Destabilizing carry trades*

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Abstract

We offer a model of currency carry trades in which carry traders generate self-sustained excess returns if they coordinate on supplying excessive capital to a target economy. The interest-rate differential between their funding currency and the target currency is their coordination device. Such self-fulfilling profitable currency trades arise when the central bank of the target economy ignores the impact of carry-trade inflows on domestic asset prices, and responds only to their effect on inflation. We solve for a unique equilibrium that exhibits the classic pattern of the carry-trade recipient currency appreciating for extended periods, punctuated by sharp falls.

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1 Introduction

A currency carry trade consists in selling a low interest rate currency to fund the purchase of a high interest rate currency. The high Sharpe ratio generated by carry-trade strategies is one of the most enduring puzzles in international finance.

The international capital flows that chase such large interest-rate differentials are regularly at the forefront of the debate on global financial stability. They are often accused of unduly destabilizing local financial markets. Several observers have recently pointed at an important global component in local asset prices and bank leverage that is highly correlated with funding conditions in U.S. dollars. This component seems associated with international credit flows that may be misaligned with local macroeconomic conditions, and with the objectives of the local monetary authority (Agrippino and Rey, 2014; Bruno and Shin, 2015; Rey, 2013).

This paper offers a theory that relates excess returns on carry trades to the destabilizing consequences of their associated capital flows. We write down a model in which carry traders may earn positive excess returns (rents) if they successfully coordinate on exploiting asynchronous monetary policies. In our setup, international investors enter into carry trades by trading bonds denominated in the world currency and in the currency of a small open economy. Our results rest on three ingredients. First, markets are incomplete in that the households of this small economy, unlike carry traders, can only trade domestic bonds.

Second, the prices of the nontradable goods in this small economy are much stickier than that of the tradable goods. This is consistent with evidence documented by Burstein, Eichenbaum, and Rebelo (2005).[1]

Finally, the domestic monetary authority anchors domestic inflation expectations by committing to a textbook interest rule that responds to realized CPI inflation. In particular, the monetary authority responds to carry-trade inflows only insofar as they affect domestic inflation. It ignores the direct effect of these inflows on local asset prices.

The mechanism that leads to possible excess returns on carry trades is as follows. First, market incompleteness implies that the inflows/outflows in the domestic bond market resulting from carry-trade activity affect bond prices. Second, by ignoring that carry-trade inflows bid up asset prices and thus reduce the domestic real rate, the central bank acts as if it was inadver-

[1] They argue that the slow adjustment of the prices of nontradables explains why the large devaluations that they study are associated with little inflation, and with a large decline in the real exchange rate.

tently introducing positive policy shocks to the interest rule in response to these inflows. This implies that carry-trade inflows result in a realized inflation that is below target. Third, since the prices of nontradable goods do not adjust much, this deflationary impact of carry-trade inflows must operate through the prices of tradable goods, and thus through a large appreciation of the nominal exchange rate.

If these three effects are sufficiently important, then the carry trade generates self-justified abnormal profits, and the anticipation of future large capital inflows in (or outflows out of) the small open economy is self-fulfilling.

We illustrate this novel mechanism in two different settings. Section 2 first offers a perfect-foresight model in which these ingredients interplay in the simplest and most transparent fashion. We show that they can generate (at least) three steady-states. In one steady-state, carry traders' portfolio choice has an interior solution, uncovered interest parity (UIP) holds and there are no excess returns on the carry trade. If this steady-state is unstable, there are also steady-states in which the carry trade is a *self-fulfilling arbitrage opportunity*—a free lunch that arises if and only if carry traders seek to exploit it. In one of these steady-states, carry traders supply excessive capital to the domestic economy and benefit from a perpetual appreciation of the exchange rate, whereas in the other one they short domestic bonds too much and benefit from the domestic currency depreciating.

Section 3 then develops a more sophisticated version of the model in which the interest rate on the world currency is subject to exogenous stochastic shocks that we interpret as policy shocks. In this case, the equilibrium is unique and the interest-rate differential acts as a coordination device among carry traders. Positive shocks on the interest-rate differential set off dynamics in which capital inflows increase, and the domestic currency keeps appreciating. This generates a prolonged series of positive returns on the carry trade, that ends abruptly only after a sufficiently long series of negative shocks on the interest differential leads carry traders to coordinate on large and rapid capital outflows. Importantly, the carry trade is no longer an arbitrage opportunity in this model but only a "good deal"—a financial transaction that has a positive net present value.

This latter model with unique equilibrium yields several interesting predictions. First, it suggests that a sufficiently large interest-rate differential predicts an appreciation of the high-rate currency. Second, it also predicts the profitability of FX momentum strategies—carry trades that were profitable in the previous period are more likely to be profitable in the current one. Further, since equilibrium paths feature rare but dramatic fluctuations in carry-trade activity, finite sample paths generated by the model would likely generate a peso problem, and lead to an overestimation of the expected return on the carry trade. Finally, our model also generates a new set of yet untested predictions on the relationship between the stance of monetary policy and the patterns of carry-trade returns.

Related Literature

Note first that our theory of carry-trade returns as self-fulfilling genuine excess returns bears little relationship to the existing theories that seek to explain the return on carry trades as a compensation for (possibly mismeasured) risk. Farhi and Gabaix (2014) thorougly survey this existing literature. We do not deny that a significant fraction of carry-trade returns may reflect risk premia. We abstract from risk considerations here for tractability only, and view our theory as a complement to such risk-based considerations rather than a competing alternative.

More generally, Engel (2015) provides a comprehensive recent survey of the vast literature on exchange rate determination and on the failure of UIP. Here we focus on detailing the relationship of our paper to a small number of closely related contributions both within and outside the field of international economics.

Our approach is most closely related to models of financial instability in which speculators earn rents if they successfully coordinate on a collective course of action that triggers a policy response that benefits them. In international economics, static models of self-fulfilling currency attacks pioneered by Obstfeld (1996) have this flavor. Farhi and Tirole (2012) or Schneider and Tornell (2004) offer models of "collective moral hazard" in which the government bails out speculators if their aggregate losses are sufficiently large, which creates a coordination motive among speculators. In this paper, we invoke related arguments in order to rationalize carry-trades returns. We contribute to this literature on coordination-driven financial instability in two ways.

First, our paper is the first, to our knowledge, in which speculators seek to game an interest-rate rule that is directly borrowed from New-Keynesian textbooks. This is a useful and novel attempt at bridging the gap between the literature on destabilizing speculation and mainstream monetary economics.

Second, we formalize the dynamic coordination game among carry traders using the tools developed by Frankel and Pauzner (2000) and Burdzy, Frankel, and Pauzner (2001) in order to obtain a unique predictable outcome. We show that their setup can be adapted to the situation in which strategic complementaries among agents coexist with congestion effects. This is important because most financial models with strategic complementarities also feature congestion effects. We also show that the equilibrium paths resulting from this model square well, at least qualitatively, with many empirical patterns of carry-trade returns. He and Xiong (2012) also apply the equilibrium selection techniques developed by Burdzy, Frankel, and Pauzner in a financial context—the roll-over of short-term debt.

Finally, it is interesting to relate our approach to the literature on portfolio demand in incomplete markets. In a recent contribution, Gabaix and Maggiori (2015) introduce carry traders as financial institutions that benefit from the incompleteness of global financial markets by intermediating gains from trade between countries. Financial constraints imply that these institutions supply liquidity inelastically, which generates risk premia on carry trades. We also model carry traders as financial institutions operating in incomplete markets. In our setup, however, these institutions source funds in one country in order to destabilize another country that uses an inappropriate monetary policy. Thus we reach very different conclusions regarding the relationship between limits to arbitrage and excess returns. First, in our setup, excess returns are self-fulfilling. They may or may not be there depending on the equilibrium trading strategies on which the arbitrageurs coordinate. Second, whereas tighter financial constraints lead to larger excess returns in Gabaix and Maggiori (2015), the opposite holds in our setup. Less constrained arbitrageurs have more financial muscle to exploit the policy mistakes of central banks. Thus they have a free hand at reaping higher excess returns, and they generate more financial instability by doing so.

