A Dynamic Equilibrium Model of Imperfectly Integrated Financial Markets

Nicolas Coeurdacier*
London Business School and CEPR

Stéphane Guibaud†
London School of Economics

October 24, 2008‡

Abstract

This paper analyzes the determination of equity portfolios and country stock returns in the context of imperfectly integrated stock markets. We consider a continuous-time model of a two-country endowment economy in which the level of financial integration is captured by a proportional tax on foreign dividends. Despite the heterogeneity among investors induced by this tax, we obtain approximate closed-form expressions for asset prices and we characterize equity holdings and the joint process followed by country stock returns in equilibrium. Our model is consistent with a broad range of empirical findings on international financial integration. When the (endogenous) cross-country return correlation is high, small frictions in equity markets can generate a substantial home bias in portfolios. In the baseline version of our model, the cross-country return correlation is driven by fundamental correlation and portfolio rebalancing. In a two-good extension of the model, the adjustment of relative good prices can generate high stock return correlation even for a low level of fundamental correlation, thus magnifying the impact of the financial friction on portfolios.

Keywords: Two Trees, Asset Pricing with Heterogenous Investors, Home Bias in Portfolios, International Stock Return Correlations, Financial Integration.

JEL Codes: G15, F37, G11, G12, F36

---

*London Business School, Department of Economics, Regent's Park, London NW1 4SA, UK. e-mail: ncoeurdacier@london.edu

†London School of Economics, Department of Finance, Houghton Street, London WC2A 2AE, UK. e-mail: s.guibaud@lse.ac.uk

‡We thank Suleyman Basak, John Cochrane, Darrell Duffie, Bernard Dumas, Michael Gallmeyer, Rodrigo Guimaraes, Philippe Martin, Yves Nosbusch, Anna Pavlova, Monika Piazzesi, Richard Portes, Hélène Rey, José Scheinkman, Costis Skiadas, Nizar Touzi, Pierre-Olivier Weill and Wei Xiong for helpful comments and discussions. We are also grateful to seminar participants at the American Economic Association Meetings, Chicago GSB, CREI, the Econometric Society World Congress, LBS, LSE, the New York Fed, Oxford Saïd Business School and Princeton for their comments.

1 Introduction

Over the last decades most equity markets around the world have been liberalized and cross-border equity holdings have surged.[1] However a number of frictions remain in international equity markets: transaction costs, withholding taxes, as well as informational and agency problems, still act as impediments to cross-border investment. In a sense, the mere existence of a home bias in portfolios, initially documented by French and Poterba (1991), indicates that some frictions are still at play.[2] In 2005, US investors held 82% of their stock portfolios in domestic stocks, and the equity home bias is observed in all developed countries (Sercu and Vanpee (2007)). As a big picture, it is probably fair to describe international equity markets today as neither perfectly integrated nor totally segmented.

In this paper, we analyze the workings of international financial markets in between the polar cases of perfect financial integration and complete segmentation. We consider a two-country endowment economy with one non-storable good, one Lucas tree in each country and equity claims on national output. The friction which induces equity markets to be partially segmented takes the form of a proportional cost that shareholders have to pay on the dividends earned abroad. Naturally, the size of the home bias in portfolios depends on the size of the friction on equity markets, but it also depends on the international correlation of returns which determines the benefits of diversification. At the same time, this correlation is affected by cross-border equity holdings, since portfolio rebalancing effects can generate comovements in asset prices. Our main achievement is to determine both the joint distribution of asset returns and portfolio holdings in equilibrium for various levels of financial integration. We believe our setting is appropriate to make sense of (i) the extent of international portfolio diversification, (ii) the joint behavior of country stock markets, and (iii) how they are affected by the process of financial integration.[3]

Modeling *imperfectly integrated* financial markets is appealing for the sake of realism but it is technically challenging. Any form of financial segmentation essentially implies some heterogeneity among investors, a feature which makes the pricing of assets more complicated than under perfect

[1] See Lane and Milesi-Ferreti (2003). Quinn (1997), Bekaert and Harvey (2000) and Kaminsky and Schmuckler (2003) provide direct institutional measures of financial openness.

[2] In a standard CAPM world with *perfectly* integrated financial markets and identical preferences, all investors would hold the world market portfolio, independently of their nationality. However, even in the absence of frictions in international financial markets, deviations from purchasing power parity or the existence of non-insurable labor income shocks could induce heterogenous portfolios (for references, see the literature review below).

[3] We assume that fundamentals are not affected by the integration process, as would be the case, for instance, if access to new risksharing opportunities and new sources of finance induced inter-sectoral reallocations (e.g., Obstfeld (1994)). Empirically, Imbs (2006) does find a positive impact of financial integration on the synchronization of country outputs.

and complete markets. We manage to keep the problem tractable by capturing in a simple way the partial segmentation of international financial markets. The friction we consider essentially acts as a withholding tax on foreign dividends.[4] Withholding taxes are relevant in practice (e.g., pension funds have to bear such taxes on their foreign equity investments), but our friction can also be interpreted metaphorically as a reduced form for agency costs or informational frictions.[5] The asset pricing problem is non trivial. Indeed, since each investor has a specific "after-tax" investment opportunity set, the equilibrium allocation resulting from trade in assets is not Pareto efficient, risksharing is imperfect, and we cannot use the pricing kernel of a single representative investor holding the world market portfolio and consuming the aggregate endowment at each instant to price assets. In order to characterize the equilibrium, we need to keep track of the time-varying cross-country distribution of wealth. Asset prices can be expressed as functions of three state variables: the world endowment, the relative size of the two economies and their relative wealth which fluctuates endogenously. Working under the assumptions of logarithmic utility and lognormal endowment processes, we pin down these pricing functions and use them to derive the joint behavior of returns and equity portfolios.

An important contribution of this paper lies in the approximation method we use to solve the model. Even though we keep it very parsimonious, the dynamic asset pricing problem that we formulate, with two risky assets and heterogenous investors, translates into an infinite-horizon coupled forward-backward stochastic differential equations (FBSDE) problem (Ma and Yong (1999)). The exact solution to this problem cannot be obtained analytically. Instead, we derive approximate analytical formulas for asset prices, the (time-varying) first- and second moments of asset returns, and portfolios by taking Taylor expansions around the zero-tax case. To our knowledge, this use of approximation techniques to characterize the effect of taxes "in the small" is novel and we believe it could be applied fruitfully in other contexts.

Our solution technique allows us to characterize the impact of financial integration (which in our model means a decrease in the withholding tax on foreign dividends) through comparative statics. As the size of the friction decreases, we find that asset prices increase, the cross-country correlation of returns and cross-country equity holdings both also increase (the latter being a first-order effect, while the former is a second-order effect) and the volatility of asset returns diminishes

[4]This friction is by nature different from a *transaction* cost à la Constantinides (1986): it does not bear on transactions but instead reduces cash-flows during the holding period.

[5]Stulz (2005) analyzes the impact of moral hazard on cross-border investment. The role of informational frictions is emphasized in, e.g., Gehrig (1993) and Van Nieuweburgh and Veldkamp (2008).

(also a second-order effect). The overall impact of financial integration on the cost of funds is not clear-cut, depending on the respective size of the increase in the riskfree rate (due to lower precautionary saving) and of the decrease in the risk premium. The latter effect shows up as an extra term in a modified version of the CCAPM, where the level of friction is interacted with the relative wealth of countries. As a by-product of our analysis, we also derive a gravity equation for international trade in financial assets, giving a theoretical foundation to the use of gravity equations in empirical work on cross-border asset holdings (e.g., Portes and Rey (2005)).

