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Liquidity constrained exporters ☆

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ABSTRACT

I propose a model of international trade with liquidity constraints. If firms must pay a fixed entry cost in order to access foreign markets, and if they face liquidity constraints to finance these costs, only those firms that have sufficient liquidity are able to export. A set of firms could profitably export, but are prevented from doing so because they lack sufficient liquidity. More productive firms that generate large liquidity from their domestic sales, and wealthier firms that inherit a large amount of liquidity, are more likely to export. This model offers a potential explanation for the apparent lack of sensitivity of exports to exchange rate fluctuations. When the exchange rate appreciates, existing exporters lose competitiveness abroad, and are forced to reduce their exports. At the same time, the value of domestic assets owned by potential exporters increases. Some liquidity constrained exporters start exporting. This dampens the anti-competitiveness impact of a currency appreciation. Under some conditions, it may reverse it altogether and increase aggregate exports. In this sense, the model is able to rationalize the co-existence of competitive devaluations and competitive revaluations.

1. Introduction

Traditional trade models view the patterns of imports and exports mainly as the outcome of a competition game between producers in different countries. If goods from a given country become cheaper, provided that demand is not too inelastic, producers export larger quantities. Eventually, general equilibrium adjustments will have to take place to restore trade balance, but in the short and medium run, we should observe export expansion arising from a devaluation. The only departure from this competition mechanism may happen in the very short run, and has been described as the J-curve: if demand is inelastic in the very short run, a devaluation may have a negative impact on the current account in the short run, before the competitiveness effect comes into play and exports catch up. However, we observe relatively little response of trade flows to exchange rate fluctuations. The euro-dollar exchange rate has experienced wide fluctuations since the inception of the euro, without any significant and systematic effect on the patterns of trade between Europe and the US. In the early 2000s, the US dollar steadily depreciated vis á vis most foreign currencies, without any evidence of a reduction in the US trade deficit. In a different context, many middle income countries underwent massive devaluations of their currency without much gain in terms of current account imbalances. One way to reconcile the observed impact of exchange rate fluctuations with the existing theory of competitive trade is to assume extremely inelastic demands for foreign goods. Such elasticities are at odds with other evidence of the impact of trade barriers on trade flows.

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I propose a theory of international trade with liquidity constraints that can account for these facts, along with micro-evidence on the characteristics of exporters. The main predictions of the model are, first, that liquidity constraints are a key determinant of the export behavior of firms, and second, that exchange rate fluctuations (or more generally fluctuations of relative prices) may have the opposite effect as predicted by traditional theories. If there are fixed costs associated with exporting, then liquidity constraints at the firm level will come into play. In such a context, fewer firms will be able to export. Furthermore, an appreciation of the domestic currency, despite the negative effect on the competitiveness of exporters, will have a mild impact on aggregate exports, even if demand is elastic. It may actually have a positive impact on exports in the medium run if goods are not too substitutable.

The reason is the following. In the presence of fixed costs associated with exporting and liquidity constraints, some firms could profitably export, but they are prevented from doing because they cannot gather sufficient liquidity. Only those firms that are productive enough and generate sufficient internal liquidity from their domestic sales are able to export. If the exchange rate appreciates, potential exporters lose competitiveness abroad and therefore lose some market shares abroad. Existing exporters reduce their exports. This is the traditional competitiveness effect. But an appreciation of the exchange rate also means that the value of domestic assets abroad increases. Some firms that could not enter foreign markets because of liquidity constraints enter now that the value of their assets has appreciated. Total trade does not change much: existing exporters export less, but new firms start exporting. Under some conditions, aggregate exports may increase following an appreciation of the exchange rate.

In other words, it is the extensive margin of trade that responds differently to exchange rate fluctuations in the presence of firm heterogeneity and liquidity constraints. Following an exchange rate appreciation, some firms, due to the increased value of their domestic assets, enter the export market. This entry of exporters, the extensive margin of trade, may offset the reduction of the volumes exported by existing exporters, the intensive margin of trade. The strength of this effect depends on how competitive markets are, what fraction of fixed export costs are denominated in foreign currency, and whether firms own liquid assets and have access to developed financial markets.

This theory also accounts for the fact that few firms export, and that exporters will typically be firms that are not liquidity constrained. There is a large body of empirical evidence from micro-data that only a small fraction of firms export and exporters are more productive than non-exporters, they are larger and more capital intensive. I develop a model where the selection into the export market is similar. Only those firms that are not liquidity constrained are able to export. The capacity to overcome liquidity constraints is endogenously determined in this model. Firms may have sufficient assets, inherited from their past activities, but they can also generate sufficient cash flow from their domestic activities in order to gain access to foreign markets. In equilibrium, only a subset of firms are able to gather enough liquidity and export. The export status is the outcome of the characteristics of the firm, even though the partition between exporters and non-exporters is endogenously determined as the outcome of a competitive game.

Finally, this model has important implications for the link between financial development, macroeconomic stability and openness to trade. The model predicts that a deepening or a widening of the financial markets will increase total exports. When firms get easier access to external finance (a deepening of financial markets), or when more firms get access to cheap external finance (a widening of the financial markets), they become able to overcome barriers associated with international trade. More firms export, and total exports increase. However, the model does not predict that better financial markets will either stabilize or destabilize the current account. The predictions of the model about the volatility of exports and the degree of financial development are ambiguous. Only in the extreme case of perfect financial markets are exports more volatile than if financial markets were not perfectly developed. Exchange rate fluctuations, for instance, will cause larger movements of the volume of exports if financial markets are perfectly developed than if they are not. The reason is the following. If financial markets are not perfectly developed, there exists a fringe of liquidity constrained exporters. When the exchange rate appreciates, some of those liquidity constrained firms start exporting. This entry of new exporters dampens the negative impact of the exchange rate appreciation on existing exporters. If financial markets are perfectly developed, meaning that no firm is liquidity constrained, this dampening channel does not exist anymore. Exports will be more responsive to exchange rate fluctuations. This is the only case where the model makes a clear prediction about the link between financial development and current account volatility. For intermediate levels of development on the other hand, an improvement of financial markets will always increase total exports, but it may or may not increase export volatility. The primary purpose of this model is not to describe the link between financial development and macroeconomic instability. However it offers an interesting angle on a potential link between the volatility of some aggregates (fluctuations in the volume of exports here), and the degree of financial development. It also gives specific predictions for the impact of financial development on the volume of exports.

In the remaining part of this Introduction, I review the literature related to this model. First, recent research has documented the importance of firm heterogeneity and the role of fixed costs in international trade, both empirically and theoretically. Second, there is a large body of literature on the importance of liquidity constraints for firms. Finally, there is a growing literature on the interaction between firm level liquidity constraints and international trade.

