"Stock Market Volatility and Learning"
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• *A parti pris* of simplicity.
• Nothing exotic or non-standard in the model.
• Avoid freeing up too many parameters.
• An determined attempt to take learning models into the mainstream of macro.
General comments
• Show that during the (slow) process of convergence to rational expectations, an OLS learning model generates data that closely match the behavior of actual stock markets.
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• Mechanism: beliefs affect prices and prices affect beliefs $\Rightarrow$ possibility of \textit{momentum} effect
Objectives of the paper

- Show that during the (slow) process of convergence to rational expectations, an OLS learning model generates data that closely match the behavior of actual stock markets.
- Mechanism: beliefs affect prices and prices affect beliefs $\Rightarrow$ possibility of momentum effect
- Key result: this effect is strong during the transition to R.E., yet weak close to it.
The name of the game remains the somewhat mechanical characterization of the behavior of a stochastic non-linear difference equation.
Caveats

• The name of the game remains the somewhat mechanical characterization of the behavior of a stochastic non-linear difference equation.
• OLS learning rule is intuitive, has desirable properties, yet remains arbitrary. At the very least, the exogeneity of the gain sequence is problematic.
• Hard to disentangle which results are general, and which are specific to the OLS learning rule
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➢ What is the implication of the auxiliary assumption that agents have RE about dividends, and learning is only about prices?

• No attention paid to the fact that, in this model, the fundamental value of the asset can be perfectly computed by all investors
Under the hood

Simplifications
Pricing equation
Rational expectations
Irrational expectations
Two important results
Learning rule
More results
Moral
Fundamentals
Conclusion
• No dividend uncertainty: everyone knows $D_t = 1$ for all $t$ (→ focus on learning about *prices*).
Simplifications

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- Timing: observe $p(t)$, compute $p^E(t + dt)$, observe $p(t + dt)$ etc.
- Interest rate: $r = \delta$
- No rational bubbles (e.g., Ramsey model)
Under risk neutrality

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\frac{1 + p^E}{p} = r
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Rational expectations

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- Perfect foresight on prices: \( \dot{p}^F = \dot{p} \).
- If there are no bubbles, the RE price is \( p = 1/r \).
- Investors rationally expect \( \dot{p}^E / p = 0 \).
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- Hence both the *level* and *rate of change* of price-growth expectations determine actual growth. **General result**
Two important results

\[ \frac{\dot{p}}{p} = p \dot{\beta} = \frac{1}{r - \beta} \dot{\beta} \]

- Price growth generated by the learning model exceeds the fundamental growth rate \((\dot{p}/p > 0)\) if growth expectations are increasingly bullish \((\dot{\beta} > 0)\).
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- If the current price is high \((\beta \text{ close to } r)\), then even a small degree of bearishness \((\dot{\beta} < 0)\) can cause a huge price collapse. When the current price is low, \(\dot{\beta}\) does not have a large influence on prices.
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But these are simply results 1 and 4 in the paper!
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They are general.
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• Hence

$$\dot{\beta} = -\theta' \beta, \quad \theta' = \frac{\theta}{1 - \frac{\theta}{r - \beta}}.$$
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- But in the short run, the gain \( \theta \) is large so that we can have \( \theta' < 0 \) if \( r > \beta > r - \theta \).
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\[ r > \beta > r - \theta. \]

- If this inequality is verified, anything that throws \( \beta \) off its (zero) SS RE value is reinforced in the short run by self-referential learning: momentum, bubbles.
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The alternative (followed here) is to eliminate shocks and allow $\beta_0 \neq 0$. 
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• What the stripped-down model teaches us more generally about learning and stock prices?
  ▶ Short- and long-run stability properties of learning rules may differ a lot.
  ▶ Crucial role of decreasing gain.
  ▶ The shape of the learning rule is key to understanding how small dividend shocks interact, on impact, with learning to produce big movements in prices.
Fundamentals
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- Moral: to understand price crashes, one needs a model in which the fundamental i) must be learned and ii) can crash.
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▷ Dangerous argument: why don’t agents compare price growth (which is hard to predict) to dividend growth (which is easy to predict)? Why don’t they know it is “crazy” that prices move more than dividends?

• Moral: use economic arguments instead (TVC condition probably rules out equilibria with $\frac{\dot{p}^E}{p} = \beta > r$).
Conclusion
• A thought-provoking paper that tries to play the asset pricing game without affording itself extra degrees of freedom.
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Generalize H-J bounds to learning economies?