What size of friction do we need to generate a reasonable level of home bias? In order to get a sense of the magnitude of the effects, we calibrate our model to stock market and output data for the US and Europe. We find that small frictions akin to a proportional dividend tax of the order of 10% can generate a level of domestic exposure around 80%, close to the observed home bias for the US economy.[6]

The result that small frictions on cross-border holdings can result in substantial portfolio home bias relies on a high elasticity of asset demand and on a high level of assets substitutability. In our model, the substitutability between national assets is driven by common shocks affecting national economic fundamentals and by portfolio rebalancing. The portfolio rebalancing mechanism that induces the correlation of two assets returns to be higher than their "fundamental" correlation works as in Cochrane et al. (2007). A good shock to domestic dividends drives the price of the domestic asset up and increases its share in investors' portfolios. When financial markets are integrated, investors increase their demand for the foreign asset in order to keep the composition of their portfolios constant, which drives the price of the foreign asset up. In other words, as the share of the domestic asset in the world market portfolio increases, the required return on the foreign asset decreases because its diversification properties become more valuable. In financial autarky by contrast, a good shock to an asset drives its price up without affecting the price of the other asset and the correlation of asset returns is equal to the correlation of economic fundamentals. In-between complete segmentation and perfect integration, the lower the frictions between two markets, the higher the comovements of their stock prices, for a given level of fundamental correlation.[7] We point out that this portfolio rebalancing effect, though spectacular for low levels of fundamental

[6] As a point of comparison, the average level of withholding dividend taxes faced by a (domestically tax-exempt) US pension fund investing abroad is 14% (source: International Bureau of Fiscal Documentation).

[7] One might prefer to think in terms of stochastic discount factors (SDFs). The two agents have perfectly correlated SDFs in the perfectly integrated case, so that the two assets are discounted the same way, which increases their correlation compared to the extreme case of complete segmentation where each asset is priced using the corresponding autarkic SDF. As financial integration increases, the discount factors that are applied to national assets become closer to each other, which increases the correlation of their returns.

correlation and no friction on financial markets, is quantitatively small for a realistic calibration of the model. This result is interesting when one wants to think about the home bias from a general equilibrium perspective. Any cost bearing on foreign equity holdings has two opposite effects on portfolios: the direct effect is to reduce cross-border holdings by reducing expected returns on foreign assets; but there is also an indirect effect, which is to reduce the substitutability between national assets by reducing the correlation of their returns, thus increasing the willingness to diversify internationally. The overall quantitative impact of a friction depends on the relative size of the two effects, and the fact that the indirect effect is of small magnitude plays in favor of the result that small frictions can generate a large home bias.

We also provide an extension of our framework with two differentiated goods: each country produces one type of good and agents have CES preferences over the two goods. We show that the two-good model is completely isomorphic to the one-good case up to a simple transformation of the state variables. Hence, all our findings go through in this environment but we can endogenously generate a high level of asset substitutability without requiring an exogenously high level of fundamental correlation. The reason is that a positive domestic endowment shock increases the relative price of foreign goods and foreign dividends at market value. This Ricardian adjustment of the terms of trade (see also Cole and Obstfeld (1991) and Pavlova and Rigobon (2007)) generates high levels of stock return correlation even when endowment shocks are independent across countries, thus magnifying the impact of the financial friction on portfolios.

Finally, our analysis yields some insight on the correlation puzzle in international equity holdings, by which we refer to the empirical finding of a robust positive relationship between bilateral equity holdings and bilateral stock return correlations (see Portes and Rey (2005), Chan *et al.* (2005) and Lane and Milesi-Ferreti (2008)). As the level of financial integration between two countries affects positively both their cross-border holdings and the correlation of their returns, it could be that the positive relationship between these two variables across pairs of countries is just driven by variations in the bilateral level of financial integration. In an empirical companion paper (Coeurdacier and Guibaud (2007)), we show that once this endogeneity issue is taken into account the correlation puzzle indeed disappears: holding financial frictions constant, investors do tilt their foreign holdings towards countries which offer better diversification opportunities.

Related literature. Basak and Gallmeyer (2003) consider a dynamic asset pricing model with asymmetric dividend taxation and a unique risky asset. We follow their approach to deal with investors' heterogeneity through the introduction of a time-varying Pareto-Negishi weight.[8] But to address issues of portfolio composition or stock return correlation, we need at least two risky assets. Our analysis can be seen as a natural extension of the work of Basak and Gallmeyer (2003) to the two-asset case. But our solution method clearly differs from theirs: whereas they solve their FBSDE problem numerically by solving an equivalent quasi-linear PDE (see Ma, Protter and Yong (1994)), we provide approximate analytical formulas. Because our two-tree specification follows Cochrane et al. (2007), we can use the quasi-closed-form pricing functions provided in their paper to compute our approximations.[9]

A vast strand of the literature in international finance studies portfolios and asset prices in the context of imperfectly integrated financial markets. Two papers closely related to ours are Martin and Rey (2004) and Bhamra (2004). Martin and Rey (2004) build a static model featuring a transaction cost on international trade in assets. Much as in our paper, this cost induces a home bias and the size of the bias depends on the elasticity of the demand for foreign assets, which is related to investors' risk aversion.[10] The dynamic setup of our model allows us to explore the joint dynamic of asset prices and wealth distribution, and issues related to portfolio rebalancing and endogenous return correlation.[11] Bhamra (2004) builds a full-fledged dynamic equilibrium model of partially segmented financial markets, but he imposes constraints directly on the amount of wealth that can be invested abroad.[12] We get the home bias in a more endogenous way by relating it to small frictions in equity markets.

Black (1974), Stulz (1981), Errunza and Losq (1985, 1989), Eun and Jarakiramanan (1986) and Hietala (1989) analyze the impact of international financial barriers on portfolio holdings and on the risk-return tradeoff, characterizing how specific kinds of financial frictions lead to specific deviations from the traditional CAPM. We derive a modified version of the CCAPM in our dynamic asset pricing model, which is close in spirit to their work.

By focusing on small frictions in financial markets, we depart from a literature which tries to

[8] Stochastic weights have been used in the literature to characterize equilibrium under incomplete markets (e.g., Cuoco and He (1994), Basak and Cuoco (1998)). In our setup, like in Basak and Gallmeyer (2003), world markets are dynamically complete but deviation from perfect risksharing results from differential taxation.

[9] Menzly, Santos and Veronesi (2004), Santos and Veronesi (2006) and Martin (2007) build multi-asset dynamic asset pricing models under alternative assumptions on preferences and cash flows. Our solution method could be generalized to their setups.

[10] This effect shows up in our model through the impact of volatility.

[11] Dumas et al. (2003) and Cochrane et al. (2007) analyze the endogenous determination of asset return correlation in the context of perfectly integrated financial markets. The implications of portfolio rebalancing for the joint behavior of asset returns and the exchange rate is explored in Hau and Rey (2004).

[12] Pavlova and Rigobon (2008) analyze the impact of portfolio constraints on the international propagation of shocks.

ascribe the observed equity home bias to hedging motives. One strand of this literature focuses on the hedging of real exchange rate fluctuations, which can be induced by the imperfect integration of markets for goods and services (Adler and Dumas (1983), Dumas (1992), Cooper and Kaplanis (1994)). It explores whether portfolio biases can be related to the presence of *trade costs* (Uppal (1993), Obstfeld and Rogoff (2000), Coeurdacier (2008)) or to the presence of *non-tradable goods* (Stockman and Dellas (1989), Baxter, Jermann and King (1998), Serrat (2001), Kollmann (2006)). Another strand of this literature focuses on the hedging of *non-diversifiable labor income risk* (Baxter and Jermann (1997), Bottazzi, Pesenti and van Wincoop (1996), Heathcote and Perri (2007), Engel and Matsumoto (2008)). In our paper, these hedging motives do not operate. We focus on the implications of frictions in international financial markets.

Our theoretical predictions regarding the impact of financial integration on asset prices relate to some empirical contributions on this subject. Henry (2000) and Chari and Henry (2004) document a positive impact of financial integration on asset prices. Bekaert and Harvey (2000), Goetzmann et al. (2005) and Quinn and Voth (2008) find evidence of a positive relationship between the level of financial market integration and stock return correlations. Our results are consistent with this set of findings.

From a methodological perspective, our paper is related to Devereux and Sutherland (2006) and Tille and van Wincoop (2007). Building on Judd and Guu (2001), they independently developed an approximation approach to solve for international portfolios in general equilibrium with heterogenous agents. Their method also relies on Taylor expansions but whereas we take approximations in the size of the friction (around the frictionless case), they take approximations in the variance of shocks (around the non-stochastic steady-state).

The rest of the paper proceeds as follows. Section 2 lays out the model. Section 3 shows how to deal with investors' heterogeneity and solve for equilibrium asset prices by taking Taylor expansions around the frictionless case. The implications of imperfect market integration for asset prices, asset returns and portfolios are derived in Section 4. Section 5 presents the two-good extension. Section 6 concludes. The proofs are relegated in the Appendix.