Firm heterogeneity is a major feature of the export behavior of firms. Exporters are different from non-exporters in many respects. They tend to be more productive, larger, more capital intensive. The link between productivity and trade has been analyzed in many countries: Bernard and Jensen (1999, 2001a,b, 2002) for the US, Aw and Huang (1995) for Taiwanese and Korean firms, Clerides et al. (1998) for Colombian, Mexican and Moroccan firms, and Delgado et al. (2002) for Spanish firms, to name a few early contributions. This paper extends the study of the heterogeneity between exporters and non-exporters to the severity of the liquidity constraints they face. My model predicts that one dimension of heterogeneity along which exporters may differ from non-exporters is their ability to access financial intermediaries. Less financially constrained firms are more likely to export.

The dichotomy between exporters and non-exporters allows the extensive margin of trade, the entry and exit of firms into the export market, to play a crucial role in determining the volume of trade flows. When trade barriers change, or when the degree of competition in foreign markets evolves, some firms will go in and out of the export markets. The importance of the extensive margin of trade has been pointed out since the seminal work of Krugman (1980). Trade is as much about the quantity each exporting firm exports, as it is about how many firms export. Empirically, in the medium run, most of the adjustment for aggregate trade flows comes from entry and exit of firms into the export market. Eaton et al. (2011), using data on French exporters, disaggregated by trading partner, show that most of the variation in aggregate French exports comes from variation in the number of exporters, rather than differences of exports per firm. Helpman et al. (2004) use the zeros of trade matrices (which happen when no firm trades between two countries) to infer information about the extensive margin of trade, and derive an estimate of trade barriers between countries. Broda and Weinstein (2006) point out that in the last 30 years, the number of varieties of foreign goods available to US consumers has increased fourfold, and contributed to an additional increase in welfare worth 2.6% of GDP over 30 years.

Ruhl (2008) uses a theoretical framework with heterogeneous firms to show that the extensive margin of trade may explain the discrepancy between the short run and long run elasticities of trade with respect to trade barriers. Ruhl argues that high frequency variations in exchange rates, because exchange rates are mean reverting, will trigger only negligible adjustments of the extensive margin, which explains why exchange rate fluctuations seem to have so little impact on trade flows, whereas variations in tariffs or quotas have such a large impact. This paper builds on this literature and studies the extensive margin of trade in the presence of liquidity constraints. If financial markets are underdeveloped, the extensive margin of trade changes due to both productivity differences across firms and differences in the severity of liquidity constraints they face. Predictions of traditional models of trade may be modified, or even overturned. I offer an alternative explanation for the relatively mild impact of exchange rate fluctuations from that of Ruhl (2008). Even if the extensive margin of trade does respond to exchange rate movements, since there will be simultaneous entry and exit of firms, the impact on aggregate exports will be mild. When the exchange rate appreciates, some existing exporters lose competitiveness in the foreign market and stop exporting. At the same time, the value of domestic assets denominated in foreign currency increases, so that liquidity constrained firms start exporting. The net effect on the extensive margin is mild. While both my and Ruhl's model predict the same qualitative difference between the elasticity of trade with respect to tariffs changes versus exchange rate shocks, the mechanisms are different. By interacting exchange rate shocks with measures of external finance dependence and financial development, as well as by distinguishing the impact of exchange rate shocks on the extensive margin for small versus large incumbent exporters, one can test for the relative importance of my model versus Ruhl's in explaining the international elasticity puzzle.

I build on the Melitz model and add liquidity constraints to it. Those liquidity constraints interact with productivity heterogeneity. The most productive firms generate enough liquidity from domestic sales to overcome any liquidity constraints. However, some less productive firms would be profitable enough to export, but are prevented from doing so because they are liquidity constrained. On top of interacting liquidity constraints with productivity heterogeneity, one important contribution of this model is to break up the symmetry in country sizes and wages imposed in Melitz (2003). Exchange rate fluctuations in my model are equivalent to fluctuations in relative real wages in different countries. This formalization is similar to the one used by Atkeson and Burstein (2008).

There is a vast literature on the importance of liquidity constraints for firms, which follows the pioneering work of Stiglitz and Weiss (1981). Fazzari et al. (1988) study the importance of financing constraints for investment. The importance of the lending channel has been stressed in Holmstrom and Tirole (1997), as well as in Stein (1998). Empirically Bernanke and Gertler (1995) and Kashyap and Stein (2000) study the importance of credit constraints for banks. Gertler and Gilchrist (1994) show small firms' production contracts when money is tight, which is further evidence of the importance of liquidity constraints. Hoshi et al. (1994) offer a model where net worth determines whether to use direct or indirect finance.

Liquidity constraints in my model are introduced in the simplest way possible: a collateral constraint. This is the reduced form expression of an unmodelled game between potential financial intermediaries and potential exporters. However, in the presence of productivity heterogeneity, liquidity constraints will interact with trade barriers, exchange rates, firm level productivity in a complex way. Domestic sales may endogenously relax the liquidity constraints faced by a potential exporter. Manova (2013) tests some of the predictions from this model at the sectoral level. In particular, she finds that trade is lower in sectors that rely more heavily on external finance and in countries where financial markets are less developed. Foley and Manova (2015) give a survey on the recent literature on trade and financing frictions. Fitzgerald and Haller (2015), using a panel of Irish exporters, find that trade is more affected by tariff changes than by exchange rate shocks, and that most of the difference is accounted for by the extensive margin of trade. Amiti and Konings (2007) present evidence that imported intermediates (denominated in foreign currency) affect firms ability to export, which contribute to an attenuation of the impact of exchange rate shocks on exports. The evidence in both Fitzgerald and Haller (2015) and Amiti and Konings (2007) are consistent with both my model, and the Ruhl (2008) model. Finally, Bonfiglioli et al. (2015) propose an elegant model with financing frictions and endogenous entry, where financing frictions systematically affect the ex post distribution of firms' productivities: if firms expect financing will be costly, they will endogenously choose less risky and less productive innovations, so that the ex post distribution of productivities is endogenously less dispersed when financing frictions are severe. This prediction is in line with the evidence in Markus (2015) that firm sizes are larger and more dispersed on average in more developed economies.

The remainder of the paper is organized as follows. Section 2 introduces a simple model of trade with liquidity constraints and heterogeneous firms. Section 3 describes the impact of exchange rate fluctuations in the presence of liquidity constraints.

2. A model of trade with liquidity constrained exporters

In this section, I develop a model of international trade with liquidity constrained firms with heterogeneous firms á la Melitz (2003).

There are 2 countries, home and foreign, that produce goods using only labor. All foreign variables are denoted by an asterisk. The home country has a population L (L^* for the foreign country). There are 2 sectors. One sector provides a single homogeneous good that can be freely traded. This good is used as the numeraire, and its price is set equal to 1. It is produced under constant returns to scale. The unit labor requirement for producing the homogeneous good at home is $1/w$ ($1/w^*$ abroad). Provided that each country produces the homogeneous good, the wages will be w and w^*. I shall only consider equilibria where this assumption holds. The other sector supplies a continuum of differentiated goods. Each firm is a monopolist for the variety it produces.