2 Inflation targeting and self-justified arbitrages

Time is discrete and is indexed by $t \in \mathbb{Z}$. There are two types of agents, households populating a small open economy and "carry traders." There is a single tradable good that has a fixed unit price in the world currency.

Households

The households live in a small open economy. They use a domestic currency that trades at S_t units of the world currency per unit at date t.

At each date, a unit mass of households are born. Households live for two dates, consume when young and old, and work when old. Each household receives an initial endowment at birth with nominal value $P_t W \geq 0$, where

P_t is the domestic price level.[2] The cohort that is born at date t has quasi-linear preferences over bundles of consumption and labor (C_t, C_{t+1}, N_{t+1})

$$\ln C_t + \frac{C_{t+1} - N_{t+1}^{1+\eta}}{R}, \tag{1}$$

where $\eta, R > 0$.

Domestic consumption services C_t are produced combining the tradable good C_t^T and two nontradable goods $C_t^{N_1}$ and $C_t^{N_2}$ according to the technology

$$C_t = \frac{\left(C_t^T\right)^{\alpha} \left(C_t^{N_1}\right)^{\beta} \left(C_t^{N_2}\right)^{\gamma}}{\alpha^{\alpha}\beta^{\beta}\gamma^{\gamma}}, \tag{2}$$

where α, β, $\gamma \in (0, 1)$ and $\alpha + \beta + \gamma = 1$. Domestic firms set by old households use labor input to produce. The exact specification of the production processes are immaterial for our analysis. All that is needed is that both nontradable goods are produced in finite, non zero quantities at each date. Households collect labor income and the profits from their firms when old.

Households can trade risk-free one-period bonds in zero net supply that are denominated in the domestic currency. The nominal interest rate on these bonds is set by the domestic central bank according to a rule described below.

Carry traders

Carry traders consume outside the domestic economy. They can trade the same nominal bonds as that available to households. In addition, they also have access to investments denominated in the world currency. These investments generate an exogenous gross per period return that we denote $R^W > 0$.

In this section we are only interested in studying the circumstances under which these carry traders can generate arbitrage opportunities between two dates by forming zero-cost portfolios—portfolios such that the long and short initial positions have the same initial value. We deem such portfolios "carry trades." Arbitrage opportunities are free lunches that any agent with increasing utility over consumption demands in infinite quantity. Accordingly, we do not need to specify carry traders' preferences. We only assume that their utility is weakly increasing in the consumption of the

[2] If the endowment of young households is zero, then the carry traders introduced below cannot have an aggregate short position in domestic bonds. A strictly positive endowment plays no other role than allowing such short positions.

tradable good.[3] On the other hand, we need to impose limits on the size of their portfolios so that their positions remain finite in the presence of such arbitrage opportunities.[4] We assume that the position of each carry trader in domestic bonds must lie within $[P_t L^-, P_t L^+]$, where these limits are denominated in the domestic currency and

$$L^- > -W,$$

which ensures that households always consume positively.[5]

We now introduce in turn the two key ingredients of the model. First, the domestic central bank responds to carry trades inflows only to the extent that they affect the domestic CPI. Second, the prices of the non tradable goods are more rigid than that of the tradable good in the domestic currency.

Monetary policy

We suppose that the domestic monetary authority sets the nominal interest rate between t and $t+1$, I_{t+1}, following the interest-rate feedback rule:

$$I_{t+1} = R\left(\frac{P_t}{P_{t-1}}\right)^{1+\Phi} \tag{3}$$

where

$$\Phi > 0. \tag{4}$$

Rule (3) is a textbook interest-rate rule that follows the Taylor principle from (4). Note that we assume an inflation target equal to zero and a target real rate equal to the households' discount rate R only to save on notations, and without loss of generality. The important property of rule (3) is that these targets are constant: The central bank responds to carry-trades inflows only insofar as they affect domestic inflation. It does not respond directly to the bond price fluctuations induced by these inflows.

[3]Whether they also derive utility from consuming other goods, and the curvature of their utility function are immaterial.

[4]As is well-known, these trading limits could result, for example, from the carry traders' ability to divert cash flows at some cost.

[5]Setting lending limits in real terms simplifies the exposition but is not crucial. Nominal rigidities in trading limits would actually amplify our results.

Nominal rigidities

The other important ingredient of the model is that the prices of the nontradable goods are less flexible than that of the tradable good. We introduce rigidity in the pricing of nontradable goods in the straightforward following way. First, we suppose that the law of one price holds for the tradable good ("PPP at the docks"). Second, we suppose that the first nontradable good N_1 has a fully flexible price. A linear technology enables the transformation of each date-t unit of N_1 into F units of the tradable good, where $F > 0$.

Conversely, the second nontradable good N_2 has a fully rigid price that we normalize to 1 without loss of generality. Denoting $P_t^T, P_t^{N_1}$, and $P_t^{N_2}$ the respective prices of these three goods, these assumptions readily imply:

$$P_t^T S_t = 1, \tag{5}$$

$$P_t^{N_1} = F P_t^T, \tag{6}$$

$$P_t^{N_2} = 1. \tag{7}$$

Relation (5) states that agents must be indifferent between purchasing the tradable good using the world or the domestic currency. Relation (6) states that domestic agents must be indifferent between purchasing the tradable good or producing it out of N_1, and relation (7) accounts for the rigidity of N_2's price.

The introduction of the nontradable good with flexible price N_1 is only meant to decouple the "openness" of the economy as measured by $1 - \beta - \gamma$ and the flexibility of prices as measured by $1 - \gamma$, where β and γ are defined in (2). Only the parameter γ matters, however, for the remainder of the analysis. Note that the case $\gamma = 0$ corresponds to the fully flexible benchmark.

Perfect-foresight steady-states

We are now equipped to solve for the perfect-foresight steady-states of this economy. We denote L_t the real aggregate borrowing by young households from carry traders at date t, possibly negative. An equilibrium must be such that the domestic economy is in equilibrium and carry traders form optimal portfolios at each date.

Optimal carry trades

Let

$$\Theta_{t+1} = \frac{S_{t+1} I_{t+1}}{R^W S_t}. \tag{8}$$

That carry traders form optimal portfolios implies:

$$L_t \begin{cases} = L^+ & \text{if } \Theta_{t+1} > 1, \\ = L^- & \text{if } \Theta_{t+1} < 1, \\ \in (L^-, L^+) & \text{if } \Theta_{t+1} = 1. \end{cases} \tag{9}$$

Equilibrium in the domestic economy

The equilibrium in the domestic economy is characterized by (3), (5), (6), (7), the households' Euler equation,

$$I_{t+1} = \frac{RP_{t+1}}{(L_t + W)P_t}, \tag{10}$$

and their optimal spending across goods at each date, which implies:

$$P_t = (P_t^T)^\alpha (P_t^{N_1})^\beta (P_t^{N_2})^\gamma \tag{11}$$

$$= \left(P_t^T\right)^{1-\gamma} F^\beta, \tag{12}$$

where (12) stems from injecting (6) and (7) in (11).

These equilibrium conditions form a simple log-linear system. We introduce

$$\begin{aligned} r &= \ln R, \\ \delta &= \ln\left(\frac{R}{R^W}\right), \\ \theta_t &= \ln \Theta_t, \\ i_t &= \ln I_t, \\ s_t &= \ln S_t, \\ l_t &= \ln(L_t + W), \\ \pi_{t+1} &= \ln\left(\frac{P_{t+1}}{P_t}\right). \end{aligned}$$

As is standard, the Euler equation (10) and the interest-rate rule (3) define a linear-difference system for the path of inflation:

$$i_{t+1} = r - l_t + \pi_{t+1} \tag{13}$$

$$i_{t+1} = r + \Phi\pi_t \tag{14}$$

that has a unique non-exploding solution:

$$\pi_t = -\sum_{k\geq 0} \frac{l_{t+k}}{(1+\Phi)^{k+1}}. \tag{15}$$

Relation (15) captures the first important ingredient of the model. Current inflation reflects all the future expected "shocks" $(l_{t+k})_{k\geq 0}$ caused by carry trades on the real rate. A generic feature of New-Keynesian models is that inflation reflects anticipated future "policy shocks"(see e.g. Cochrane, 2011). The novelty here is that these shocks are not exogenous policy shocks. They are instead the equilibrium outcome of carry traders' optimal portfolio choice.