2 Model

2.1 Assumptions

We consider a continuous-time economy with an infinite horizon. There are two countries, home (H) and foreign (F), and a single non-storable good. Each country has a representative agent with time-separable expected utility and logarithmic preferences. The utility of agent i at time t is

$$U_{it} = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \log(c_{is}) ds \right], \tag{1}$$

where c_{is} is the consumption rate in country $i = H, F$, and ρ is the common rate of time preference.

Endowments. There is a Lucas tree in each country. We assume the real endowments (dividends) follow geometric Brownian motions:

$$\frac{dD_i(t)}{D_i(t)} = \mu_{D_i} dt + \boldsymbol{\sigma}_{D_i}^T d\boldsymbol{W}(t), \qquad i = H, F. \tag{2}$$

All uncertainty is generated by the bi-dimensional standard Wiener process $\boldsymbol{W}(t)$. We call η the instantaneous correlation of the two dividend growth rates, which we henceforth refer to as the "fundamental" correlation. Throughout, we use bold cases for vectors and matrices and $\boldsymbol{A}^T$ to denote the transpose of $\boldsymbol{A}$.

From (2), the world endowment $D \equiv D_H + D_F$ follows a diffusion process whose drift and diffusion coefficients are weighted averages of those of D_H and D_F, with a time-varying weight depending on the size of each economy's endowment relative to the world endowment. We can write

$$\frac{dD(t)}{D(t)} = \underbrace{[\delta(t)\mu_{D_H} + (1-\delta(t))\mu_{D_F}]}_{\equiv \mu_D(t)} dt + \underbrace{[\delta(t)\boldsymbol{\sigma}_{D_H}^T + (1-\delta(t))\boldsymbol{\sigma}_{D_F}^T]}_{\equiv \boldsymbol{\sigma}_D^T(t)} d\boldsymbol{W}(t), \tag{3}$$

where $\delta(t) \equiv D_H(t)/\left(D_H(t) + D_F(t)\right)$ captures the relative size of the domestic economy. Using the dynamics of D_H and D_F and applying Itô's lemma, one can write

$$d\delta/\delta = \mu_\delta dt + \boldsymbol{\sigma}_\delta^T d\boldsymbol{W}, \tag{4}$$

with

$$\mu_\delta = (1-\delta)\left[\mu_{D_H} - \mu_{D_F} - \delta(\boldsymbol{\sigma}_{D_H}.\boldsymbol{\sigma}_{D_H}) + (1-\delta)(\boldsymbol{\sigma}_{D_F}.\boldsymbol{\sigma}_{D_F}) + (2\delta-1)(\boldsymbol{\sigma}_{D_H}.\boldsymbol{\sigma}_{D_F})\right], \tag{5}$$

$$\boldsymbol{\sigma}_\delta = (1-\delta)(\boldsymbol{\sigma}_{D_H} - \boldsymbol{\sigma}_{D_F}). \tag{6}$$

Menu of assets. The menu of financial assets consists of stocks that are claims on the two Lucas trees (each stock being in constant net supply normalized to one) and a frictionless international bank deposit (in zero net supply). We will note S_H and S_F the two stock prices and r the riskfree interest rate. Their processes will be determined as part of the equilibrium.

Frictions on equity markets. We assume investors have to pay a proportional cost $\tau \in (0,1)$ on the dividends they earn abroad.[13] For instance, a domestic agent who holds a unit of foreign stock receives the instantaneous dividend $(1-\tau)D_F$. No cost is paid on the domestic dividends.

One way to think about this τ is that it captures literally differences in the taxation of domestic and foreign dividends. Such kind of fiscal discrimination is relevant in practice (see, for instance, Gordon and Hines (2002)): it can be due to withholding taxes on foreign dividends,[14] or to tax credits that are extended to shareholders (based on their domestic holdings only), in principle to avoid the double taxation of dividends at the corporate and at the personal level.[15] But our proportional cost could be given other interpretations: it could capture for instance higher fees required by mutual funds investing in international stocks, or it could be micro-founded as an agency cost in a model with moral hazard on cross-border investment. In what follows though, for simplicity, we refer to τ as a tax. When $\tau = 0$, financial markets are perfectly integrated.

We follow Basak and Gallmeyer (2003) and assume that taxes are redistributed in the economy as lump sum transfers, each agent continuously receiving transfers $e_i(t)dt$. This assumption allows us to write the market clearing condition for goods in a simple way, keeping the aggregate consumption equal to aggregate dividend at each instant. The particular redistribution scheme under consideration does not matter much for our results. One could assume for instance that each agent receives the taxes paid by the other investor.[16] In that case,

$$e_H(t) = \tau\alpha_{FH}(t)D_H(t), \tag{7}$$

$$e_F(t) = \tau\alpha_{HF}(t)D_F(t). \tag{8}$$

where α_{ij} denotes the quantity of claim on country j output held by the representative investor in country i.

[13] Our analysis could easily be extended to the case where these costs differ between countries.

[14] Though the payment of these taxes to foreign fiscal authorities often gives a right to tax credits at home, withholding taxes constitute a real cost for tax-exempt investors like pension funds.

[15] These "dividend imputation schemes" are quite common. An extreme example is the "avoir fiscal" in France: Until a recent reform, a French investor would receive from the French fiscal authorities an amount equal to 50% of the dividends perceived on stocks held in a PEA, i.e., a *tax-exempt* saving account! Only domestic stocks were eligible in PEAs, which created a powerful incentive to invest domestically.

[16] We assume all investors act competitively. Therefore, the redistribution of taxes does not give rise to any kind of strategic behavior.

2.2 Individual optimization

Investor i is endowed with an initial share $\alpha_{ij}(0)$ of each stock j. At each point in time, given the price processes S_H and S_F, the interest rate process r, her wealth X_i and a transfer process e_i, she chooses consumption c_i and asset holdings $\boldsymbol{\alpha}_i = (\alpha_{iH}, \alpha_{iF})^T$ in order to maximize her intertemporal utility (1). The induced process for financial wealth X_i is given by

$$dX_i(t) = [r(t)X_i(t) + \boldsymbol{\alpha}_i^T(t)\boldsymbol{I}_S(t)(\boldsymbol{\mu}_i(t) - \boldsymbol{r}(t)) + e_i(t) - c_i(t)]dt + \boldsymbol{\alpha}_i^T(t)\boldsymbol{I}_S(t)\boldsymbol{\sigma}^T(t)d\boldsymbol{W}(t), \quad (9)$$

with $\boldsymbol{I}_S$ a diagonal matrix that has S_H and S_F as diagonal coefficients, $\boldsymbol{\mu}_i$ the vector of expected returns from the perspective of investor i, and $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_H \ \boldsymbol{\sigma}_F]$ the diffusion matrix of stock prices.[17]

2.3 Definition of equilibrium

Given preferences, initial endowments and a tax reallocation rule, a competitive equilibrium is a set of adapted processes for asset prices, consumption c_i and asset holdings $\boldsymbol{\alpha}_i$ such that $(c_i, \boldsymbol{\alpha}_i)$ is a solution to investor i's optimization problem, and all markets clear at all dates, i.e., for all $t \geq 0$

$$\begin{aligned} c_H(t) + c_F(t) &= D_H(t) + D_F(t) = D(t), \\ \boldsymbol{\alpha}_H(t) + \boldsymbol{\alpha}_F(t) &= \mathbf{1}, \\ X_H(t) + X_F(t) &= S_H(t) + S_F(t). \end{aligned}$$

Imposing that the aggregate financial wealth be equal to the world market capitalization is equivalent to imposing that the aggregate position on the bank deposit be zero.

3 Equilibrium

In this section, we start with a brief description of the equilibrium in the benchmark case of perfect integration (i.e., $\tau = 0$), recalling the quasi-closed-form expressions obtained for asset prices in that case (Cochrane et al. (2007), Martin (2007)). This is a good starting point to understand, by contrast, the impact of the friction we introduce. Moreover, we later make use of the frictionless solution by deriving Taylor approximations for asset prices around the case of perfect integration.

[17] The instantaneous expected returns as well as the four coefficients of $\boldsymbol{\sigma}$ are stochastic processes to be determined in equilibrium.