2.1. Demand

The workers are the only consumers, each endowed with one unit of labor. They all share the same CES preferences over the differentiated good. A consumer that consumes q_o units of the homogeneous good, $q(x)$ units of each variety x of the differentiated good, for all varieties x in the set X (to be determined in equilibrium) has a utility U:

$$U \equiv q_o^{1-\mu}\left(\int_{x \in X} q(x)^{\frac{\sigma-1}{\sigma}}\, dx\right)^{\frac{\sigma}{\sigma-1}\mu}$$
$$\text{with } \sigma > 1$$

where σ is the elasticity of substitution between two varieties of the differentiated good.

If all varieties in the set X are available domestically, at a price $p(x)$ each, the ideal price index for differentiated goods is:

$$P = \left(\int_{x \in X} p(x)^{1-\sigma}\, dx\right)^{\frac{1}{1-\sigma}} \tag{1}$$

The representative consumer has an isoelastic demand function for each differentiated variety. She spends $r(x)$ on each variety x:

$$r(x) = \mu w L \left(\frac{p(x)}{P}\right)^{1-\sigma} \tag{2}$$

where μwL is the total expenditure spent on differentiated goods.

2.2. Production and trade

There are two types of trade barriers, a fixed cost and a variable cost. If a firm exports, it must pay a fixed cost, composed of domestic and foreign labor. The foreign labor part of the fixed cost is denoted $\phi C_f w^*$, measured in units of the numeraire, and the domestic part $(1-\phi)C_f w$. The parameter ϕ is an index of the foreign labor content of fixed export costs, which may vary across sectors or across countries. The assumption that at least part of the entry cost into the foreign market is denominated in foreign labor is important. An exporter must cover costs both in domestic and in foreign labor. I only need to assume that the part of the fixed entry cost denominated in foreign labor is positive ($\phi > 0$). Campa and Goldberg (2010) show evidence that between 50 and 70% of the cost of entering foreign markets are denominated in foreign currency. This part consists of the cost of acquiring local information, setting up a local distribution network, and customizing goods to fit the local market. The second, variable, trade cost takes the form of an "iceberg" transportation cost. If one unit of any differentiated good is shipped abroad, only a fraction $1/\tau$ arrives. The rest melts on the way. The higher τ, the higher the variable trade cost.

Each country has access to the same technology. The marginal product of labor is constant. In order to start production, a firm must pay a fixed entry cost C_d in terms of domestic labor, at a cost wC_d in terms of the numeraire. The presence of fixed entry cost means that firms operate under increasing returns to scale. Each firm in the differentiated sector draws a random unit labor productivity $x \geq 0$. For a firm with productivity x, the cost of producing q_d units of good for the home market is

$c_d(q_d)$, and the cost of producing q_f units for the foreign market is $c_f(q_f)$:

$$c_d(q_d) = q_d \frac{w}{x} + wC_d$$
$$c_f(q_f) = q_f \frac{\tau w}{x} + (1-\phi)wC_f + \phi w^* C_f$$

Firms are price setters. Given that demand functions are isoelastic, the optimal price is a constant mark-up over the unit cost (including transportation costs),

$$p_d(x) = \frac{\sigma}{\sigma - 1} \times \frac{w}{x} \text{ at home,} \quad p_f(x) = \frac{\sigma}{\sigma - 1} \times \frac{\tau w}{x} \text{ abroad}$$

Given these pricing strategies, more productive firms are able to charge lower prices, capture a larger market share, and generate larger profits, both at home and abroad. A firm with productivity x potentially generates profits $\pi_d(x)$ in the domestic market, and $\pi_f(x)$ in the foreign market:

$$\pi_d(x) = \frac{r_d(x)}{\sigma} - wC_d = \frac{\mu}{\sigma} wL \left(\frac{\sigma}{\sigma - 1} \frac{w}{xP} \right)^{1-\sigma} - wC_d$$
$$\pi_f(x) = \frac{r_f(x)}{\sigma} - (1-\phi)wC_f - \phi w^* C_f = \frac{\mu}{\sigma} w^* L^* \left(\frac{\sigma}{\sigma - 1} \frac{\tau w}{xP^*} \right)^{1-\sigma} - (1-\phi)wC_f - \phi w^* C_f$$

Only those firms that can profitably produce domestically will survive, and only those firms that can profitably produce for the export market could export. $\overline{x}_d$ is the productivity threshold for survival on the domestic market, and $\overline{x}_f$ for profitable entry into the foreign market, absent any additional constraint. Only those firms that generate non-negative profits from domestic sales survive, and only those firms that generate non-negative profits from selling in the foreign market could export. The productivity thresholds are defined by,[1]

$$\pi_d(\overline{x}_d) = 0 \quad \text{and} \quad \pi_f(\overline{x}_f = 0 \tag{3}$$

Absent any other friction, all firms with a productivity above $\overline{x}_f$ would export. But for the potential asymmetry between countries, this model is almost identical to the Melitz (2003) model of international trade. Among other things, the only reason why the export status of a firm is correlated to the size of its domestic sales is that more productive firms sell more *and* are more likely to export. There is no direct link between what a firm exports, how many countries it exports to, and what it does at home.

In the next section, I introduce liquidity constraints. Financial imperfections create a link between different markets, and modify the adjustment to changes in exogenous variables.

2.3. Liquidity constraints

With perfect financial markets, any firm that could profitably export will find some investors to finance the entry cost into the foreign market. However, there are practical reasons to believe that such investments may not be easy to finance. The nature of the contracting and informational environment is different from a similar entry cost investment made domestically. This is mainly due to two factors. First, export activities are essentially riskier than domestic ones. Part of it is due to the objective added risks, such as foreign exchange risk. Existing financial hedging products such as swaps and options may not be available, or available at a prohibitive cost for most potential exporters. Information about foreign markets is more difficult and more expensive to acquire, and is less verifiable. Since such information is more difficult to verify than similar information on domestic markets, a potential investor may not be willing to trust a would-be exporter. Second, the contracting environment for international transactions is relatively weak. For example, it may be more difficult for investors to collect the proceeds of such sales in case of disagreement. In other words, a potential exporter cannot pledge much of its foreign activities as collateral, and this translates into ex ante under-investment.

A symmetric problem applies for foreign investors: informational asymmetries and contract incompleteness plague such relations. A foreign investor has little information on foreign firms. If she does enter in a relationship with a foreign firm, and if the terms of their contract are violated, she will find it difficult to seize the firm's assets. Arguably, trade credits will alleviate some of these issues. However, trade credits are typically offered to existing exporters, that is firms with a known and verifiable history of exports into a given market. Such a firm arguably has already covered most of the entry cost into foreign markets.

In the remaining part of this paper, I take an extreme view on the limitations of financial markets that potential exporters face. A firm may find investors for any investment regarding domestic activities, but none whatsoever for exporting activities. Therefore, a firm must rely on its own existing liquidity to cover entry costs into foreign markets. Moreover, firms inherit an exogenous amount of assets. Those assets vary depending on the firm's history of accumulating cash, on the

[1] Note that $(\overline{x}_f/\overline{x}_d)^{\sigma-1} = (\tau^{\sigma-1}C_d/C_f) \times (L/L^*) \times (P/P^*)$. I assume that trade barriers are always sufficiently high ($\tau^{\sigma-1}C_d/C_f$ sufficiently high) so that $\overline{x}_f > \overline{x}_d$ always holds. Only a subset of firms are able to export, and no firm is able to sell abroad but not domestically.

pledgeability of its assets, on the current market price of these assets. In Chaney et al. (2012), we explore for instance variations in local real estate prices as a source of variation of the market, and collateral, value of a firm's real estate holdings. This is an extreme and oversimplified view of liquidity constraints. These assumptions are designed to carry two properties. First, liquidity constraints are more severe for international trade than for domestic trade. Second, firms are more or less severely hampered by liquidity constraints, and how much constraints they face is not perfectly correlated with their current productivity.