Using (12) and (5), one has

$$s_{t+1} - s_t = -\frac{1}{1-\gamma}\pi_{t+1}. \tag{16}$$

Relation (16) reflects the second important ingredient of the model—the rigidity of nontradables' prices implies that the nominal exchange rate is very sensitive to inflation expectations.

Plugging (16) and (10) in (8) yields

$$\begin{aligned} \theta_{t+1} &= s_{t+1} - s_t + i_{t+1} - \ln R^W, \\ &= -\frac{1}{1-\gamma}\pi_{t+1} + \pi_{t+1} - l_t + \delta, & (17) \\ &= \frac{\gamma}{1-\gamma}\sum_{k\geq 0}\frac{l_{t+k+1}}{(1+\Phi)^{k+1}} - l_t + \delta. & (18) \end{aligned}$$

We now determine the steady-states in which the debt level l is constant over time. We introduce

$$\underline{l} \equiv \ln(W + L^-), \tag{19}$$

$$\bar{l} \equiv \ln(W + L^+). \tag{20}$$

We have:

Proposition 1. *Suppose there exists $l^* \in (\underline{l}, \bar{l})$ such that*

$$\frac{\gamma - \Phi(1-\gamma)}{(1-\gamma)\Phi} l^* + \delta = 0. \tag{21}$$

Then $l = l^*$ is a steady-state in which uncovered interest parity (UIP) holds and the carry trade earns no excess return ($\theta = 0$).

If $\Phi(1-\gamma) > \gamma$, this is the only steady-state. If $R^W = R$, then $l^* = 0$, and the nominal exchange rate and the domestic price level are constant in this steady-state.

If $\Phi(1-\gamma) < \gamma$, there also exists a steady-state with maximum inflows ($l = \bar{l}$) in which the excess return on the carry trade is positive. There also exists a steady-state with maximum outflows ($l = \underline{l}$), and a negative excess return on the carry trade. If $R^W = R$, then the nominal exchange rate perpetually appreciates in the former steady-state and depreciates in the latter.

Proof. For l fixed, expression (18) becomes

$$\theta = \frac{\gamma - \Phi\left(1-\gamma\right)}{\left(1-\gamma\right)\Phi} l + \delta.$$

Condition (21) implies that there exists a unique steady-state such that $\theta = 0$. It is the only steady-state if $\Phi(1-\gamma) > \gamma$, whereas $l = \bar{l}$ and $l = \underline{l}$ can be sustained otherwise.■

The situation $\Phi(1-\gamma) < \gamma$ in which there are multiple steady-states may be interpreted as one that is prone to destabilizing speculation. In this situation, the stable steady-state in which UIP holds and there is no free lunch is unstable. Because current and future capital inflows reinforce each other, there is also the possibility that carry traders create and exploit a self-justified arbitrage opportunity. Note that the no-arbitrage/UIP steady-state is unique regardless of the monetary rule when prices are fully flexible ($\gamma = 0$).

The intuition behind Proposition 1 is best seen from the return on carry trade as given in equation (17). This expression decomposes the impact of current lending l_t and that of future lending $(l_{t+k})_{k\geq 1}$ on the current excess return on carry trade θ_{t+1}. Current lending has a negative impact on the current return on carry trade simply because it makes bonds expensive (term $-l_t$ in (17)). In contrast, the return on carry trade increases in future lending for the following reason. First, future anticipated carry trades are deflationary, and this reduces the nominal domestic interest rate and thus the profitability of the current carry trade (term π_{t+1} decreasing in future lending). This is more than offset, however, by the impact of future lending on nominal exchange rate appreciation (term $\frac{-\pi_{t+1}}{1-\gamma}$ increasing in future lending). Future lending leads to a current nominal exchange rate appreciation that is larger than the reduction in the CPI because of the assumption that nontradables' prices are less flexible than that of the tradable good.

In sum, there are multiple steady-states when lending by other carry traders makes lending more appealing to each carry trader. Market incompleteness—households have no access to foreign investments—combined with a monetary rule that is not too aggressive leads to self-justified arbitrage opportunities. This result is novel, to the best of our knowledge. The goal of this perfect-foresight model is to present it in the simplest and most transparent fashion. This setup has two important limitations, however:

1. If carry traders hold the same position forever, then the central bank should end up adjusting its real-rate target and the producers of non-tradable goods should end up adjusting their prices.

2. In this environment, the interest-rate differential plays no role in setting off steady-states with extreme capital inflows or outflows. Thus the model has no prediction regarding the correlation between the interest rate on a currency and its appreciation, nor on returns on the carry trade.

The next section develops a model that addresses these issues. We write down a version of the model in which carry traders switch from maximum to minimum positions at points that are uniquely determined by the paths of a stochastic interest-rate differential. This addresses the first limitation because the assumption of passive monetary policy and price setting is more natural when the carry trade size oscillates around a long-term average. This may stem for example from adjustment costs or noisy data. Regarding the second limitation, that the interest-rate differential is the coordination device among carry traders in a unique equilibrium will imply that this differential predicts the expected return on carry trades.

3 Destabilizing carry trades

We now assume that time is continuous. The fixed integer dates of the previous section are replaced by the arrival times of a Poisson process with intensity 1. Namely, at each arrival time T_n, a new cohort of households are born, and die at the next arrival time T_{n+1}. They value consumption and labor only at these two dates, with preferences that are the same as that in the previous section:

$$\ln C_{T_n} + \frac{1}{R} E_{T_n}\left[C_{T_{n+1}} - N_{T_{n+1}}^{1+\eta}\right].$$

At each arrival date T_n, the central bank sets a nominal rate $I_{T_{n+1}}$ between T_n and T_{n+1} according to the rule:

$$I_{T_{n+1}} = R\left(\frac{P_{T_n}}{P_{T_{n-1}}}\right)^{1+\Phi}. \tag{22}$$

Replacing integer dates with dates that arrive at a constant rate is not essential, and only for tractability. It will entail that the carry traders' problem studied below is time homogeneous.

In Section 2, carry trades were (possibly) textbook arbitrage opportunities. Thus we only needed to assume that carry traders had increasing utility without further detailing their preferences. In this section, carry trades will be profitable on average, but will have negative payoffs with a non-zero probability. They will be mere "good deals" rather than arbitrage opportunities. Thus we now need to be more explicit about carry traders' preferences in order to characterize their trading behavior. We follow Gabaix and Maggiori (2015) and model carry traders as financial institutions as follows.

Carry traders

Carry traders are a unit-mass continuum of financial institutions that can form zero-cost portfolios in bonds denominated in either currency at each date T_n with size within $[P_{T_n}L^-, P_{T_n}L^+]$ (in units of the domestic currency). The date-T_n trade is unwound at the next date T_{n+1} and the realized profit or loss is paid to the old households at this date. Each firm maximizes the expected value of future consumption paid to all future households discounted at the households' rate R.[6]

The two following modifications to the model in Section 2 are key to generate equilibrium uniqueness.

First, we assume that the interest rate at which carry traders borrow in the world currency between two arrival dates T_n and T_{n+1} is given by

$$R^W_{T_{n+1}} = R\left(1 - w_{T_n}\right), \tag{23}$$

where w_t is a Wiener process with no drift and volatility σ^2. In other words, we introduce an exogenous stochastic component in the interest rate differential.

Second, we assume that the capital supplied by carry traders is slow-moving in the following sense. Each carry trader can revise its trading strategy only at switching dates that are generated by a Poisson process with intensity λ. These switching dates are independent across carry traders. In between two switching dates, each carry trader commits to a trading strategy and thus to lending a fixed real amount $L_t \in [L^-, L^+]$ to each new cohort of households at each arrival date T_n (if any).