3.1 Benchmark case: no friction

When $\tau = 0$, all investors face the same opportunity set. Since they have identical preferences, they choose the same portfolio composition – every investor holds the world market portfolio. In this case, one can use the pricing kernel of a logarithmic representative agent consuming the world endowment at every instant to price each asset as the expected present value of appropriately discounted future dividends:

$$S_{i0}(t) = \mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \frac{D(t)}{D(s)} D_i(s) ds \right], \qquad i = H, F.$$

Using the definition of δ, one can rewrite the price of each stock as follows:

$$\begin{aligned} S_{H0}(t) &= D(t)\mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} \delta(s) ds \right], \\ S_{F0}(t) &= D(t)\mathbb{E}_t \left[\int_t^\infty e^{-\rho(s-t)} (1-\delta(s)) ds \right]. \end{aligned}$$

Letting $y_H(\delta) \equiv \mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)} \delta(s) ds \right| \delta(t) = \delta \right]$ and $y_F(\delta) = \frac{1}{\rho} - y_H(\delta)$, one obtains:

$$S_{H0}(t) = D(t) y_H(\delta(t)), \tag{10}$$

$$S_{F0}(t) = D(t) y_F(\delta(t)). \tag{11}$$

The equation for S_{H0} says that the price of the home country asset at time t is equal to the world endowment at time t, $D(t)$, times the conditional expectation at time t of the discounted future values of δ. Because δ is a Markov process, this conditional expectation can be written as a function y_H of $\delta(t)$. The expression for S_{F0} is similar, so that both stock prices are functions of only two state variables: D and δ. As pointed out by Cochrane et al. (2007), the function y_H turns out to be the standard hypergeometric function (see details in Appendix A).

The consumption allocation in the benchmark case is straightforward. The relative consumption ratio is constant over time, both agents consuming a constant fraction of the world endowment according to their relative wealth ratio. There is perfect risksharing. Besides, due to the logarithmic utility assumption, both agents' consumption-to-wealth ratios are constant, equal to their common rate of time preference ρ.

3.2 Heterogeneity and imperfect risksharing

The immediate impact of a tax on foreign dividends is that all investors do not get the same after-tax returns; in that sense they face different investment opportunities. In the presence of taxes, we have a model with heterogenous investors.[18]

Wedge in perceived expected returns. We will now pin down precisely the heterogeneity among investors, taking the returns on asset H as an example. The total instantaneous expected payoff on this asset is $D_H(t)dt+\mathbb{E}_t dS_H(t)$ for a domestic investor and $(1-\tau)D_H(t)dt+\mathbb{E}_t dS_H(t)$ for a foreign investor. The difference in the expected payoff on asset H for home and foreign investors comes from the dividends, which are lower for the foreign investor because of the tax. From this, we can define the total instantaneous expected rates of return on asset H, which we respectively note μ_H for the home investor and $\mu_{F,H}$ for the foreign investor:

$$\mu_H(t)dt = \mathbb{E}_t\left[\frac{D_H(t)dt + dS_H(t)}{S_H(t)}\right], \qquad \mu_{F,H}(t)dt = \mathbb{E}_t\left[\frac{(1-\tau)D_H(t)dt + dS_H(t)}{S_H(t)}\right].$$

Obviously, μ_H is greater than $\mu_{F,H}$, the wedge between the two being equal to the tax rate τ multiplied by the dividend-price ratio of asset H:

$$\mu_H(t) - \mu_{F,H}(t) = \tau\frac{D_H(t)}{S_H(t)}. \tag{12}$$

Analogously, we get

$$\mu_F(t) - \mu_{H,F}(t) = \tau\frac{D_F(t)}{S_F(t)}, \tag{13}$$

where $\mu_{H,F}$ and μ_F respectively denote the total instantaneous expected rates of return on asset F for home and foreign investors. These expressions for the wedges characterize tightly the heterogeneity induced by taxes.

Investor-specific state prices. Investors being heterogenous, we have to solve their individual optimization problems separately. Since both investors face dynamically complete markets, we use the solution technique of Cox and Huang (1989) and Karatzas et al. (1987). Therefore, we introduce the investor-specific (after-tax) market prices of risk $\boldsymbol{\theta}_H$ and $\boldsymbol{\theta}_F$:

$$\boldsymbol{\theta}_H(t) \equiv \left(\boldsymbol{\sigma}^T(t)\right)^{-1}\begin{pmatrix}\mu_H(t)-r(t)\\ \mu_{H,F}(t)-r(t)\end{pmatrix}, \qquad \boldsymbol{\theta}_F(t) \equiv \left(\boldsymbol{\sigma}^T(t)\right)^{-1}\begin{pmatrix}\mu_{F,H}(t)-r(t)\\ \mu_F(t)-r(t)\end{pmatrix}. \tag{14}$$

[18]This subsection and the next draw extensively on Basak and Gallmeyer (2003).

The difference between the market prices of risk relevant for the two representative agents follows directly from the wedges characterized in (12) and (13):

$$\boldsymbol{\theta}_H(t) - \boldsymbol{\theta}_F(t) = \left(\boldsymbol{\sigma}^T(t)\right)^{-1} \begin{pmatrix} \tau \frac{D_H(t)}{S_H(t)} \\ -\tau \frac{D_F(t)}{S_F(t)} \end{pmatrix}. \tag{15}$$

We can now define investor i's state-price deflator ξ_i as

$$\xi_i(t) = \exp\left(-\int_0^t r(s)ds\right) \exp\left(-\int_0^t \boldsymbol{\theta}_i(s)d\boldsymbol{W}(s) - \frac{1}{2}\int_0^t \boldsymbol{\theta}_i^T(s)\boldsymbol{\theta}_i(s)ds\right), \qquad i = H, F.$$

$\xi_i(\omega, t)$ can be understood as the price (faced by agent i) of a security paying dt at time t in state ω. Each ξ_i satisfies the following stochastic differential equation:

$$\frac{d\xi_i(t)}{\xi_i(t)} = -r(t)dt - \boldsymbol{\theta}_i^T(t)d\boldsymbol{W}(t). \tag{16}$$

Using these state prices, each individual dynamic optimization problem can be restated as a static problem, which consists in choosing a vector of contingent consumption rates under a single budget constraint:

$$\max_{\{c_i(t)\}} \quad \mathbb{E}\left[\int_0^\infty e^{-\rho t}\log(c_i(t))dt\right]$$

subject to

$$\mathbb{E}\left[\int_0^\infty \xi_i(t)c_i(t)dt\right] \leq X_i(0) + \mathbb{E}\left[\int_0^\infty \xi_i(t)e_i(t)dt\right],$$

where the initial wealth $X_i(0)$ depends on the initial distribution of property rights on the equity claims.

Imperfect risksharing. Letting Ψ_i denote the Lagrange multiplier on investor i's budget constraint, the first-order conditions of the above problem can be stated as

$$e^{-\rho t}\frac{1}{c_i(t)} = \Psi_i \xi_i(t), \quad \forall t,\ \forall i = H, F. \tag{17}$$

This implies

$$\frac{c_F(t)}{c_H(t)} = \frac{\Psi_H \xi_H(t)}{\Psi_F \xi_F(t)} \equiv \lambda(t), \quad \forall t. \tag{18}$$

We know from (15) and (16) that ξ_H and ξ_F follow different dynamics, therefore (18) implies that the consumption ratio c_F/c_H is not constant: imperfect risksharing prevails. Using the market clearing condition on the goods market, we can write

$$c_H(t) = \frac{1}{1+\lambda(t)} D(t) \qquad \text{and} \qquad c_F(t) = \frac{\lambda(t)}{1+\lambda(t)} D(t). \tag{19}$$

The consumption of each agent is a function of the total endowment D and of λ, which acts as a time-varying relative Pareto-Negishi weight for agent F. This is reminiscent of equilibria under incomplete markets *à la* Cuoco and He (1994). In our case, markets are complete but the deviation from the Pareto-efficient allocation is induced by asymmetric taxation.

These results have to be contrasted with the case where $\tau = 0$. In a frictionless environment, the two investors face the same state prices, ξ_H/ξ_F is constant, the relative consumption ratio is constant and each agent consumes a constant fraction of the world endowment. In that case, λ is exactly equal to the constant wealth ratio X_F/X_H. When it comes to asset prices, the impact of the deviation from perfect risksharing which materializes in the time-varying relative weight λ is to increase the volatility of asset returns by adding a source of volatility in the stochastic discount factors and to decrease the correlation between asset returns. The reason for this latter effect is that in the frictionless case, both assets are priced by a same SDF, whereas when $\tau \neq 0$, the effective SDFs underlying the pricing of each asset (which can be thought of as linear combinations of the intertemporal rate of substitutions of the two investors, with weights depending on the relative size of their asset holdings) are no longer the same.