Liquidity constraints are formalized in the following way. Firms only face liquidity constraints for accessing foreign markets. A firm that wants to export needs to have enough liquidity on its own to cover the fixed cost of entering foreign markets.

I further assume that each firm is endowed with pledgeable assets A. A is denominated in units of domestic labor, and has a value wA in terms of the numeraire. The profits generated from domestic sales, $\pi_d(x)$, can also be used as a source of internal financing for the export fixed cost. So more productive and larger firms tend to be less liquidity constrained (they generate more cash). The correlation between size and liquidity constraints is not perfect, as firms with the same domestic cash flow stream $\pi_d(x)$ may own assets of different pledgeable values. (A,x) is drawn from a joint distribution with c.d.f. $F(A,x)$ over $\mathbb{R}^+ \times \mathbb{R}^+$, and $F_x(x) \equiv \lim_{A\to\infty} F(A,x)$ over $\mathbb{R}^+$. I remain agnostic on the shape of the joint distribution $F(A,x)$. This object can typically be observed in the data, and so the model generates sharp empirical prediction, for any given joint distribution $F(A,x)$.

I also assume that the total mass of firms is proportional to the size of the country, L.[2]

In order to export, a firm must have enough liquidity to cover the fixed entry cost, $(1-\phi)wC_f + \phi w^* C_f$. The firm generates some cash flows from domestic sales, $\pi_d(x)$, and it has access to some additional exogenous pledgeable assets wA. So an exporter is subject to the following liquidity constraint,

$$\pi_d(x) + wA \geq (1-\phi)wC_f + \phi w^* C_f \tag{4}$$

More productive firms generate larger profits at home, and therefore are less dependent on external finance. $\overline{x}(A)$ is defined as the lowest productivity below which firms with liquidity A cannot gather enough liquidity to enter the foreign market. $\overline{x}(A)$ is defined by,

$$\pi_d(\overline{x}(A)) + wA = (1-\phi)wC_f + \phi w^* C_f$$

All firms with a productivity below $\overline{x}(A)$ are prevented from exporting because of liquidity constraints, even if they could profitably export.

2.4. Open economy equilibrium

To focus on what happens in the home country, I assume that foreign firms face no liquidity constraint, and make one additional simplifying assumption: price indices only depend on prices set by local firms. In other words, prices set by foreign exporters have a negligible impact on the general price index domestically. Formally, the price index equation (1) is approximated by,

$$P \approx \left(\int_{x \geq \overline{x}_d} p_d(x)^{1-\sigma} L \, dF_x(x) \right)^{\frac{1}{1-\sigma}} \tag{5}$$

It will be convenient to define the function $g(\cdot)$ in the following way:

$$g(\cdot): \overline{x}^{\sigma-1} = \left(\frac{\sigma}{\mu} \int_{x \geq \overline{x}} x^{\sigma-1} \, dF_x(x) \right) \times C \Leftrightarrow \overline{x} = g(C) \tag{6}$$

It is straightforward to prove that $g' > 0$. Rearranging the conditions for the productivity thresholds in Eq. (3) and the liquidity constraints condition in Eq. (4),

$$\overline{x}_d = g(C_d) \tag{7}$$

$$\overline{x}_f = \left(\frac{\tau w}{w^*} \right) \left(\frac{C_f}{C_d^*} \left(\phi + (1-\phi) \frac{w}{w^*} \right) \right)^{\frac{1}{\sigma-1}} g(C_d^*) \tag{8}$$

$$\overline{x}(A) = \left(\frac{C_d + (1-\phi)C_f + \frac{w^*}{w} \phi C_f - A}{C_d} \right)^{\frac{1}{\sigma-1}} g(C_d) \tag{9}$$

All the firms with a productivity above $\overline{x}_d$ produce and sell their output domestically. Only those firms with a productivity above $\max\{\overline{x}_f, \overline{x}(A)\}$ are able to export.

What are the determinants of the liquidity constraint, $\overline{x}(A)$, in Eq. (9)? $\overline{x}(A)$ is a downward sloping schedule. Firms that only own a small amount of pledgeable assets, A small, must have a high level of productivity in order to generate sufficient liquidity on their

[2] Bonfiglioli et al. (2015) propose an elegant model with financing frictions and endogenous entry, where the presence of financing frictions systematically affects the ex post distribution of productivities.

own and enter foreign markets. Firms with a large amounts of pledgeable assets on the other hand, A large, do not require much additional liquidity, and do not need a high productivity in order to be able to export. The higher the entry cost into the foreign country, C_f, the higher the curve $\overline{x}(A)$. The fixed overhead production cost, C_d, has an ambiguous impact on the curve $\overline{x}(A)$.[3] An increase of the domestic fixed cost C_d eats up part of a firm's liquidity, and reduces its ability to enter foreign markets. However, an increase in C_d also makes it harder for firms to survive, and forces some domestic firms out of business. This softens competition, increases the market share of each surviving firm, increases profits, and hence increases available liquidity. Which force dominates depends on the underlying distribution of firm productivity. The higher the exchange rate, that is the lower w^*/w, the lower the curve $\overline{x}(A)$. The reason is straightforward: if the exchange rate appreciates (w^*/w falls), the value of domestic assets in terms of foreign prices increases, and less domestic liquidity is required to enter the foreign market. The more substitutable the goods, that is the higher σ, the flatter $\overline{x}(A)$. This is because when goods are highly substitutable, any small difference in productivity implies large differences in profits, and therefore in the liquidity generated by domestic sales.

Interestingly, only a subset of firms are potentially subject to liquidity constraints. Firms with a very high productivity, no matter how little exogenous liquidity they own, are able to export. Formally, $\overline{x}(0)$ is bounded, so that firms with a productivity above $\overline{x}(0)$ do not need any exogenous liquidity. At the other extreme, firms with a sufficiently large amount of exogenous liquidity do not need to generate any additional liquidity from domestic sales. If their exogenous liquidity endowment A is sufficiently large to cover the fixed entry cost into both the domestic and the foreign market, even without any domestic sales, those firms could export.[4] In formal terms, $\overline{x}\left(C_d+(1-\phi)C_f+\frac{w^*}{w}\phi C_f\right)=0$. From a social efficiency point of view, there is no need to allocate exogenous liquidity beyond $C_d+(1-\phi)C_f+\frac{w^*}{w}\phi C_f$. We will see in the next section that profitability conditions imply that the maximum liquidity a firm could ever need to enter foreign markets is actually below $C_d+\frac{w^*}{w}C_f$. We can already see that the distribution of liquidity among firms (and how that correlates with a firm's productivity) will have important implications for the ability of firms to export.