[6]In particular carry traders' objective increases in the consumption of the tradable good as was assumed in Section 2.

This model of slow-moving capital has a key property that will yield equilibrium uniqueness: Every carry trader knows that some other carry traders will revise their trading strategy almost surely between his current switching date and the next one.

Remark. It is important to stress that the exogenous trading limits $[P_{T_n}L^-, P_{T_n}L^+]$ fulfill a very different role from that played in Section 2. In the previous section, it was necessary to impose such limits regardless of carry traders' preferences because carry trades were (possibly) textbook arbitrage opportunities. In this section, carry-trade portfolios generate losses with a non-zero probability because of the stochastic world interest rate. Thus, any risk-averse agent would demand them in finite quantities. Trading limits here only play the role of a very tractable substitute for risk aversion that is commonplace in models in which agents' attitude towards risk is not the main focus.[7]

Local risk-neutrality implies that carry-traders choose corner portfolios. We deem "active" a carry trader who committed to maximum lending L^+ at his last switching date, and "inactive" one who committed to the minimum lending L^-. Suppose that a carry trader has a chance to revise his position at a date t such that

$$T_{n-1} < t < T_n. \tag{24}$$

Denoting T_λ his next switching date, its expected unit return from the carry trade—the expected value from committing to lend one additional real unit to each future cohort until T_λ—is

$$\Theta_t = E_t\left[\sum_{m\geq 0} \frac{1_{\{T_\lambda > T_{n+m}\}}}{R^m} \frac{P_{T_{n+m}}S_{T_{n+m}}}{P_{T_{n+m+1}}S_{T_{n+m+1}}}\left(\frac{S_{T_{n+m+1}}I_{T_{n+m+1}}}{S_{T_{n+m}}R} - 1 + w_{T_{n+m}}\right)\right]. \tag{25}$$

Expression (25) states that the carry trader earns the carry-trade return associated with each cohort that borrows until he gets a chance to revise his position.

We let x_t denote the fraction of active carry traders at date t. Note that the paths of the process $(x_t)_{t\in\mathbb{R}}$ must be Lipschitz continuous, with a Lipschitz constant smaller than λ. The aggregate real lending L_{T_n} taking place at an arrival date T_n is then equal to

$$L_{T_n} = x_{T_n}L^+ + (1 - x_{T_n})L^-. \tag{26}$$

[7]In recent work, Albagli, Hellwig, and Tsyvinski (2013) use a similar assumption to generate a high tractability and thus new insights in a standard noisy REE asset-pricing model.

The evolution of the economy is fully described by two state variables, the exogenous state variable w_t and the endogenous state variable x_t. The exogenous state variable affects only the expected return on carry trade Θ_t while the endogenous one affects both the carry trade return and the equilibrium variables $(L_{T_n}, I_{T_n}, P_{T_n}, S_{T_n})$ of the domestic economy. We are now equipped to define an equilibrium.

Definition. *An equilibrium is characterized by a process x_t that is adapted to the filtration of w_t and has Lipschitz-continuous paths such that:*

$$L_{T_n} = x_{T_n} L^+ + (1 - x_{T_n}) L^-, \tag{27}$$

$$I_{T_{n+1}} = R \left(\frac{P_{T_n}}{P_{T_{n-1}}} \right)^{1+\Phi}, \tag{28}$$

$$P_{T_n} = \left(P_{T_n}^T \right)^{1-\gamma} F^\beta, \tag{29}$$

$$P_{T_n}^T S_{T_n} = 1, \tag{30}$$

$$E_{T_n} \left[\frac{I_{T_{n+1}} P_{T_n}}{P_{T_{n+1}}} \right] = \frac{R}{L_{T_n} + W}, \tag{31}$$

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \textit{if } \Theta_t < 0, \\ \lambda(1 - x_t) & \textit{if } \Theta_t > 0. \end{cases} \tag{32}$$

Equations (28) to (31) state that the domestic economy is in equilibrium given the paths of x_t. Equation (32) states that carry traders make optimal individual decisions. They become active at switching dates at which the expected return on the carry trade is positive (or remain active if this was their previous positions), and inactive if this is negative.

Notice that relations (28) to (30) are identical to their counterparts in the perfect foresight case except for the re-labelling of dates. They are in particular log-linear. Conversely, the Euler equation (31) now features an expectation over the inverse of inflation given the stochastic environment. We will assume for the remainder of the paper that $W + L^-$ and $W + L^+$ are sufficiently close to 1 that $\underline{l}$ and $\bar{l}$ defined in (19) and (20) are sufficiently close to 0, so that we can approximate

$$\ln E_t \left[\frac{P_{T_n}}{P_{T_{n+1}}} \right] \simeq -E_t \left[\ln \frac{P_{T_{n+1}}}{P_{T_n}} \right]. \tag{33}$$

This implies of course that we restrict the analysis to the impact of relatively small capital inflows. Up to this log-linearization, we have

Proposition 2. *Suppose that*

$$\gamma > \Phi\left(1-\gamma\right). \tag{34}$$

For λ sufficiently small, there exists a unique equilibrium defined by a decreasing Lipschitz function f such that

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \text{if } w_t < f(x_t), \\ \lambda(1-x_t) & \text{if } w_t > f(x_t). \end{cases} \tag{35}$$

Figure 1 illustrates the equilibrium dynamics described in Proposition 2.

[Figure 1 here]

The frontier f divides the (w, x)-space into two regions. Proposition 2 states that in the unique equilibrium, any trader decides to be active when the system is to the right of the frontier f at his switching date, and inactive when it is on the left of the frontier. Thus, lending positions (and therefore the exchange rate) will tend to rise in the right-hand region, and tend to fall in the left-hand region, as indicated by the arrows in Figure 1.

The expected return on the carry trade at date t is zero if and only if $w_t = f(x_t)$. It is positive if (w_t, x_t) is on the right of the frontier f in the (w, x)-space and negative if it is on the left of f.

The dynamics of x_t implied by the unique equilibrium are given by:

$$dx_t = \lambda\left(1_{\{w_t > f(x_t)\}} - x_t\right) dt, \tag{36}$$

where $1_{\{.\}}$ denotes the indicator function that takes the value 1 when the condition inside the curly brackets is satisfied. These processes are known as *stochastic bifurcations,* and are studied in Bass and Burdzy (1999) and Burdzy et al. (1998). These mathematics papers establish in particular that for almost every sample path of w_t, there exists a unique Lipschitz solution x_t to the differential equation (36) defining the price dynamics for f Lipschitz decreasing.

The main features of these dynamics can be seen from Figure 1. Starting on the frontier, a positive shock on w will pull the system on the right of it. Unless the path of w_t is such that a larger negative shock brings it back on the frontier immediately, a more likely scenario is that lending grows for a while so that x_t becomes close to 1, in which case $\frac{dx_t}{dt}$ becomes close to 0. If cumulative negative shocks on w eventually lead the system back to the left of the frontier, then there are large outflows

$$\frac{dx_t}{dt} \simeq -\lambda.$$

Condition (34) is the same as the one that generates multiple steady-states in the perfect-foresight case. It is worthwhile commenting on the additional condition that capital move sufficiently slowly (λ sufficiently small). This condition guarantees that the frontier f is decreasing, and thus that carry trades are destabilizing. To better grasp its role, notice that if a carry trader expects other carry traders to become active in the future, then he expects the exchange rate to appreciate. This implies that on one hand, the currency will be expensive when he will purchase it to lend. On the other hand, it will keep appreciating over the duration of the loan, thereby generating a positive return. The former effect is akin to a congestion effect. Other traders make the trade more expensive and thus less desirable. Conversely, the latter effect is destabilizing as future carry trades make becoming active more appealing. That λ be sufficiently small ensures that this latter effect offsets the former congestion effect because aggregate lending does not converge too quickly to its maximum value. Thus a carry trader with a current switching date is more likely to have a chance to lend before the currency becomes too expensive, and its upside potential too small. This congestion effect is the salient difference between our setup and that studied by Burdzy, Frankel, and Pauzner.