3.3 Asset prices and relative wealth: an FBSDE problem

When $\tau \neq 0$, the distribution of wealth captured by the stochastic weight λ plays as a state variable in addition to D and δ. From the expressions for individual consumption given in (19), we can get the pricing kernels of both agents and use them to price the two assets.

Lemma 1. *The two asset prices at time t can be written as*

$$S_H(t) = S_H(D(t), \delta(t), \lambda(t)) = D(t) \frac{1}{1+\lambda(t)} h(\delta(t), \lambda(t)) \tag{20}$$

$$S_F(t) = S_F(D(t), \delta(t), \lambda(t)) = D(t) \frac{\lambda(t)}{1+\lambda(t)} f(\delta(t), \lambda(t)) \tag{21}$$

where h and f are defined on $(0,1)\times(0,\infty)$ *as*

$$h(\delta(t),\lambda(t)) \equiv \mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(s-t)}\left[1+\lambda(s)\right]\delta(s)ds\right], \tag{22}$$

$$f(\delta(t),\lambda(t)) \equiv \mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(s-t)}\left(1+\frac{1}{\lambda(s)}\right)(1-\delta(s))ds\right]. \tag{23}$$

Stock prices at time t can be written as functions of $D(t)$, $\delta(t)$ and $\lambda(t)$: this is enough information to form expectations on the future dividends of both assets and on the pricing kernels of both agents. It is noticeable that, though they do not share the same pricing kernels (because risksharing is imperfect), the two investors agree on asset prices. They have to do so and what makes it possible is the fact that they do not face exactly the same assets. Indeed, the dividend flows *net of taxes* are different for the two investors. Another way to put it is that investors have different perceptions both of dividends and risk: for a given investor, the bad characteristic of an investment abroad in terms of expected returns is exactly compensated by the good diversification property of that investment.

To complete the characterization of equilibrium, we need to determine the functions h and f. The conditional expectations involve future values of δ and λ. The process for δ is exogenous (Eq. (4)). The dynamic of the relative weight λ is determined endogenously. From the definition of λ in (18) and from the dynamic of ξ_i in (16), Itô's lemma implies:

$$\frac{d\lambda(t)}{\lambda(t)} = (\boldsymbol{\theta}_F(t)-\boldsymbol{\theta}_H(t))^T\boldsymbol{\theta}_F(t)dt + (\boldsymbol{\theta}_F(t)-\boldsymbol{\theta}_H(t))^T d\boldsymbol{W}(t). \tag{24}$$

The drift and diffusion coefficients driving the dynamics of λ themselves depend on the investor-specific market prices of risk. The latter can be shown to verify the following equilibrium restriction.

Lemma 2. *The market prices of risk, as perceived by home and foreign investors, satisfy*

$$\boldsymbol{\theta}_H(t) = \boldsymbol{\sigma}_D(t) + \tau\frac{\lambda(t)}{1+\lambda(t)}\left(\boldsymbol{\sigma}^T(t)\right)^{-1}\begin{pmatrix} \frac{D_H(t)}{S_H(t)} \\ -\frac{D_F(t)}{S_F(t)} \end{pmatrix}, \tag{25}$$

$$\boldsymbol{\theta}_F(t) = \boldsymbol{\sigma}_D(t) + \tau\frac{1}{1+\lambda(t)}\left(\boldsymbol{\sigma}^T(t)\right)^{-1}\begin{pmatrix} -\frac{D_H(t)}{S_H(t)} \\ \frac{D_F(t)}{S_F(t)} \end{pmatrix}. \tag{26}$$

In these expressions, the first term corresponds to the market prices of risk in the frictionless case. Indeed, when $\tau = 0$, investors face the same market prices of risk, which are equal to $\boldsymbol{\sigma}_D$, the vector of diffusion coefficients in the world endowment process (Eq. (3)). The second term captures the impact of taxes, interacted with the dividend yields. Using (20)-(21) and (25)-(26), we can rewrite (24) as $d\lambda/\lambda = \mu_\lambda dt + \boldsymbol{\sigma}_\lambda d\boldsymbol{W}$, with

$$\begin{aligned}\mu_\lambda &= \tau\left[-\frac{\delta(1+\lambda)}{h(\delta,\lambda)}\ \frac{(1+\lambda)(1-\delta)}{\lambda f(\delta,\lambda)}\right]\boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D \\ &\quad + \tau^2\frac{1}{1+\lambda}\left[-\frac{\delta(1+\lambda)}{h(\delta,\lambda)}\ \frac{(1+\lambda)(1-\delta)}{\lambda f(\delta,\lambda)}\right]\left(\boldsymbol{\sigma}^T\boldsymbol{\sigma}\right)^{-1}\begin{pmatrix}-\frac{\delta(1+\lambda)}{h(\delta,\lambda)}\\ \frac{(1+\lambda)(1-\delta)}{\lambda f(\delta,\lambda)}\end{pmatrix}, \qquad (27)\\ \boldsymbol{\sigma}_\lambda &= \tau\left(\boldsymbol{\sigma}^T\right)^{-1}\begin{pmatrix}-\frac{\delta(1+\lambda)}{h(\delta,\lambda)}\\ \frac{(1+\lambda)(1-\delta)}{\lambda f(\delta,\lambda)}\end{pmatrix}. \qquad (28)\end{aligned}$$

In our economy, asset prices depend on the distribution of wealth (captured by λ) and vice-versa. Technically, the infinite-horizon backward SDEs defining h and f are coupled with a forward SDE for λ, so that our pricing problem reduces to a forward-backward SDE problem (Ma and Yong (1999)).

3.4 Approximation strategy

Our strategy will be to derive an approximated solution to this problem around the case of perfect financial integration. We use the fact that to μ_λ and $\boldsymbol{\sigma}_\lambda$ are functions of τ, which we denote by writing $\mu_\lambda(\delta,\lambda;\tau)$ and $\boldsymbol{\sigma}_\lambda(\delta,\lambda;\tau)$. In the benchmark frictionless case, λ is constant, so that $\mu_\lambda(\delta,\lambda;0) = 0$ and $\boldsymbol{\sigma}_\lambda(\delta,\lambda;0) = \mathbf{0}$ for all (δ,λ). For τ close to zero, we can therefore derive first-order Taylor expansions for μ_λ and $\boldsymbol{\sigma}_\lambda$ as

$$\begin{aligned}\mu_\lambda(D,\delta,\lambda;\tau) &= \tau\left[-\frac{\delta}{y_H(\delta)}\ \frac{1-\delta}{y_F(\delta)}\right]\boldsymbol{\sigma}_0^{-1}(\delta)\boldsymbol{\sigma}_D(\delta) + o(\tau), \qquad (29)\\ \boldsymbol{\sigma}_\lambda(D,\delta,\lambda;\tau) &= \tau(\boldsymbol{\sigma}_0^T)^{-1}(\delta)\begin{pmatrix}-\frac{\delta}{y_H(\delta)}\\ \frac{1-\delta}{y_F(\delta)}\end{pmatrix} + \boldsymbol{o}(\tau). \qquad (30)\end{aligned}$$

Subscripts 0 refer to values prevailing when $\tau = 0$ (see Section 3.1). In particular, the diffusion matrix $\boldsymbol{\sigma}_0$ is obtained as $\boldsymbol{\sigma}_0 = [\boldsymbol{\sigma}_{H0}\ \boldsymbol{\sigma}_{F0}]$, with

$$\boldsymbol{\sigma}_{H0} = \boldsymbol{\sigma}_D + \frac{y_H'}{y_H}\delta\boldsymbol{\sigma}_\delta, \qquad (31)$$

$$\boldsymbol{\sigma}_{F0} = \boldsymbol{\sigma}_D + \frac{y_F'}{y_F}\delta\boldsymbol{\sigma}_\delta. \qquad (32)$$

In Appendix C, we also obtain second-order approximations for μ_λ and $\boldsymbol{\sigma}_\lambda$. We use these approximations to derive Taylor expansions in τ for the pricing functions h and f.

4 Findings

In this section, we give a full description of international financial markets equilibrium in the neighborhood of the frictionless case. Section 4.1 gives first and second order approximations for asset prices. Section 4.2 explores asset returns volatility and cross-country return correlations. Section 4.3 gives expressions for risk premia and the riskfree rate. Finally, we display results on the composition of portfolios in Section 4.4.

