.6</td>
<td>-4.7</td>
<td></td>
</tr>
</tbody>
</table>

The result is that country 1, hit by the asymmetric shock, should have an additional fiscal expansion, whereas country 2 should have a contraction by almost the same amount. All gaps are driven to zero, the spread falls less than the average in country 1 and more than the average in country 2.

Note that in the absence of QE, i.e. if each country were to use the sole fiscal policy, the result would be $u^*_{g1} = 1.78$, $u^*_{g2} = 0.45$ indicating a larger fiscal expansion in both countries, associated with higher, instead of lower, spreads. Therefore, as pointed out above, QE contributes to country stabilisation by allowing less recourse to fiscal policy.

The optimal fiscal accommodation in each country may of course be attainable to the extent that the budget constraint set by the current Treaties is not violated. If the constraint is binding for one or more countries, the imbalances left over by the centralised QE would remain uncorrected implying further adjustments that we leave for further analysis.

5. Conclusions
The EZ model presented in this paper supports the ECB official view that QE is both necessary and more effective when persistent deflationary expectations are in place and risky asset substitutability is impaired. We have shown that a QE programme of the appropriate size can succeed for the EZ as a whole, with some caveats. The complete success of QE depends on closing the negative output gaps at the EZ level as well as at the country level. This condition may materialise thanks to a new "divine coincidence": 1) all countries are alike, 2) the shock originating deflation is symmetric, 3) the cross-country distribution of QE is symmetric. Otherwise, the old "one size does not fit all" curse will materialise: violation of any of the previous conditions implies that QE (if large enough) will work for the EZ on average, but not for the single countries. In practice we know that the ECB has some control on the distribution of QE, by and large proportional to the countries' shares in its capital. Yet this distribution criterion is utterly unrelated to the problem to be solved.

President Draghi's repeated warning is right: monetary policy alone may be insufficient; coordinated national fiscal stimuli may be necessary. We have shown that such a coordinated solution exists, taking the country effects of QE as given, and that QE indeed mitigates the extent of fiscal deficits that would otherwise be necessary. This is clearly important, given the normative or market limitations to fiscal policy, which may by themselves make QE necessary in the first place. By contrast, with normative limitations to the coordinated fiscal accommodation still binding, QE may be doomed to failure at the country level with possibly further repercussions that deserve to be examined by additional research.

References


Woodford M. (2008), "How Important is Money in the Conduct of Monetary Policy?", *Journal of Money, Credit and Banking*, 40, pp.1561-1598.
Appendix

A1. Portfolio foundation of the LM function

In order to stylise some key features of the current financial environment of the EZ, we distinguish between domestic and global investors (see also Blanchard et al., 2015). Domestic investors are mostly concerned with optimising their portfolios in view of their non-financial transactions in the domestic economy. To this end, the domestic investors in each country \( (i = 1, 2) \) can combine zero-interest money \( (M_i) \) with an interest-bearing domestic representative asset \( (A_i) \) while taking into account the price level \( (P_i) \) and the volume of non-financial transactions \( (Y_i) \). Global investors are instead concerned with optimising pure financial portfolios all across the EZ by combining the representative asset of each country. Hence, portfolio choices of global investors generate capital movements across countries.21

Risk characteristics of domestic and global investors may possibly be different, but this detail is unnecessary here, so we assume a single typical exponential utility function of wealth, with constant absolute risk aversion \( \rho \in [0,1] \). Provided that returns to assets are normally distributed \( N \sim (R_i, \sigma^2_i) \), each investor maximises his/her expected wealth \( E(W) \) when the function

\[
F = E(W) - (\rho/2)\sigma^2_W
\]

is maximal.

The money demand and capital movement functions in the text can be understood as rates of variation around the optimal portfolio allocations to be derived below, for a given constant wealth endowment.

Domestic investors

Domestic investors own a real amount of wealth given by

\[
W_i/P_i = A_i/P_i + M_i/P_i
\]

Given \( P_i \), its expected value is given by the expected return to the asset stock \( R_iA_i \) net of the costs of non-financial transactions \( Y_i \). These are assumed to be quadratic in the difference between \( Y_i \) and real money holdings \( M_i/P_i \). Therefore, the optimal money holding results from

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21 The country location of global investors is immaterial because portfolio shifts imply that sales of asset \( i \) (capital outflows from country \( i \)) are matched by purchases of asset \( j \) (capital inflows in country \( j \)).
maximising the function $F$ net of transaction costs under the wealth constraint, i.e.

$$\max F_i = R_i(W_i / P_i - M_i / P_i) - \frac{1}{2} (Y_i - M_i / P_i)^2 - \frac{\rho_i^2}{2} (W_i / P_i - M_i / P_i)^2$$

which yields

$$M^*_i / P_i = \frac{1}{1 + \rho \sigma_i^2} \left( Y_i - R_i + \rho \sigma_i^2 W_i / P_i \right)$$

$$A^*_i / P_i = \frac{1}{1 + \rho \sigma_i^2} \left( R_i - Y_i + W_i / P_i \right)$$