Note that even if the frontier is increasing, it is still the case that small shocks w_t can have a large impact on carry-trade activity provided it is sufficiently steep in the (w, x) plane. We focus on the case of a decreasing frontier

Proof of Proposition 2

The proof of Proposition 2 essentially extends to this stochastic environment the logic leading to the perfect-foresight results in Proposition 1. In a first step, we solve for the nominal exchange rate and domestic interest rate as a function of future capital inflows. This will yield an expression of the expected return on carry trades (25) as a function of these inflows and of the interest-rate differential w_t that is the stochastic counterpart of equation (18). Second, we use this expression to solve for a Lipschitz process that satisfies (32). This latter step is the equivalent of the one that consisted in solving for feasible steady-states given the expected return for carry traders (18) under perfect foresight.

More precisely, the first step consists in using relations (28) to (31) to express the nominal exchange rate and interest rate as functions of the expected future paths of capital inflows L_t. This yields in turn a relatively simple expression for the expected return on the carry trade Θ_t as a function of these expected capital inflows:

Lemma 3. *At first-order, the expected return on the carry trade is*

$$\Theta\left(w_t, x_t\right) = \int_0^{+\infty} \left(\left(\frac{\chi\omega}{\omega-\rho-\lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi\omega}{\omega-\rho-\lambda} e^{-\omega v} \right) E_t\left[l_{t+v}\right] dv + \frac{w_t}{\lambda+\rho}, \tag{37}$$

where

$$l_t = \ln(L_t + W) \simeq x_t \bar{l} + (1-x_t)\,\underline{l}, \tag{38}$$

$$\rho = 1 - \frac{1}{R}, \tag{39}$$

$$\omega = \frac{\Phi}{1+\Phi}, \tag{40}$$

$$\chi = \frac{\gamma}{(1-\gamma)\,\Phi}. \tag{41}$$

Proof. See the Appendix. ■

The factor that discounts future capital inflows in (37):

$$\left(\frac{\chi\omega}{\omega-\rho-\lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi\omega}{\omega-\rho-\lambda} e^{-\omega v}, \tag{42}$$

is first negative, then positive as v spans $[0, +\infty)$.[8] This formalizes the above comment that future active traders create congestion effect for the current trader. The earliest inflows have a negative impact on Θ because they make the domestic currency expensive. The more remote inflows are desirable as the current trader is more likely to have lent before they push up the exchange rate. The following lemma establishes conditions under which the congestion effect is not too important.

Lemma 4. *Suppose that $\chi > 1$. There exists $\overline{\lambda}$ such that for all $\lambda \leq \overline{\lambda}$, the following is true. Suppose that two processes x_t^1 and x_t^2 satisfy*

$$0 < x_0^1 \leq x_0^2 < 1,$$
$$\textit{For } i = 1,2,\ dx_t^i = \lambda \left(1_{\left\{w_t > f^i\left(x_t^i\right)\right\}} - x_t^i \right) dt,$$

[8]Notice that this is so regardless of the sign of $\omega - \lambda - \rho$.

where f^i is decreasing Lipschitz and $f^2 \leq f^1$. Then

$$\Theta(w_t, x_0^2) \geq \Theta(w_t, x_0^1). \tag{43}$$

The inequality is strict if $f^1 \neq f^2$ and/or $x_0^1 \neq x_0^2$.

Proof. See the Appendix. ■

Lemma 4 states that if (34) holds and λ is sufficiently small, then future carry trades make current carry trades more attractive because the reinforcing effect overcomes the congestion effect. In the balance of the paper, we suppose that the conditions in Lemma 4 are satisfied. We now show that there is in this case a unique Lipschitz process x_t that satisfies the equilibrium conditions.

First, the proof of Lemma 4 also shows that the case in which x_t obeys $\frac{dx_t}{dt} = -\lambda x_t$ for all $u \geq 0$ corresponds to a lower bound on the expected carry-trade return. When x_t obeys such dynamics, there exists a frontier f_0 such that

$$w_t = f_0(x_t) \implies \Theta(x_t, w_t) = 0 \tag{44}$$

The frontier f_0 is decreasing from Lemma 4 (with $f^1 = f^2 = +\infty$) and is clearly affine and thus Lipschitz.[9] Thus an admissible equilibrium process must be such that traders who have a chance to switch when the system is on the right of f_0 become active.

Define now f_1 such that

$$w_t = f_1(x_t) \implies \Theta(x_t, w_t) = 0 \tag{45}$$

if for all $u \geq 0$,

$$\frac{dx_{t+u}}{du} = \begin{cases} -\lambda x_{t+u} & \text{if } w_{t+u} < f_0(x_{t+u}), \\ \lambda(1 - x_{t+u}) & \text{if } w_{t+u} > f_0(x_{t+u}). \end{cases} \tag{46}$$

That is, f_1 is such that a carry trader is indifferent between being active or inactive when the system is on f_1 at his switching date if he believes that other traders become active if and only if they are on the right of f_0. This function f_1 must be decreasing. Suppose otherwise that two points (w, x) and (w', x') on f_1 satisfy

$$x' > x,$$
$$w' \geq w.$$

[9] The frontier simply obtains from writing $E_t [l_{t+v}] = \underline{l} + \left(\bar{l} - \underline{l}\right) x_t e^{-\lambda v}$ in (37).

Then applying Lemma 4 with $f^2 = f_0, f^1 = f_0 + w' - w$ contradicts that both points generate the same expected carry trade return. We also show in the appendix that f_1 is Lipschitz, with a Lipschitz constant smaller than that of f_0.

By iterating this process, we obtain a limit f_∞ of the sequence of frontiers $(f_n)_{n\geq 0}$ that is decreasing Lipschitz as a limit of decreasing Lipschitz functions with decreasing Lipschitz constants. The process

$$\frac{dx_t}{dt} = \begin{cases} -\lambda x_t & \text{if } w_t < f_\infty(x_t), \\ \lambda(1 - x_t) & \text{if } w_t > f_\infty(x_t). \end{cases} \tag{47}$$

is an admissible equilibrium since by construction, if all traders switch to inactivity to the left of f_∞ and to activity to the right, the indifference point for a trader also lies on f_∞. We now show that this is the only equilibrium process.

Consider a translation to the left of the graph of f_∞ in (w, x) so that the whole of the curve lies in a region where w_t is sufficiently small that inactivity is dominant regardless of the dynamics of x_t. Call this translation f_0'. To the left of f_0', inactivity is dominant. Then construct f_1' as the rightmost translation of f_0' such that a trader must choose inactivity to the left of f_1' if he believes that other traders will play according to f_0'. By iterating this process, we obtain a sequence of translations to the right of f_0'. Denote by f_∞' the limit of the sequence. Refer to Figure 2.

[Figure 2 here]

The boundary f_∞' does not necessarily define an equilibrium strategy, since it was merely constructed as a translation of f_0'. However, we know that if all others were to play according to the boundary f_∞', then there is at least one point A on f_∞' where the trader is indifferent. If there were no such point as A, this would imply that f_∞' is not the rightmost translation, as required in the definition.

We claim that f_∞' and f_∞ coincide exactly. The argument is by contradiction. Suppose that we have a gap between f_∞' and f_∞. Then, choose point B on f_∞ such that A and B have the same height - i.e. correspond to the same x. But then, since the shape of the boundaries of f_∞' and f_∞ and the values of x are identical, the paths starting from A must have the same distribution as the paths starting from B up to the constant difference in the initial values of w. This contradicts the hypothesis that a trader is indifferent between the two actions both at A and at B. If he were indifferent at A, he would strictly prefer maximum lending at B, and if he is indifferent at B, he would strictly prefer minimum lending when in A. But we constructed A and B so that traders are indifferent in both A and B. Thus, there is only one way to make everything consistent, namely to conclude that $A = B$. Thus, there is no "gap", and we must have $f'_\infty = f_\infty$. ■

Proposition 2 shows that adding exogenous shocks w_t to the carry return eliminates the indeterminacy of the perfect-foresight case. More precisely, equilibrium uniqueness stems from the interplay of these shocks with the fact that each carry trader, when he receives a switching opportunity, needs to form beliefs about the decisions of the carry traders that will have an opportunity to switch between now and his next switching date. Suppose that (w_t, x_t) is close to a dominance region in which carry traders would prefer a course of action for sure, but just outside it. If w_t was fixed, it may be possible to construct an equilibrium for both actions, but when w_t moves around stochastically, it will wander into the dominance region between now and the next opportunity that the trader gets to switch with some probability. This gives the trader some reason to hedge his bets and take one course of action for sure. But then, this shifts out the dominance region, and a new round of reasoning takes place given the new boundary, and so on.