4.1 Asset prices

Proposition 1. *To the first order, S_H and S_F are given by*

$$S_H(D,\delta,\lambda;\tau) = \left[1-\tau\frac{\lambda}{1+\lambda}\right] S_{H0}(D,\delta)+o(\tau), \tag{33}$$

$$S_F(D,\delta,\lambda;\tau) = \left[1-\tau\frac{1}{1+\lambda}\right] S_{F0}(D,\delta)+o(\tau). \tag{34}$$

The first-order effect of imperfect market integration is to reduce equilibrium asset prices: frictions in financial markets translate into lower prices by reducing expected income streams on domestic shares received by foreigners. Note that the decrease in domestic asset prices is higher when λ is higher. This makes sense since λ is a proxy for the relative wealth of the foreign investors: as λ increases, the relative influence of foreign investors in the pricing of assets becomes higher, which has a negative impact on the domestic asset price, since foreigners are willing to pay a lower price because of the tax they pay on dividends.

Proposition 2. *Let* $\boldsymbol{\Gamma} \equiv (\boldsymbol{\sigma}_0^T)^{-1}\begin{pmatrix} -\frac{D_H}{S_{H0}} \\ \frac{D_F}{S_{F0}} \end{pmatrix}$. *Second-order approximations for S_H and S_F are:*

$$S_H(D,\delta,\lambda;\tau) = \left[1-\tau\frac{\lambda}{1+\lambda}\right] S_{H0}(D,\delta)+\tau^2\frac{\lambda}{(1+\lambda)^2}D\left[y_H(\delta)+\varphi_H(\delta)\right]+o(\tau^2), \tag{35}$$

$$S_F(D,\delta,\lambda;\tau) = \left[1-\tau\frac{1}{1+\lambda}\right] S_{F0}(D,\delta)+\tau^2\frac{\lambda}{(1+\lambda)^2}D[y_F(\delta)+\varphi_F(\delta)]+o(\tau^2), \tag{36}$$

where φ_H and φ_F are solutions of the following ODEs [19]

$$\begin{aligned}
\rho\varphi_H(\delta) - \delta\mu_\delta(\delta)\varphi'_H(\delta) - \frac{1}{2}\delta^2\boldsymbol{\sigma}_\delta^T(\delta)\boldsymbol{\sigma}_\delta(\delta)\varphi''_H(\delta) &= y_H(\delta)\boldsymbol{\Gamma}(\delta).\boldsymbol{\Gamma}(\delta), \\
\rho\varphi_F(\delta) - \delta\mu_\delta(\delta)\varphi'_F(\delta) - \frac{1}{2}\delta^2\boldsymbol{\sigma}_\delta^T(\delta)\boldsymbol{\sigma}_\delta(\delta)\varphi''_F(\delta) &= y_F(\delta)\boldsymbol{\Gamma}(\delta).\boldsymbol{\Gamma}(\delta),
\end{aligned}$$

with boundary conditions

$$\begin{aligned}
\varphi_H(0) &= 0 \\
\varphi_H(1) &= \lim_{\delta\to 1}\frac{1}{\rho^2}\boldsymbol{\Gamma}(\delta).\boldsymbol{\Gamma}(\delta) \\
\varphi_F(0) &= \lim_{\delta\to 0}\frac{1}{\rho^2}\boldsymbol{\Gamma}(\delta).\boldsymbol{\Gamma}(\delta) \\
\varphi_F(1) &= 0.
\end{aligned}$$

To make sense of the second-order price effect of integration, one needs to understand the impact of integration on the riskless rate and on the variance-covariance matrix of returns. As we will see, the riskless rate and the return correlation are both decreasing in τ, which induces a positive impact on asset prices through the risk-adjusted discount factor.

4.2 Volatility and correlation of asset returns

Proposition 3. *Second-order approximations of asset prices diffusion coefficients $\boldsymbol{\sigma}_H$ and $\boldsymbol{\sigma}_F$ are:*

$$\boldsymbol{\sigma}_H(\delta,\lambda) = \boldsymbol{\sigma}_{H0}(\delta) + \tau^2\frac{\lambda}{(1+\lambda)^2}\left\{-\boldsymbol{\Gamma}(\delta) + \left[\frac{\varphi'_H(\delta)}{y_H(\delta)} - \lambda\frac{y'_H(\delta)}{y_H(\delta)} - \frac{\varphi_H(\delta)y'_H(\delta)}{y_H^2(\delta)}\right]\delta\boldsymbol{\sigma}_\delta(\delta)\right\} + \boldsymbol{o}(\tau^2), \tag{37}$$

$$\boldsymbol{\sigma}_F(\delta,\lambda) = \boldsymbol{\sigma}_{F0}(\delta) + \tau^2\frac{\lambda}{(1+\lambda)^2}\left\{\boldsymbol{\Gamma}(\delta) + \left[\frac{\varphi'_F(\delta)}{y_F(\delta)} - \frac{1}{\lambda}\frac{y'_F(\delta)}{y_F(\delta)} - \frac{\varphi_F(\delta)y'_F(\delta)}{y_F^2(\delta)}\right]\delta\boldsymbol{\sigma}_\delta(\delta)\right\} + \boldsymbol{o}(\tau^2). \tag{38}$$

We obtain the instantaneous volatility and correlation of asset returns from the instantaneous variance-covariance matrix $\boldsymbol{\sigma}^T\boldsymbol{\sigma}$, where $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_H\ \boldsymbol{\sigma}_F]$. A noticeable feature of Eqs. (37)-(38) is that withholding taxes have no first-order impact on asset returns second-order moments.

Parameter values. In order to illustrate our results, we will assume symmetric fundamentals, taking the following parameters: $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H,1} = \sigma_{D_F,2} = 0.145$ and

[19] We solve these boundary value problems using Chebychev polynomial approximations.

$\sigma_{D_H,2} = \sigma_{D_F,1} = 0.039$.[20] This calibration is meant to match some dimensions of US stock market data: on an annual basis, the S&P500 volatility after World War II is 0.15 and the dividend yield (equal to ρ in the symmetric case of perfect integration) is around 0.04. Our fundamental correlation η is equal to 0.5, which is consistent with the empirical stock return correlation of 0.58 between the US and a non-US synthetic world index over the period 1980-2000.[21] In what follows, we focus on the impact of τ and fix the state variables at $\delta = 0.5$ and $\lambda = 1$ (symmetric state).

Impacts of financial integration on return volatility and correlation. As illustrated in Figure 1 and Figure 2, we find that return volatility decreases with financial integration, while return correlation increases. In order to understand the impact of the degree of market integration on the equilibrium correlation of returns, a good starting point is to contrast the two polar case of perfect integration and complete segmentation. When markets are completely segmented, a good dividend shock in one country has no impact on the price of assets in another country. However, the story goes differently when investors can hold assets everywhere. The reason is that following the rise in the domestic price induced by a good domestic shock, the share of asset H in the world market portfolio increases, making country F's asset more appealing because the diversification opportunities it offers are suddenly more valuable. Therefore, the required excess return on asset F decreases and its price increases.[22] For a small but positive τ, the same mechanism is at work but dampened due to investors heterogeneity. Indeed, a good shock to D_H affects each investor differently: the home investor is the most affected since his portfolio is biased towards domestic assets; but because he is reluctant to rebalance his portfolio towards taxed foreign assets, the increase in S_F is attenuated compared to the case of perfect integration. The result that stock return correlations between countries increase when cross-border impediments to foreign equity holdings are relaxed is consistent with the empirical findings of Bekaert and Harvey (2000) who show that, following episodes of stock market liberalization in emerging countries, the stock market indices of these countries became more correlated with a global index.

[Figures 1 and 2 here]

Sensitivity analysis. Table 1 shows the magnitude of asset return correlation η_S conditional on three structural parameters: the degree of market integration (inversely related to τ), the level of

[20] This corresponds to $\sigma_D = 0.15$ and to a fundamental correlation $\eta = 0.5$. This calibration allow us to match the moments of stock returns in the US at the expense of the moments observed for the fundamentals. It is well known that the volatility of stock markets is well above the volatility of GDP.

[21] The empirical stock return correlation is calculated using monthly returns in US$ for both indices.