We have seen that a firm's productivity may allow it to overcome liquidity constraints without the need to accessing financial markets. Are liquidity constraints going to be binding for any firm? In other words, is any firm profitable enough to be a viable exporter, but prevented from accessing foreign markets because of liquidity constraints? The following proposition gives conditions under which there will be a set of liquidity constrained exporters.

Proposition 1. *If* $\left(\frac{C_d^*}{C_d}\frac{C_d+(1-\phi)C_f+\frac{w^*}{w}\phi C_f}{\phi C_f+(1-\phi)\frac{w}{w^*}C_f}\right)^{\frac{1}{\sigma-1}}\frac{g(C_d)}{g(C_d^*)}>\frac{\tau w}{w^*}$, *then there is a non-empty set of liquidity constrained firms (denoted* Ω*).*

These firms could profitably export, but are prevented from doing so because they lack sufficient liquidity.

Proof. See Appendix B. □

From now on, I assume that the condition in Proposition 1 holds, so that the set of liquidity constrained firms, Ω, is non-empty.

No firm with a productivity below $\overline{x}_f$ could profitably export. These firms have a productivity too low to allow them to generate enough profits in the foreign market to recover the fixed entry cost. Firms with a productivity above $\overline{x}(0)$ export no matter how little pledgeable assets they hold: they are competitive enough in the foreign market to generate positive profits, and they generate sufficient liquidity from their domestic activities to cover the entry cost into the foreign market, without the need for external financing. Firms with an intermediate productivity, $\overline{x}_f \le x < \overline{x}(0)$, could profitably export, but do not generate sufficient liquidity from their sales on the domestic market. They need extra liquidity. Without pledgeable assets, despite being profitable, they would be prevented from exporting. This is shown in Fig. 1. The dark shaded area Ω corresponds to liquidity constrained firms: these firms are willing to export ($x \ge \overline{x}_f$), but cannot do so because they lack sufficient liquidity ($x \le \overline{x}(A)$).

It is interesting to note that the distribution of liquidity among firms does matter. As can be seen graphically, firms with more than $\overline{A}$ (with $\overline{x}(\overline{A})=\overline{x}_f$) pledgeable assets have "too much" liquidity. $\overline{A}$ corresponds to the amount of liquidity the least productive exporter would need to enter foreign markets. Any firm with a productivity below $\overline{x}_f$ could not export, and therefore would have no use for exogenous liquidity. Following a similar reasoning, no firm with a productivity above $\overline{x}(0)$ has the need for any pledgeable assets. Such high productivity firms are already able to generate sufficient liquidity from their domestic sales, and do not need additional source of funding. Only firms with an intermediate levels of productivity (between $\overline{x}_f$ and $\overline{x}(0)$) must have access to some exogenous source of liquidity in order to overcome financial constraints. However none of these firms would need more than a maximum $\overline{A}$ of liquidity. I describe in the next section how the distribution of wealth matters for aggregate exports in the next section.

Proposition 1 is testable. It states that financially constrained firms cannot export. Financially constrained firms are firms that both lack sufficient pledgeable assets, and are not productive enough to generate sufficient liquidity on their own. Campa and Shaver (2001) find that more liquidity constrained firms are less likely to export. They define financially constrained firms as firms for which investment is correlated with cash flows. In my model, financially constrained firms in my model, that is firms that both lack pledgeable assets and generate insufficient liquidity from their sales, correspond to Campa and Shaver's category of financially constrained firms. Therefore there is some evidence supporting Proposition 1: there exists a set of financially constrained firms that are prevented from exporting.

[3] See Appendix A for a formal proof of this statement.

[4] Note however that such wealthy firms may not be able to survive in the domestic market, or profitably export. This is precisely described in the next section.

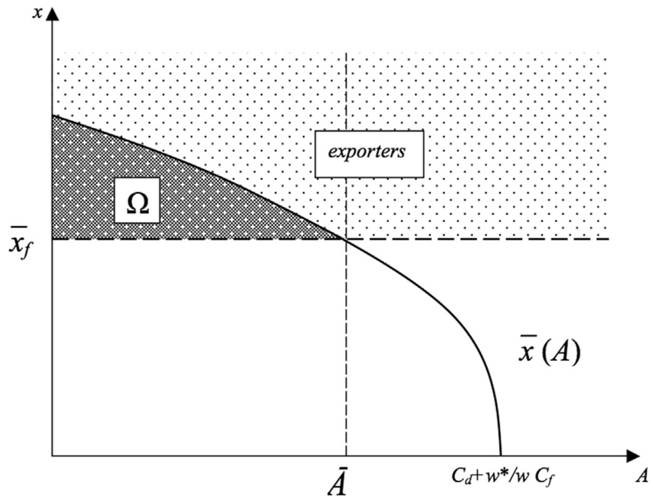

Fig. 1. Liquidity constrained exporters. *Notes*: A is the firm's exogenous liquidity, and x its productivity. No firm with a productivity below $\overline{x}_f$ can profitably export. On top of this, firms below the curve $\overline{x}(A)$ are liquidity constrained. All firms in the area Ω between $\overline{x}_f$ and $\overline{x}(A)$ are liquidity constrained exporters. They could profitably export, but are prevented from doing so because of liquidity constraints.

2.5. Liquidity constraints and missing trade

If it does export, the total value of exports (f.o.b.) by a firm with productivity x is $r_f(x)$. Using the expressions for the productivity thresholds in Eqs. (7)–(9), plugging those and the price index Eq. (5) back into the revenue Eq. (2),

$$r_f(x) = \sigma\big((1-\phi)wC_f + \phi w^* C_f\big)\left(\frac{w^*}{\tau w} \times \frac{x}{\overline{x}_d^*}\right)^{\sigma-1} \tag{10}$$

All firms with a productivity above $\max\{\overline{x}_f, \overline{x}(A)\}$ export. All firms in the set Ω are prevented from exporting. The total volume of missing trade (f.o.b.) from these constrained exporters, T_{missing}, and the total volume of exports (f.o.b.), T_{total}, are given by,

$$T_{\text{missing}} = L \iint_{(A,x)\in\Omega} r_f(x)\, dF(A,x)$$
$$T_{\text{total}} = L \int_{x \geq \overline{x}_f} r_f(x)\, dF_x(x) - T_{\text{missing}}$$

The total volume of missing trade depends on several parameters of the distribution of productivity and liquidity shocks. It depends both on the average liquidity available economy wide and on the distribution of this liquidity. If only highly productive firms are liquidity constrained, there will not be any missing trade. If only those low productivity firms that would not export anyway are liquidity constrained, there will not be any missing trade. To get a better understanding of the magnitude of this missing trade, I consider a special case for the distribution of liquidity and productivity shocks $F(A, x)$.