Remark 1. We model the interest-rate differential as a Brownian motion for expositional simplicity. It is easy to see that we could write it as $d(w_t)$, where w_t is a standard Brownian motion, and d a Lipschitz increasing function, possibly bounded as long as there are still dominant actions for w_t sufficiently large or small.

Remark 2. While a strong persistence in target rates is undoubtedly realistic, extensions of this framework can also accommodate for various forms of mean-reversion (Burdzy, Frankel, and Pauzner, 2001, or Frankel and Burdzy, 2005).

Remark 3. The condition that λ be sufficiently small seems particularly relevant for the carry trades that involved many retail investors, such as those targeting New Zealand dollar or Icelandic krona. The glacier bonds denominated in Icelandic krona or the uridashi bonds used by Japanese investors to invest in New Zealand had a typical maturity of 1 to 5 years, and were principally purchased by retail investors. More generally, Bacchetta and van Wincoop (2009) claim an average two-year rebalancing frequency to be plausible in FX markets in general, and assume it in order to quantitatively explain the forward discount bias. Also, well-documented price pressure and illiquidity in currency markets, especially for small currencies, may force professional FX speculators to build-up or unwind large positions more gradually than they would like to.[10]

The case of small shocks

The limiting case in which the volatility σ of the interest-rate differential tends to zero yields useful insights. It is possible to characterize the shape of the frontier f in this case.

In this section we denote the frontier f_σ to emphasize its dependence on σ. Suppose the economy is in the state $(f_\sigma(x_t), x_t)$ at date t. That is, it is on the equilibrium frontier. For some arbitrarily small $\varepsilon > 0$, introduce the stopping times

$$\begin{aligned} T_1 &= \inf_{u \geq 0} \{x_{t+u} \notin (\varepsilon, 1-\varepsilon)\}, \\ T_0 &= \sup_{0 \leq u < T_1} \{w_{t+u} \neq f_\sigma(x_{t+u})\}. \end{aligned}$$

In words, T_1 is the first date at which x_t gets close to 0 or 1, and T_0 is the last date at which x_t crosses the frontier before T_1. If T_0 is small in distribution, it means that the economy is prone to bifurcations. That is, it never stays around the frontier for long. Upon hitting it, it quickly heads towards extreme values of x. The next proposition shows that this is actually the most likely scenario when σ is small. This, in turn, yields a simple explicit determination of the frontier.

Proposition 5.

1. *As $\sigma \to 0$, T_0 converges to 0 in distribution, and the probability that $\frac{dx_t}{dt} > 0$ (respectively $\frac{dx_t}{dt} < 0$) over $[T_0, T_1]$ converges to $1 - x_t$ (x_t respectively).*

2. *As $\sigma \to 0$, the frontier f_σ tends to an affine function. For λ sufficiently small, the slope of this function is increasing in Φ and decreasing in γ.*

Proof. See the Appendix.■

First, Proposition 5 clears the concern that in equilibrium, x would only exhibit small fluctuations around a fixed value because Brownian paths cross the frontier too often. As σ becomes smaller, the system exhibits more frequent bifurcations towards extremal values of x. When the system reaches the frontier, it is all the more likely to bifurcate towards capital outflows

[10] In fact, our model is identical to one in which a single large carry trader can move his capital only at the rate λ.

when cumulative inflows have been large (x large). Thus the model does generate "destabilizing carry trades," whereby carry traders generate durable self-justified excess returns on the carry trade followed by large reversals.

The second point in Proposition 5 relates the slope of the frontier f_σ to the monetary parameters of the model Φ and γ in this case of small shocks. The slope of the frontier affects the dynamics of capital inflows and in turn the exchange-rate dynamics. If the graph of the frontier is closer to being horizontal in the (w, x) plane, then the system should cross the frontier less often, and thus do so only for more extreme values of x. Carry-trade returns should in this case exhibit more serial correlation and fatter tails. Point 2 states that, at least for λ sufficiently small, the frontier is flatter when Φ is smaller, and γ larger. In other words, if the central bank fails to respond to inflows by sufficiently reducing its official rate, then carry trade returns should exhibit more skewness.

4 Empirical content

The model generates a rich set of qualitative empirical predictions. This suggests that a coordination motive among carry traders may be a common force behind several well-documented empirical findings on carry-trade returns.

Profitability of FX momentum strategies

Proposition 5 shows that as $\sigma \to 0$, the system often bifurcates in one direction. This implies that, at least at a sufficiently short horizon, returns are positively autocorrelated, so that momentum strategies in FX markets should generate a positive excess return.

It is important to stress that the profitability of momentum strategies is not a mechanical consequence of the assumption of slow-moving capital (λ sufficiently small). Returns on the carry trade are still positively autocorrelated if the system bifurcates quickly towards extreme values of activity x. The key economic force behind this profitability of momentum strategies is that once carry traders coordinate on a course of action, they stick to it until a sufficiently large reversal of the interest-rate differential leads them to switch to a different strategy. Such a rationalization of momentum returns with coordination motives is novel to our knowledge.

Profitability of FX carry trades

Lemma 4 implies that the equilibrium expected return on the carry trade $\Theta(w,x)$ increases with respect to x. It also implies that $\Theta(w,x)$ increases with respect to w, because an increase in w is equivalent to a leftward translation of the frontier f in the graph (w,x). On the other hand, the interest-rate differential increases in w and decreases in x. We have indeed:

Lemma 6. *At first-order, the interest-rate differential at a given arrival date T_n is given by*

$$R\left(w_{T_n} - l_{T_n} - \frac{1}{1+\Phi}\int_0^{+\infty} e^{-\omega s} E_{T_n}\left[l_{T_n+s}\right] ds\right). \tag{48}$$

Proof. See the Appendix.■

The interest-rate differential increases w.r.t. w but decreases w.r.t. l (and thus x) because the current domestic real rate is lower and future deflation more likely when l is large. Thus the expected return on the carry trade is not unambiguously increasing in the interest-rate differential. For $\underline{l}$, $\bar{l}$ sufficiently small, however, most of the interest-rate differential is due to the exogenous component w rather than to the endogenous actions of the carry traders l. In this case, when the interest-rate differential is sufficiently large in absolute terms, it must be that the system is on the right (left) of the frontier when the differential is positive (negative). In other words, we have the following interesting prediction:

A positive (negative) interest-rate differential predicts a positive (negative) return on the carry-trade only for sufficiently large absolute differentials. The exchange rate must be more volatile when the interest-rate differential is small.

When w is small, so is the interest-rate differential, and the differential may correspond to values of (w,x) that are either on the left or on the right of the frontier. The expected return on the carry trade is thus unclear. Since the system is closer to the frontier in this case, future crossings of the frontier are more likely and thus the exchange rate should be more volatile. This is because close to the frontier, for a finite σ, it takes more time to carry traders to coordinate on a given course of action and bifurcate in one direction. This nonlinear impact of the interest-rate differential on the carry-trade return has not been tested to our knowledge.