[22] The increase of S_H is also lower than under full segmentation. Note that this reasoning only holds when the market share of H is not "too small" to start with.

fundamental correlation η and the rate of time preference ρ.[23] For given η and ρ, the correlation of asset returns is monotonously decreasing in τ. It is noticeable that the ratio η_S/η decreases with the exogenous level of fundamental correlation: the endogenous component of asset returns comovements becomes relatively less important. This is because when the fundamental correlation is higher, high dividends in one country are often accompanied by high dividends in the other country, reducing the incentives to rebalance the portfolio.

[Table 1 here]

4.3 Risk premia and riskfree rate

Proposition 4. *To the first order, "before-tax" excess returns on assets H and F are*

$$\mu_H - r = \boldsymbol{\sigma}_{H0}.\boldsymbol{\sigma}_D + \tau \frac{\lambda}{1+\lambda} \frac{D_H}{S_{H0}} + o(\tau), \tag{39}$$

$$\mu_F - r = \boldsymbol{\sigma}_{F0}.\boldsymbol{\sigma}_D + \frac{\tau}{1+\lambda} \frac{D_F}{S_{F0}} + o(\tau). \tag{40}$$

Proposition 4 is a modified version of the continuous-time consumption-based CAPM. With logarithmic utility, in the benchmark case without taxes, the vector of expected excess returns for the two assets is $\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_D$: the risk premia are equal to the covariance of asset returns with aggregate consumption growth.[24] The first-order impact of τ is to drive the risk premia above their benchmark level. This is because both assets are partly held by taxed investors who require a higher pre-tax excess return to compensate for taxation.[25] The prediction that an increase in financial markets integration (a decrease in τ) reduces the required excess return is consistent with the empirical evidence (Bekaert and Harvey (2000), Henry (2000), Chari and Henry (2004)). The term in τ that appears in Proposition 4 is interacted with the dividend-price ratio and the relative wealth of countries. This suggests a potential way of testing our international version of the CCAPM, by testing for the significance of a proxy for this term in the pricing equation.

When we go to the second order, two additional effects on the risk premia show up, through the level of asset prices and through asset returns volatilities. First, since dividend-price ratios are

[23] The return correlation decreases with the rate of time preference ρ. When $\tau = 0$ for instance, inspection of Eqs. (31)-(32) shows that the covariance of returns decreases with ρ while return volatility increases with ρ. In the limit case of complete myopia, the portfolio rebalancing behavior that induces extra endogenous comovements between asset prices no longer operates.

[24] A nice feature of our model is that it exhibits time-varying risk premia (cf. Cochrane et al. (2007)). But it obviously does a poor job at matching the observed level of equity premium.

[25] It is straightforward to see that the presence of taxes lowers the "after-tax" risk premia $\mu_{H,F} - r$ and $\mu_{F,H} - r$.

higher under imperfect integration, the effect of the friction on the risk premium is amplified.[26] The decrease in the correlation of stock returns with aggregate output plays in the opposite direction, driving the risk premium down.

Proposition 5. *The second-order approximation of the riskfree rate is*

$$r = \rho + \mu_D - \boldsymbol{\sigma}_D.\boldsymbol{\sigma}_D - \tau^2 \frac{\lambda}{(1+\lambda)^2} \left[-\frac{D_H}{S_{H0}} \; \frac{D_F}{S_{F0}} \right] \left(\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_0 \right)^{-1} \begin{pmatrix} -\frac{D_H}{S_{H0}} \\ \frac{D_F}{S_{F0}} \end{pmatrix} + o(\tau^2). \tag{41}$$

In the fully integrated case ($\tau = 0$), we obtain the standard interest rate formula with logarithmic utility: the riskfree rate is determined by the rate of time preference and by the mean and variance of aggregate consumption growth. When markets are imperfectly integrated, the interest rate is below its level of perfect integration. This can be seen from the fact that $(\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_0)^{-1}$ is definite positive and this is to be interpreted as an effect of higher precautionary savings, due to the fact that risksharing is imperfect.

Total cost of capital. A decrease in τ causes both an increase in the riskless rate and a decrease in the equilibrium excess returns. Therefore, the overall impact of financial integration on the cost of capital is not clear-cut, depending on the relative strength of these two effects. There could be non-monotonous effects.

4.4 Portfolios

We now turn to the extent of international portfolio diversification in our imperfectly integrated financial markets. For that matter, we let $\pi_{ij} \equiv \frac{\alpha_{ij} S_j}{X_i}$ denote the share of equity j in the wealth of investor i.

Proposition 6. *To a first order, portfolio shares are given by*

$$\begin{bmatrix} \pi_{HH} \\ \pi_{HF} \end{bmatrix} = \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_D + \tau \frac{\lambda}{1+\lambda} \left(\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_0 \right)^{-1} \begin{bmatrix} \frac{D_H}{S_{H0}} \\ -\frac{D_F}{S_{F0}} \end{bmatrix} + \boldsymbol{\epsilon}_H + \boldsymbol{o}(\tau), \tag{42}$$

$$\begin{bmatrix} \pi_{FH} \\ \pi_{FF} \end{bmatrix} = \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_D + \tau \frac{1}{1+\lambda} \left(\boldsymbol{\sigma}_0^T \boldsymbol{\sigma}_0 \right)^{-1} \begin{bmatrix} -\frac{D_H}{S_{H0}} \\ \frac{D_F}{S_{F0}} \end{bmatrix} + \boldsymbol{\epsilon}_F + \boldsymbol{o}(\tau). \tag{43}$$

Portfolios can be decomposed into three components. In (42) and (43), the sum of the first two terms approximates $\boldsymbol{\sigma}^{-1} \boldsymbol{\theta}_i = \left(\boldsymbol{\sigma}^T \boldsymbol{\sigma} \right)^{-1} [\boldsymbol{\mu}_i - \boldsymbol{r}]$, i.e., the standard portfolio composition of a

[26] Basak and Gallmeyer (2003) also note that asymmetric taxation impairs risksharing, which causes an increase in the risk premium.

logarithmic investor with financial wealth only. The first term, $\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_D$, is the portfolio held by both investors when $\tau = 0$. For an investor in country H, τ reduces the demand for foreign stocks by reducing after-tax expected returns on these stocks. Symmetrically, due to market clearing, τ increases the domestic demand for domestic shares to compensate for the lower demand by foreign investors.[27] The third term $\boldsymbol{\epsilon}_i$ comes from the redistribution of taxes: for instance, if e_H is positively correlated with D_H, this will create a demand for foreign shares in order to hedge this additional income risk. In the Appendix, we derive a first-order approximation for this term in the case where $e_i = \tau\alpha_{ji}D_i$. But because this hedging component depends very much on the assumed redistribution scheme and is small when the two countries are not too asymmetric, we neglect the effect of this term in the following expressions.

Let the elements of the variance-covariance matrix of stock returns be denoted as

$$\boldsymbol{\sigma}^T\boldsymbol{\sigma} \equiv \begin{pmatrix} \sigma_H^2 & \eta_S\sigma_H\sigma_F \\ \eta_S\sigma_H\sigma_F & \sigma_F^2 \end{pmatrix}.$$

From (42), the portfolio shares of a domestic investor approximately satisfy

$$\pi_{HH} \simeq \frac{S_{H0}}{S_{H0}+S_{F0}} + \tau\frac{\eta_S}{1-\eta_S^2}\frac{\lambda}{1+\lambda}\frac{1}{\sigma_H\sigma_F}\frac{D_F}{S_F} + \frac{\tau}{1-\eta_S^2}\frac{\lambda}{1+\lambda}\frac{1}{\sigma_H^2}\frac{D_H}{S_H}, \tag{44}$$

$$\pi_{HF} \simeq \frac{S_{F0}}{S_{H0}+S_{F0}} - \frac{\tau}{1-\eta_S^2}\frac{\lambda}{1+\lambda}\frac{1}{\sigma_F^2}\frac{D_F}{S_F} - \tau\frac{\eta_S}{1-\eta_S^2}\frac{\lambda}{1+\lambda}\frac{1}{\sigma_H\sigma_F}\frac{D_H}{S_H}. \tag{45}$$

These expressions show explicitly how portfolios deviate from the frictionless world market portfolio. The impact of the friction on portfolios goes through expected returns, both directly and indirectly *via* market clearing. The size of the bias in portfolios is inversely related to $(1-\eta_S^2)$: when assets are closer substitutes, the effect of the friction on equity holdings is amplified.