A simplified form for the joint distribution of productivity and liquidity shocks, $F(A, x)$ allows simple predictions for the export behavior of liquidity constrained firms. Assume that the liquidity shocks and the productivity shocks are uncorrelated. Further assume that a fraction θ of firms are liquidity constrained (they hold A pledgeable assets, with $A < \overline{A}$), and the remaining $(1-\theta)$ is not liquidity constrained (they hold pledgeable assets above $\overline{A}$). $\overline{A}$ is defined as the minimum liquidity above which financial constraints are not binding, and the only constraint is the profitability constraint: $\overline{x}(\overline{A}) = \overline{x}_f$. Firms with a high enough positive liquidity shock will always be able to generate enough liquidity on the home market. They will export only if exporting is profitable. In this special case, T_{missing} and T_{total} are given by,

$$T_{\text{missing}} = \theta L \int_{\overline{x}(A)}^{\overline{x}_f} r_f(x)\, dF_x(x) \tag{11}$$

$$T_{\text{total}} = L \int_{x \geq \overline{x}_f} r_f(x)\, dF_x(x) - T_{\text{missing}} \tag{12}$$

Definition 2. A deepening of financial markets corresponds to an increase of A, the amount of pledgeable assets available to financially constrained firms. A widening of financial markets corresponds to a reduction in θ, the fraction of financially constrained firms.

Proposition 3. *Both a deepening and a widening of financial markets have a positive impact on total trade exports of differentiated goods.*

Proof. See Appendix C. □

This model predicts that financial constraints faced by potential exporters have a negative impact on trade flows. Both the absolute amount of liquidity and the distribution of liquidity among firms matter for the total volume of trade in differentiated goods. These predictions are testable. The model predicts that more financially constrained industries should have lower trade flows. Sectors where the distribution of liquidity is more unequal should have lower trade flows. Moreover, financial constraints matter more when entry costs to foreign markets are larger (C_f large). If those entry costs increase with distance, more financially constrained sectors should have a larger distance elasticity of trade (in absolute value).

3. The ambiguous impact of exchange rate shocks on trade flows

In this section, I describe the impact of exchange rate shocks on trade. If the domestic currency appreciates vis á vis the foreign currency, domestic producers lose competitiveness in the foreign market. This is a classic term of trade effect. However, an appreciation of the domestic currency relaxes the liquidity constraint faced by potential exporters. The value of domestic assets in terms of the foreign currency increases. Liquidity constrained firms are now more likely to be able to pay foreign denominated part of the fixed export cost and start exporting. Simultaneously, existing exporters lose competitiveness and export less. The least productive among the non-liquidity constrained exporters exit the export market altogether, both because their marginal cost of production appreciates with the domestic currency, and because the domestic currency denominated part of the fixed export cost increases. The intensive margin of trade is negatively affected by an appreciation of the exchange rate, whereas the extensive margin is both negatively and positively affected.

I model exchange rate shocks as a shock on relative wages (in terms of the numeraire). An appreciation of the domestic currency is defined as an increase in the productivity in the homogeneous sector at home, which leads to an increase in the domestic wages w, all else equal. This definition is similar to the one used by Atkeson and Burstein (2008). If the domestic wage increases, the value of domestic assets (wA and $\pi_d(x)$) increases, whereas potential exporters lose competitiveness in the foreign market ($p_f(x)/P^*$ increases). These effects are exactly equivalent to an appreciation of the domestic currency vis á vis the foreign currency.

Proposition 4. *An appreciation of the exchange rate has* 3 *effects*:

(i) *Existing exporters lose market shares abroad and reduce their exports*: $\frac{\partial r_f(x)}{\partial w} < 0$.
(ii) *The least productive non-constrained exporters are forced out of the export market*: $\frac{\partial \bar{x}_f}{\partial w} > 0$.
(iii) *The most productive constrained firms start exporting*: $\frac{\partial \overline{x}(A)}{\partial w} < 0$.

Proof. (i) The first effect is the classic impact of a loss of competitiveness for exporters. As the value of domestic inputs increases, domestic exporters have to charge higher prices in order to maintain mark-ups, and therefore they lose market shares in the foreign market. This loss of market shares implies a reduction in exports. Formally, differentiating Eq. (10) with respect to w,

$$\frac{\partial r_f(x)}{\partial w} = -(\sigma-1)\frac{r_{f(x)}}{w} < 0$$

(ii) The second effect is the natural corollary of the first effect. Exporters lose profits for two reasons. First, they lose market share and their variable profits fall. Second, the increase in the domestic currency denominated part of the fixed export cost eats some of their profits.[5]

$$\frac{\partial \pi_f(x)}{\partial w} = -\frac{\sigma-1}{\sigma}\frac{r_{f(x)}}{w} - (1-\phi)C_f < 0$$

So the least productive firms, earning smaller profits, cannot cover the entry cost into the foreign market anymore. The productivity threshold $\bar{x}_f$ goes up. Formally, differentiating Eq. (8) with respect to w,

$$\frac{\partial \bar{x}_f}{\partial w} = \frac{\bar{x}_f}{w}\left(1 + \frac{\frac{C_f}{C_d^*}(1-\phi)\frac{w}{w^*}}{\frac{C_d + (1-\phi)C_f + \frac{w^*}{w}\phi C_f - A}{C_d}}\right) > 0$$

[5] I am grateful to Mario Crucini for suggesting to introduce a domestic denominated part to the fixed export cost.

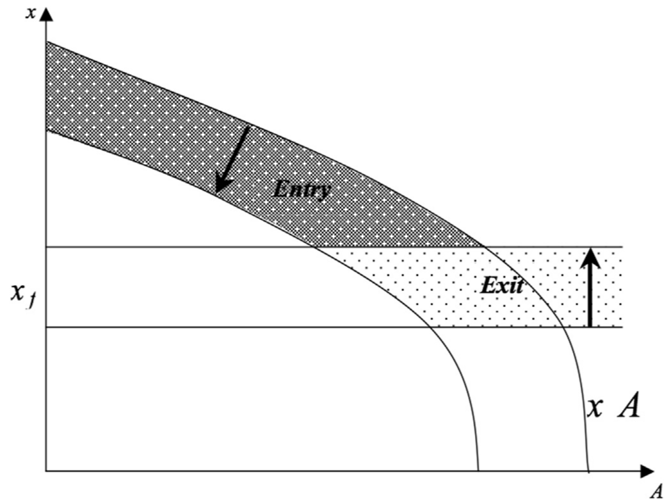

Fig. 2. The ambiguous impact of an exchange rate appreciation. *Notes*: A is the firm's exogenous liquidity, and x its productivity. An appreciation of the domestic currency erodes the competitiveness of exporters, and forces the least productive exporters to exit (light shaded area). At the same time, it relaxes the liquidity constraint, and allows some liquidity constrained firms to enter the export market (dark shaded area).