Peso problem

A large literature argues that the return on the carry trade partly reflects a risk premium for rare and extreme events that may not show in finite samples (see, e.g., Burnside, Eichenbaum, Kleshchelski, and Rebelo, 2011, Farhi and Gabaix, 2014, Jurek, 2014, or Lewis, 2007, and the references herein.) We closely connect to this literature as follows. Fix $\epsilon > 0$ small. The expected return on the carry trade is 0 starting both from $(f(\epsilon), \epsilon)$ and $(f(1-\epsilon), 1-\epsilon)$ in the (w, x) plane. Yet from Proposition 5, as σ becomes small, most paths starting from $(f(\epsilon), \epsilon)$ will exhibit long periods of appreciation of the domestic currency ended with rare (and large) depreciations, while paths starting from $(f(1-\epsilon), 1-\epsilon)$ will feature a symmetric prolonged depreciation. The interest-rate differential is positive in the former case and negative in the latter. Thus, finite samples should yield that a positive interest-rate differential predicts a positive excess return on the carry trade even when the unconditional return is zero.

Leverage and currency appreciation predict financial crises

Gourinchas and Obstfeld (2011) find that credit expansion and appreciation of the domestic currency predict financial crises. The build up of leverage and currency appreciation correspond to paths in which x increases for a long time in our model. Such paths are the ones in which sharp deleveraging and important capital outflows are most likely to occur soon other things being equal.

Monetary policy and carry-trade returns

In addition to relating to the above existing empirical findings, the model also generates a new range of predictions on the relationship between the stance of monetary policy and the distribution of the returns on momentum and carry trade strategies. Proposition 5 suggests that the frontier is flatter when Φ is smaller and γ larger. In words, the frontier is flatter when the central bank is more reluctant to respond to a surge in carry-trade activity with a large reduction in the official rate. This is in turn more likely to be the case when the prices of nontradables are very sticky. Otherwise stated, if an economy is such that the CPI is not too sensitive to the exchange rate, and/or the central bank not too aggressive, then this economy should be more prone to large fluctuations in carry-trade activity because it will experience more prolonged bifurcations. Thus the returns on carry-trade and momentum strategies should have fatter tails. These predictions are novel, to our knowledge.

Concluding remarks

As a conclusion, we briefly discuss two interesting avenues for future research.

- *More general preferences.* Assuming that households are risk neutral over late consumption dramatically simplifies the analysis, because it implies that the impact of capital inflows on the real rate is straightforward. With strictly concave preferences, the current real rate would depend on consumption growth, so that we could no longer abstract from the impact of foreign lending on quantities and thus production in the domestic economy as we are able to do here. We find it useful to derive our novel mechanism for self-fulfilling profitable carry trades in a highly tractable framework that delivers clear intuitions. An interesting avenue for future research is the study of the impact of such carry trades on quantities under more standard preferences. For such a study, one should also introduce a more standard modelling of price adjustment.

- *Repelling carry traders.* We assume here that the domestic central bank does not use an appropriate rule. An interesting avenue for future research consists in explicitly modelling the commitment issues or welfare costs that prevent the monetary authority from using a larger Φ. This would pave the way to a normative analysis. Notice that the central bank can repell carry traders in this framework in three other ways: using a measure of inflation that is tilted towards tradables, adding a term that is sufficiently decreasing in the exchange rate appreciation to the interest-rate rule, or simply targeting the realized real rate $r - l_t$. It is easy to see from the perfect-foresight model that these three measures are strictly equivalent in this simple environment, because they all amount to sufficiently reducing the official rate in response to carry-trade activity, therereby discouraging it. These different policies would probably each come with distinctive costs in a more general environment. In any case, a clear implication from this framework is that a decrease in the official rate is the appropriate response when foreign speculative inflows bid up domestic asset prices.

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Appendix

Proof of Lemma 3

Using the first-order approximation (33) in (31), relations (28) and (31) yield domestic inflation as a function of future expected inflows as in the perfect-foresight case:

$$\ln \frac{P_{T_n}}{P_{T_{n-1}}} = -\sum_{k\geq 0} \frac{E_{T_n}\left[l_{T_{n+k}}\right]}{(1+\Phi)^{k+1}}, \tag{49}$$

where $l_t = \ln(L_t + W)$. As in the perfect-foresight case, (29) and (30) yield in turn:

$$E_{T_n}\left[\ln \frac{S_{T_{n+1}} I_{T_{n+1}}}{RS_{T_n}}\right] = \frac{\gamma}{1-\gamma} \sum_{k\geq 0} \frac{E_{T_n}\left[l_{T_{n+k+1}}\right]}{(1+\Phi)^{k+1}} - l_{T_n} \tag{50}$$

One can write (25) as

$$\Theta_t = E_t\left[\sum_{m\geq 0} \frac{1_{\{T_\lambda > T_{n+m}\}}}{R^m} E_{T_{n+m}}\left[\frac{P_{T_{n+m}} S_{T_{n+m}}}{P_{T_{n+m+1}} S_{T_{n+m+1}}} \left(\frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{RS_{T_{n+m}}} - 1 + w_{T_{n+m}}\right)\right]\right]. \tag{51}$$

At first-order w.r.t. l_t,

$$E_{T_{n+m}}\left[\frac{P_{T_{n+m}} S_{T_{n+m}}}{P_{T_{n+m+1}} S_{T_{n+m+1}}} \left(\frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{RS_{T_{n+m}}} - 1\right)\right] = E_{T_{n+m}}\left[\ln \frac{S_{T_{n+m+1}} I_{T_{n+m+1}}}{RS_{T_{n+m}}}\right] \tag{52}$$

$$= \frac{\gamma}{1-\gamma} \sum_{k\geq 0} \frac{E_{T_{n+m}}\left[l_{T_{n+m+k+1}}\right]}{(1+\Phi)^{k+1}} - l_{T_{n+m}}. \tag{53}$$

Thus,

$$\Theta_t = E_t\left[\int_0^{+\infty} \sum_{m\geq 1} \left(\frac{s}{R}\right)^{m-1} \frac{e^{-(\lambda+1)s}}{(m-1)!} \left[\begin{array}{l} \int_0^{+\infty} \frac{\gamma}{1-\gamma} \sum_{k\geq 1} \frac{u^{k-1}}{(k-1)!} \frac{e^{-u}}{(1+\Phi)^k} l_{t+s+u} du \\ -l_{t+s} + w_t \end{array}\right] ds\right], \tag{54}$$

$$= E_t\left[\int_0^{+\infty} e^{-(\lambda+\rho)s} \left(\int_0^{+\infty} \chi\omega e^{-\omega u} l_{t+s+u} du - l_{t+s} + w_t\right) ds\right], \tag{55}$$

$$= \int_0^{+\infty} e^{-(\lambda+\rho)v} \left(\chi\omega \int_0^{v} e^{-(\omega-\lambda-\rho)u} du - 1\right) E_t\left[l_{t+v}\right] dv + \frac{w_t}{\lambda+\rho}, \tag{56}$$

and integrating yields the result.

Proof of Lemma 4

Suppose $\chi > 1$. Consider two processes x_t^1 and x_t^2 that satisfy the conditions stated in Lemma 4 with $x_0^1 < x_0^2$. Lemma 2 in Burdzy, Frankel and Pauzner (1998) states that almost surely,

$$x_t^2 \geq x_t^1 \text{ for all } t \geq 0. \tag{57}$$

This implies in particular that whenever traders switch to being active along a sample path of (w_t, x_t^1), so do they along the sample path of (w_t, x_t^2) that corresponds to the same sample path of w_t. This is because it must be that (w_t, x_t^2) is on the right of the frontier f^2 whenever (w_t, x_t^1) is on the right of the frontier f^1. Thus, the process

$$y_t = x_t^2 - x_t^1 \tag{58}$$

satisfies

$$0 < y_0 < 1, \tag{59}$$

$$\frac{dy_t}{dt} = \lambda(\epsilon_t - y_t), \tag{60}$$

where $\epsilon_t \in \{0; 1\}$.

In order to prove the Lemma, we only need to find $\overline{\lambda}$ such that for all $\lambda \leq \overline{\lambda}$,

$$\Delta = \int_0^{+\infty} \left(\left(\frac{\chi\omega}{\omega - \rho - \lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi\omega}{\omega - \rho - \lambda} e^{-\omega v} \right) y_v dv \geq 0. \tag{61}$$

for all deterministic process y_t that obeys (59) and (60). The result then obtains from taking expectations over all paths of w_t.