We thus obtain the standard money demand function which is homogenous of degree 1 in the price level, increasing in the volume of non-financial transactions, decreasing in the expected rate of return to the asset (interest rate for short), plus a positive wealth effect. As said in the text, money-asset substitutability, or the responsiveness of money demand to the interest rate, falls as the riskiness of the asset and/or the investors' risk aversion rise. Note that in this particular formulation the non-financial transactions and the interest rate have the same coefficient. In the text we have posited the more general case in which the two coefficients may be different.

**Global investors**

Global investors aim to maximise the value of their wealth given by

$$W = A_1 + A_2$$

It is sufficient to consider nominal wealth because by assumption global investors' geographical location is irrelevant (and hence so are the location of their personal consumption and the specific price level in any location). Maximisation of the function $F$ under the constraint $W$ yields the demands for the two asset stocks:

$$A^*_1 = \frac{1}{a+b} (R_1 - R_2) + \frac{b}{a+b} W$$

$$A^*_2 = \frac{1}{a+b} (R_2 - R_1) + \frac{a}{a+b} W$$

where $a = \rho(\sigma_{1}^2 - \sigma_{12}), b = \rho(\sigma_{2}^2 - \sigma_{12})$.

Again, we obtain the standard portfolio result whereby the demand for each asset is proportional to its own interest-rate differential plus a wealth effect. Cross-country asset substitutability is determined by the parameters $a$ and $b$, i.e. the risk parameters of the two assets and the degree of risk.
aversion of investors. The responsiveness of each asset to its own interest-rate differential is symmetric, which implies that as $R_1$ ($R_2$) rises relative to $R_2$ ($R_1$) the demand for $A_1$ increases (decreases) and that for $A_2$ decreases (increases) by the same amount. This change in asset holdings generate a capital movement from country 1 to country 2.

On this account, it is interesting to note that the optimal asset holdings imply the following interest-rate differential

$$R_1 - R_2 = aA^*_1 - bA^*_2$$

Hence any non-zero spread may develop according to combinations of: (i) the sign and size of the risk factors $a$ and $b$, (ii) the outstanding stocks of the two assets, i.e. the so-called relative supply effect. Though various combinations are possible, typically asset $A_1$ will pay a spread over asset $A_2$ to the extent that it is more risky ($\sigma_1^2 > \sigma_2^2 \rightarrow a > b$) and/or it is in larger supply.

### A2. Parameters of the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_p = 0.73$</td>
<td>Private expenditure/GDP EZ average value 2000-14, Eurostat, AMECO database</td>
</tr>
<tr>
<td>$\alpha_g = 0.23$</td>
<td>Public sector contribution to GDP EZ average value 2000-14, Eurostat, AMECO database</td>
</tr>
<tr>
<td>$\alpha_x = 0.04$</td>
<td>Foreign sector contribution to GDP EZ average value 2000-14, Eurostat, AMECO database</td>
</tr>
<tr>
<td>$\theta = 0.66$</td>
<td>Intra-EZ share of foreign trade EZ average value 2000-14, Eurostat, AMECO database</td>
</tr>
<tr>
<td>$\tau = 0.45$</td>
<td>Total revenue/GDP General government, EZ average value 2000-14, Eurostat, AMECO database</td>
</tr>
<tr>
<td>$\sigma = 0.2$</td>
<td>Interest-rate elasticity of private demand Garnier and Wilhelmsen (2005)</td>
</tr>
<tr>
<td>$\beta = 0.99$</td>
<td>Discount factor Standard value in literature</td>
</tr>
<tr>
<td>$\eta = 0.086$</td>
<td>Output-gap elasticity of inflation Implied by the Calvo equation, given $\beta$ and 75% of non-adjusted prices (e.g. Smets and Vouters (2003), Luk and Vines (2015))</td>
</tr>
<tr>
<td>$p = 0, 0.05$</td>
<td>Probability of output gap persistence</td>
</tr>
<tr>
<td>$b_s/(m_s+b_s)$ = 0.4</td>
<td>Coefficient of the spread in country $i$ w.r.t. the spread in country $j$ Ehrman and Fratzscher (2015), $i = \text{Italy, } j = \text{Germany}$</td>
</tr>
<tr>
<td>$m_y$=1</td>
<td>Income elasticity of money Calza et al. (2001), Beyer (2009)</td>
</tr>
</tbody>
</table>
\begin{tabular}{l|l}
\text{demand} & \text{Interest-rate semi-elasticity} \\
\text{$m_g = 0.8$} & \text{of money demand} \\
\end{tabular}