(iii) The last effect is novel and comes from the relaxation of the liquidity constraint. As the domestic currency appreciates, the value of domestic assets (both exogenous liquidity and endogenous domestic profits) in terms of foreign currency increases. Since the entry cost into the foreign market is paid in foreign currency, this means a relaxation of the liquidity constraint for constrained exporters. Formally, differentiating Eq. (9) with respect to w,

$$\frac{\partial \overline{x}(A)}{\partial w} = -\left(\frac{1}{\sigma-1}\right)\frac{\frac{w^*}{w}\phi C_f}{\left(C_d+(1-\phi)C_f+\frac{w^*}{w}\phi C_f - A\right)}\frac{\overline{x}(A)}{w} < 0 \ \square$$

An appreciation of the exchange rate causes both entry and exit. Non-liquidity constrained firms with a low productivity are forced out of the export market because they lose competitiveness in the foreign market and they face higher export costs (the domestic currency denominated part). Liquidity constrained firms with a high productivity (close to $\overline{x}(A)$) face a relaxed liquidity constraint and enter the export market.

This can be seen in Fig. 2. Low productivity non-constrained firms, in the light shaded area, exit the export market. This is due to the fact that they lose competitiveness in the foreign market and face a higher fixed export cost, and therefore earn less profits. They cannot cover the fixed trade barrier any more, and exit the export market. At the same time, high productivity constrained firms, in the dark shaded area, enter the export market. These firms are sufficiently productive to export (even after the currency appreciation), but they were prevented from doing so because of liquidity constraints. The appreciation of their currency increases the value of their domestic assets, relaxes their liquidity constraint, and allows them to start exporting. Depending both on the strength of the liquidity constraints (the overall scarcity of liquidity in the economy) and on the number of liquidity constrained firms, either effect can dominate. If there are relatively many liquidity constrained firms, there will be a net entry of firms following an appreciation of the exchange rate.

The presence of liquidity constraints introduces an investment motive in international trade in goods. Exports do not depend only on the competitiveness of exporters, it also depends on the value of domestic assets relative to the "cost" of exporting. In the same way as an exchange rate appreciation will make investment abroad more accessible, it makes exporting more likely for a group of firms.

Proposition 5. *If competition is soft* (σ *close to* 1), *an appreciation of the exchange rate will have a positive impact on exports of differentiated goods.*

Proof. See Appendix D □

This model of international trade with liquidity constrained exporters predicts that, under some conditions, an appreciation of the exchange rate, despite its negative impact on the competitiveness of exporters, may have a strictly positive impact on exports of differentiated goods. If competitiveness does not have too large an impact on the size of market shares, that is if goods are very differentiated (σ low), then the entry of liquidity constrained exporters following an appreciation of the exchange rate will dominate. Total exports of differentiated goods increase after an appreciation of the exchange rate.

More generally, even if an appreciation of the exchange rate has an negative impact on exports, the negative impact of an exchange rate appreciation will be milder (or even become positive) the more unequal the distribution of liquidity within the sector (θ low). If liquidity is unequally shared among firms (θ low), many healthy and productive firms are liquidity constrained. This means that many firms could profitably export, but they lack sufficient access to financial markets to cover the entry cost into the foreign market. If the exchange rate appreciates, the liquidity constraint faced by all those firms is relaxed. A fraction of these firms will then start exporting, despite the loss of competitiveness. The more unequal the distribution of wealth among firms, the more firms will start exporting, and the more positive the impact of a exchange rate appreciation.

Liquidity constraints for the access to foreign markets allow effective policy interventions. There is room for a temporary competitive revaluation. A temporary revaluation of the domestic currency, by increasing the value of domestic assets, may allow liquidity constrained firms to start exporting. The sunk cost nature of a fraction of fixed costs associated with exporting implies some asymmetry for the response of trade to exchange rate fluctuations. Once liquidity constrained firms have started exporting, insofar as they do not have to pay this fixed cost again, they will continue exporting, even after a devaluation of their currency.

It may also be possible to extend this model to describe phenomena of amplification and contagion in international trade. If a firm exports to a given foreign market, it generates some liquidity, in addition to domestic profits. Such extra liquidity will give this firm an edge for entering other foreign markets. This corresponds to an amplification mechanism: an increase in exports by a firm may trigger additional exports, since it relaxes its liquidity constraint. In a multi-country dynamic setting, firms may be able to accumulate liquidity from both their domestic sales, and past sales to other foreign markets. Firms that have already entered many foreign markets are more likely to have sufficient liquidity to enter new and less accessible markets. Shocks that affect exports with one trading partner may influence the volume of exports with other trading partners, even absent any direct link between those countries. If trade links with a given trading partner are severed, the liquidity streams generated from exporting to this country cease, and some exporters may be forced to pull out of other markets as well. Moreover, if markets in the same region are characterized by similar trade barriers, they will attract similar exporters. Modifying the access to one of these markets will affect trade with all other countries in the same region. Hence, liquidity constraints may artificially generate contagion phenomena in international trade.

4. Conclusion

I have shown in this paper that liquidity constraints may modify fundamentally the behavior of exporters, and the patterns of aggregate exports. If firms face liquidity constraints when accessing foreign markets, some firms are prevented from exporting. They could profitably enter foreign markets, but are prevented from doing so because they lack the ability to access financial markets and cover entry costs into foreign markets. The main prediction of the model is that financial underdevelopment hinders exports. Both the total amount of liquidity available and the distribution of this liquidity matter for trade. The model also predicts that the presence of liquidity constraints will reduce the sensitivity of trade flows to exchange rate fluctuations. When the exchange rate appreciates, exporters lose competitiveness, and they reduce their exports and the least productive among them exit altogether. However, since the value of domestic assets in terms of foreign prices increases, liquidity constraints for accessing foreign markets are relaxed. Some firms start exporting. This entry of liquidity constrained exporters dampens the negative competitiveness effect of an exchange rate appreciation. Under some conditions, an exchange rate appreciation may even have a positive impact on exports. Liquidity constraints also create artificial links between different markets, and thus generate amplification and contagion phenomena.

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Appendix A. Proof: monotonicity of $\overline{x}(A, C_d)$

Proposition 6. *$\overline{x}(A)$ is increasing in C_d.*

Proof. The function $g(C)$ is steeper than $C^{\frac{1}{\sigma-1}}$. This can be seen simply by rearranging the definition of the function $g(\cdot)$ in Eq. (6). We can define the function $\xi(C)$ in the following way:

$$\xi(C) \equiv \frac{g(C)}{C^{\frac{1}{\sigma-1}}} = \frac{\sigma}{\mu} \int_{x > g(C)} x^{\sigma-1}\, dF_x(x)$$

Since $g(\cdot)$ is increasing in C, $\xi(\cdot)$ is decreasing in C. How steep it is depends on the underlying distribution of productivity F_x. Depending on this distribution, $\xi(\cdot)$ can be arbitrarily steep, or arbitrarily flat.

We can now plug in this new function $\xi(\cdot)$ into the formula for $\overline{x}(A)$ in Eq. (9):

$$\overline{x}(A)=\left(C_d+\frac{w^*}{w}C_f-A\right)^{\frac{1}{\sigma-1}}\times\xi(C_d)$$

When C_d increases, the first term of the product goes up, and the second term goes down. The net effect can go either way, depending on how steep the function $\xi(\cdot)$ is.

We can describe the intuition behind each term of the product. When the domestic fixed cost goes up, it eats up the liquidity of all firms. This pushes up the minimum productivity required for entering foreign markets, at any level of A. This is the increase in the first term of the product.