To prove (61), we introduce the function ζ that satisfies

$$\begin{cases} \frac{d\zeta(v)}{dv} = -\left(\left(\frac{\chi\omega}{\omega-\rho-\lambda} - 1 \right) e^{-(\lambda+\rho)v} - \frac{\chi\omega}{\omega-\rho-\lambda} e^{-\omega v} \right), \\ \lim_{+\infty} \zeta = 0. \end{cases}$$

Integrating by parts, we have

$$\Delta = \zeta(0) y_0 + \int_0^{+\infty} \zeta(v) \frac{dy_v}{dv} dv, \tag{62}$$

$$= \zeta(0) y_0 + \lambda \int_0^{+\infty} \zeta(v)(\epsilon_v - y_v) dv. \tag{63}$$

Further,

$$y_v = y_0 e^{-\lambda v} + \lambda \int_0^v e^{-\lambda(v-u)} \epsilon_u du, \tag{64}$$

and thus,

$$\Delta = y_0 \left(\zeta(0) - \lambda \int_0^{+\infty} \zeta(v) e^{-\lambda v} dv \right) \tag{65}$$

$$+ \lambda \left[\int_0^{+\infty} \epsilon_v \left(\zeta(v) - \lambda \int_v^{+\infty} \zeta(u) e^{-\lambda(u-v)} du \right) \right]. \tag{66}$$

We have

$$\lim_{\lambda \to 0} \zeta(0) = \frac{\chi - 1}{\rho} > 0, \tag{67}$$

ζ is increasing then decreasing beyond a value that stays bounded as λ tends to zero, and $\int_0^{+\infty} \zeta$ converges. Thus for λ sufficiently small,

$$\zeta(v) - \lambda \int_v^{+\infty} \zeta(u) e^{-\lambda(u-v)} du \tag{68}$$

is positive for all $v \geq 0$, which yields that Δ is positive, and concludes the proof.

Complement to the proof of Proposition 2

We prove here that f_1 is Lipschitz with a constant that is smaller than that of f_0, that we denote K_0. Suppose by contradiction that two points (w_t, x_t) and (w_t', x_t') on f_1 satisfy

$$x' > x, \tag{69}$$

$$\frac{x_t' - x_t}{w_t - w_t'} < \frac{1}{K_0}. \tag{70}$$

We compare the paths x_{t+u}' and x_{t+u} corresponding to pairs of paths of w_{t+u}' and w_{t+u} that satisfy for all $u \geq 0$

$$w_{t+u} - w_{t+u}' = w_t - w_t'. \tag{71}$$

It must be that for such pairs of paths:

$$x_{t+u}' - x_{t+u} \leq (x_t' - x_t) e^{-\lambda u}. \tag{72}$$

Otherwise it would have to be the case that (w', x') can be on the right of f_0 when (w, x) is not. Let T denote the first time at which this occurs. It must be that

$$K_0 e^{-\lambda T}(x'_t - x_t) \geq w_{t+T} - w'_{t+T} = w_t - w'_t, \tag{73}$$

a contradiction with (70).

Thus along such paths of $w'_{t+u} - w_{t+u}$, $x'_{t+u} - x_{t+u}$ shrinks at least as fast as when traders switch to inactivity all the time. Together with (70), this implies that the expected return on the carry trade cannot be the same in (w_t, x_t) and (w'_t, x'_t), a contradiction.

Proof of Proposition 5

The first point is a particular case of Theorem 2 in Burdzy, Frankel, and Pauzner (1998). To prove the second point, notice that as $\sigma \to 0$, starting from a point on the frontier,

$$E_t\left[x_{t+v}\right] \simeq (1 - x_t)\left(1 - (1 - x_t)e^{-\lambda v}\right) + x_t^2 e^{-\lambda v} \tag{74}$$

because the system bifurcates upwards with probability $1 - x_t$ and downwards with probability x_t in the limit. Plugging this in (37) and writing that the expected return is zero yields a slope of the frontier equal to

$$-(\bar{l} - \underline{l})(\lambda + \rho)\begin{bmatrix}\left(\frac{\chi\omega}{\omega-\lambda-\rho} - 1\right)\left(\frac{2}{2\lambda+\rho} - \frac{1}{\lambda+\rho}\right) \\ -\frac{\chi\omega}{\omega-\lambda-\rho}\left(\frac{2}{\omega+\lambda} - \frac{1}{\omega}\right)\end{bmatrix}, \tag{75}$$

which tends to

$$-(\bar{l} - \underline{l})(\chi - 1) \tag{76}$$

as $\lambda \to 0$. This means that the absolute value of the slope of the frontier varies as χ w.r.t. γ, Φ for σ, λ sufficiently small.

Proof of Lemma 6

We have

$$I_{T_n+1} - R(1-w_{T_n}) = R\left(\left(\frac{P_{T_n}}{P_{T_{n-1}}}\right)^{1+\Phi} - 1 + w_{T_n}\right), \tag{77}$$

$$\simeq R\left(w_{T_n} - E_{T_n}\left[\sum_{k\geq 0}\frac{l_{T_n+k}}{(1+\Phi)^k}\right]\right), \tag{78}$$

$$= R\left(w_{T_n} - l_{T_n} - \int_0^{+\infty}\sum_{k\geq 1}\frac{s^{k-1}e^{-s}}{(k-1)!(1+\Phi)^k}E_{T_n}\left[l_{T_n+s}\right]ds\right), \tag{79}$$

$$= R\left(w_{T_n} - l_{T_n} - \frac{1}{1+\Phi}\int_0^{+\infty} e^{-\omega s}E_{T_n}\left[l_{T_n+s}\right]ds\right). \tag{80}$$

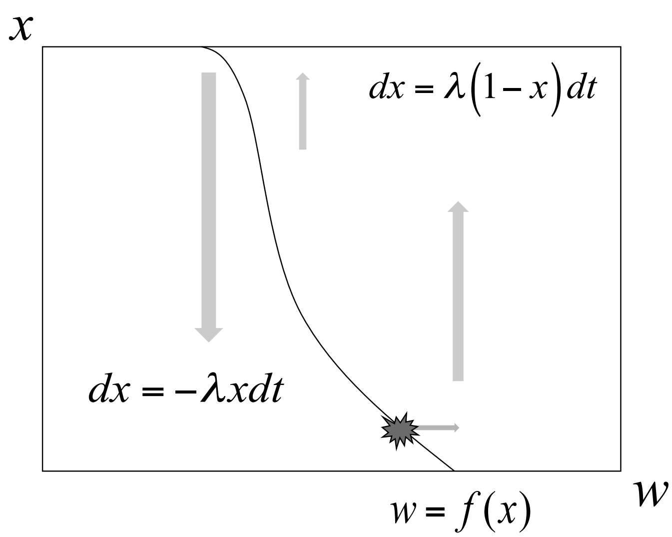
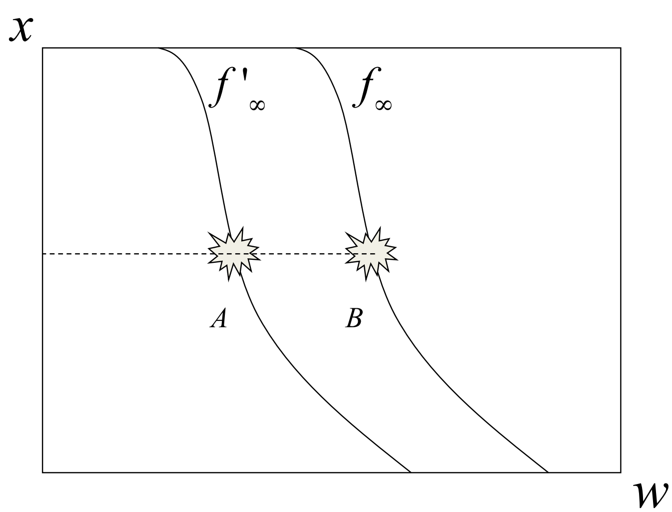


Figure 1



$$f'_\infty = f_\infty$$

Figure 2