At the same time, due to the increase in the domestic fixed cost, some firms are pushed out of business. Some firms die, competition softens among survivors, and profits increase. Each surviving firm now generates more liquidity from its domestic sales. The minimum productivity required for entering foreign markets falls, at any level of A. This is the reduction in the second term of the product. □

Appendix B. Proof of Proposition 1

Proposition 1 (reminded). If $\left(\frac{C_d^*}{C_f}+\frac{w^*C_d^*}{wC_d}\right)^{\frac{1}{\sigma-1}}\frac{g(C_d)}{g(C_d^*)}>\frac{\tau w}{w^*}$, then there is a non-empty set of liquidity constrained firms (denoted Ω). These firms could profitably export, but are prevented from doing so because they lack sufficient liquidity.

Proof. All firms below $\overline{x}(A)$ are liquidity constrained, and cannot export no matter how profitable their exporting would be. All firms above $\overline{x}_f$ could profitably export, if they have sufficient liquidity. I want to prove that $\Omega\neq\varnothing$, with $(A,x)\in\Omega$ iif $\overline{x}_f\le x<\overline{x}(A)$. Firms in Ω could profitably export ($x\ge\overline{x}_f$), but they are prevented from doing so because they are liquidity constrained ($x<\overline{x}(A)$). A necessary and sufficient condition for Ω to be non-empty is that $\overline{x}(0)>\overline{x}_f$. I know that:

$$\overline{x}_f=\left(\frac{\tau w}{w^*}\right)\left(\frac{C_f}{C_d^*}\left(\phi+(1-\phi)\frac{w}{w^*}\right)\right)^{\frac{1}{\sigma-1}}g(C_d^*)$$
$$\overline{x}(A)=\left(\frac{C_d+(1-\phi)C_f+\frac{w^*}{w}\phi C_f-A}{C_d}\right)^{\frac{1}{\sigma-1}}g(C_d)$$

so that $\overline{x}(0)>\overline{x}_f$ iif,

$$\left(\frac{C_d^*}{C_d}\frac{C_d+(1-\phi)C_f+\frac{w^*}{w}\phi C_f}{\phi C_f+(1-\phi)\frac{w}{w^*}C_f}\right)^{\frac{1}{\sigma-1}}\frac{g(C_d)}{g(C_d^*)}>\frac{\tau w}{w^*}$$

If $\left(\frac{C_d^*}{C_d}\frac{C_d+(1-\phi)C_f+\frac{w^*}{w}\phi C_f}{\phi C_f+(1-\phi)\frac{w}{w^*}C_f}\right)^{\frac{1}{\sigma-1}}\frac{g(C_d)}{g(C_d^*)}>\frac{\tau w}{w^*}$, Ω is non-empty, and there are liquidity constrained firms.□

Appendix C. Proof of Proposition 3

Proposition 3 (reminded). *Both a deepening and a widening of financial markets have a positive impact on total trade flows.*

Proof. A deepening of the financial markets corresponds to a relaxation of the liquidity constraint of constrained firms (a reduction of A). A widening of financial markets corresponds to a reduction in the number of liquidity constrained firms (a reduction of θ). Differentiating the expressions for total trade and missing trade in Eqs. (12) and (11),

$$\frac{\partial T_{\text{total}}}{\partial\theta}=-L\int_{\overline{x}(A)}^{\overline{x}_f}r_f(x)\,dF_x(x)$$
$$\frac{\partial T_{\text{total}}}{\partial A}=-\theta T\frac{\partial\overline{x}(A)}{\partial A}r_f(\overline{x}(A))\,\frac{\partial F_x}{\partial x}(\overline{x}(A))$$

Differentiating Eq. (9), $\frac{\partial\overline{x}(A)}{\partial A}=-\left(\frac{1}{\sigma-1}\right)\frac{\overline{x}(A)}{C_d+(1-\phi)C_f+\frac{w^*}{w}\phi C_f-A}<0$, which insures that a relaxation of the liquidity constraint has a positive impact on trade. Therefore,

$$\frac{\partial T_{\text{total}}}{\partial\theta}<0\quad\text{and}\quad\frac{\partial T_{\text{total}}}{\partial A}>0\ □$$

Appendix D. Proof of Proposition 5

Proposition 5 (reminded). *If competition is soft* (σ close to 1), *an appreciation of the exchange rate will have a positive impact on exports.*

Proof. I can rewrite total exports in Eq. (12) in the following way,

$$T_{\text{total}} = \int_{x \geq \overline{x}(A)} r_f(x) L \, dF_x(x) + (1-\theta) \int_{\overline{x}_f}^{\overline{x}(A)} r_f(x) L \, dF_x(x)$$

Differentiating this equation with respect to w, and using Leibnitz rule, and applying Lebesgues' theorem of monotone convergence to insure existence of the integrals,

$$\frac{\partial T_{\text{total}}}{\partial w} = \int_{x \geq \overline{x}(A)} \frac{\partial r_f(x)}{\partial w} L \, dF_x(x) + (1-\theta) \int_{\overline{x}_f}^{\overline{x}(A)} \frac{\partial r_f(x)}{\partial w} L \, dF_x(x) - (1-\theta) \frac{\partial \overline{x}_f}{\partial w} r_f(\overline{x}_f) L \frac{\partial F_x(\overline{x}_f)}{\partial x} - \theta \frac{\partial \overline{x}(A)}{\partial w} r(\overline{x}(A)) L \frac{\partial F_x(\overline{x}(A))}{\partial x}$$

Using the formulas for $\partial r_f(x)/\partial w$, $\partial \overline{x}_f/\partial w$, and $\partial \overline{x}(A)/\partial w$ from the previous proof,

$$\frac{\partial T_{\text{total}}}{\partial w} = -\left(\frac{\sigma-1}{w}\right) T_{\text{total}} - (1-\theta) \frac{\overline{x}_f}{w} \left(1 + \frac{\frac{C_f}{C_d^*}(1-\phi)\frac{w}{w^*}}{\frac{C_d + (1-\phi)C_f + \frac{w^*}{w}\phi C_f - A}{C_d}} \right) r_f(\overline{x}_f) L \frac{\partial F_x(\overline{x}_f)}{\partial x}$$
$$+ \left(\frac{\theta}{\sigma-1}\right) \frac{\frac{w^*}{w}\phi C_f}{\left(C_d + (1-\phi)C_f + \frac{w^*}{w}\phi C_f - A\right)} \frac{\overline{x}(A) r_f(\overline{x}(A))}{w} L \frac{\partial F_x(\overline{x}(A))}{\partial x}$$

The first two terms in the sum are negative (the loss of competitiveness of existing exporters, and the exit of non liquidity constrained exporters). The last term is positive (the entry of liquidity constrained exporters).

The first two terms are bounded, and the last term converges to infinity as σ converges to 1. In other words, provided that the loss in competitiveness is not too severe (σ close to 1), the entry of liquidity constrained exporters will dominate the exit of low productivity unconstrained exporters, and the reduction in exports by existing exporters.□

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