Comparative statics in a simple symmetric case. In the symmetric case where $\mu_{D_H} = \mu_{D_F}$ and $\sigma_H = \sigma_F \equiv \sigma$, when $\delta = \frac{1}{2}$, dividend yields under perfect integration are equal to ρ, so that

$$\pi_{HH} \simeq \frac{1}{2} + \tau\frac{\lambda}{1+\lambda}\frac{\rho}{\sigma^2(1-\eta_S)} \quad \text{and} \quad \pi_{HF} \simeq \frac{1}{2} - \tau\frac{\lambda}{1+\lambda}\frac{\rho}{\sigma^2(1-\eta_S)}.$$

We can derive a number of comparative statics:

$$\frac{\partial\pi_{HF}}{\partial\tau} = -\frac{\lambda}{1+\lambda}\frac{\rho}{\sigma^2(1-\eta_S)} < 0, \tag{46}$$

$$\frac{\partial\pi_{HF}}{\partial\eta_S} = -\tau\frac{\lambda}{1+\lambda}\frac{\rho}{\sigma^2(1-\eta_S)^2} < 0, \tag{47}$$

[27] This general equilibrium effect is relevant empirically. Chan et al. (2005) find that countries imposing high withholding taxes to foreign shareholders exhibit a higher home bias.

and $\left|\frac{\partial \pi_{HF}}{\partial \tau}\right|$ is increasing in η_S. These expressions capture the impact of frictions, assets substitutability and the interaction of the two on the extent of portfolio diversification. When investments are riskier (higher σ), the motive for risksharing increases and portfolios are more diversified:

$$\frac{\partial \pi_{HF}}{\partial \sigma^2} = \tau \frac{\lambda}{1+\lambda} \frac{\rho}{\sigma^4(1-\eta_S)} > 0. \tag{48}$$

Finally, we get

$$\frac{\partial \pi_{HH}}{\partial \lambda} = \tau \frac{1}{(1+\lambda)^2} \frac{\rho}{\sigma^2(1-\eta_S)^2} > 0. \tag{49}$$

A high λ means the relative wealth of foreign investors is high, which strengthens their influence in the pricing of assets and increases the negative impact of the friction on the price of the domestic asset. As a consequence, the larger λ, the lower the price of the domestic asset and the higher the incentive for domestic investors to stay invested domestically.[28]

Level of home bias. For symmetric fundamentals and in the symmetric state $\delta = 0.5$ and $\lambda = 1$, Figure 3 illustrates the share of wealth invested abroad as a function of τ and as a function of the fundamental correlation η, taking into account the endogeneity of stock returns first and second moments. For $\tau = 10\%$ and $\eta = 0.65$, we obtain $\pi_{HF} = 20\%$: a reasonable level of friction on cross-border equity holdings, coupled with a high level of return correlation, can generate a realistic level of domestic exposure of equity portfolios. In Section 5, we show that, in a setting with two differentiated goods, one can generate high return correlation (and therefore a substantial level of home bias) for a lower value of the fundamental correlation η. Note that our computation also hinges on a low level of risk aversion, which implies a rather high elasticity of asset demands to expected returns.[29]

[Figure 3 here]

A gravity equation for bilateral equity holdings. Our model can also be used to give some theoretical ground to the use of gravity equations in empirical work on bilateral equity holdings.

[28]This prediction of our model that the home bias in portfolios should be larger in countries whose relative wealth is smaller is consistent with scarce evidence in Chan *et al.* (2005). The lowest three values taken by their measure of home bias are for US, UK and Japan, and the highest four are for New Zealand, Norway, Portugal and Greece.

[29]Assuming power utility with relative risk aversion higher than one would have two effects: (i) for given return correlation, a higher risk aversion would imply more portfolio diversification; (ii) at the same time, decreasing the elasticity of intertemporal substitution would increase return correlation by inducing more common discount factor shocks (Dumas, Harvey and Ruiz (2003)). The latter effect would dampen the impact of higher risk aversion on the extent of portfolio diversification.

Indeed, when we turn from portfolio shares to the value of equity holdings, we have :

$$\log(\alpha_{HF}S_F) = \log X_H + \log(\rho y_F(\delta)) - \tau \frac{1}{1-\eta_S^2}\frac{\lambda}{1+\lambda}\frac{S_H+S_F}{\sigma_F S_F}\left(\frac{1}{\sigma_F}\frac{D_F}{S_F} + \eta_S\frac{1}{\sigma_H}\frac{D_H}{S_H}\right), \tag{50}$$

where the first two terms are the mass terms in the gravity equation.[30] As shown by Portes and Rey (2005), gravity equations give a good description of patterns of international asset holdings. In their work, they use the market capitalizations of origin and destination countries as proxies for the mass terms of the equation. Our model clarifies which variables should be used: for the origin country, one should use the aggregate wealth, whereas for the destination country, market capitalization can be used as a proxy for the present value of current and future foreign dividend streams. Moreover, Portes and Rey (2005) propose to interact variables capturing financial frictions between countries with the degree of substitutability between assets, which is measured here by $\left(1-\eta_S^2\right)^{-1}$. Our model provides a theoretical foundation for this procedure.

5 Two-good extension

In this section, we extend our analysis to the case where the goods produced in each country are imperfect substitutes. All the findings of Section 4 go through. Indeed, we show that the two-good model is isomorphic to the case of perfect substitutability. However, this specification has the potential to generate realistic portfolio predictions without requiring too high cross-country correlations between endowments. When goods are imperfect substitutes, relative prices depend on relative quantities: the relative price of a good is positively related to its relative scarcity, so that a positive output shock in one country is accompanied by a counteracting relative price change. This "terms of trade effect"makes asset cash flows and asset prices evolutions more synchronized. This mechanism is emphasized in Pavlova and Rigobon (2007).

We assume that each country produces one (tradable) good and the representative agent in each country consumes both goods. Endowments in each country, D_H and D_F, follow geometric Brownian motions, as specified in (2). The two representative agents have the same log-CES preferences. Let $c_{ij,t}$ denote agent i's consumption of goods from country j at date t. Agent i's consumption aggregate at time t is given by

$$C_{it} = \left[c_{ii,t}^{\frac{\phi-1}{\phi}} + c_{ij,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}. \tag{51}$$

[30] $y_F(\delta) = \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\left(1-\delta(s)\right)ds\right]$ captures the present value of country F's contribution to world output.

The parameter $\phi > 0$ denotes the elasticity of substitution between Home and Foreign goods ($\phi = \infty$ corresponds to the one-good case). Agent i's utility at time t is

$$U_{it} = \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(s-t)} \log(C_{is}) ds \right]. \tag{52}$$

Let $p_{H,t}$ and $p_{F,t}$ denote the prices of the two goods, normalized by taking the consumption index as numeraire:

$$\left[p_{H,t}^{1-\phi} + p_{F,t}^{1-\phi} \right]^{\frac{1}{1-\phi}} = 1. \tag{53}$$

Optimal (intra-temporal) consumption allocation implies

$$c_{ij} = p_j^{-\phi} C_i, \qquad \text{for all } (i,j). \tag{54}$$

Resource constraints for the two goods are:

$$c_{Hj} + c_{Fj} = D_j, \quad j = H, F. \tag{55}$$

Eqs. (54) and (55) pin down the terms of trade as a function of relative quantities. The ratio of outputs evaluated at market prices (i.e., the ratio of dividend cash-flows paid by the two risky assets) is

$$\frac{p_H(t) D_H(t)}{p_F(t) D_F(t)} = \left(\frac{D_H(t)}{D_F(t)} \right)^{\frac{\phi-1}{\phi}}. \tag{56}$$

The strength of the terms of trade effect increases as goods become less substitutable. For an elasticity of substitution below one, the effect is so strong that, following a good domestic shock, the cash-flows of domestic assets are lower than the ones of foreign assets. In the special case of an elasticity of substitution equal to one (i.e., Cobb-Douglas preferences), the change in relative prices exactly compensates the change in relative quantities, so that cash flows on domestic and foreign assets are perfectly correlated. In that case, portfolios are indeterminate (Cole and Obstfeld (1991)). We assume $\phi \neq 1$ in our analysis.

We redefine the state variable D as the world endowment in the composite good:

$$D(t) \equiv \left[D_{H,t}^{(\phi-1)/\phi} + D_{F,t}^{(\phi-1)/\phi} \right]^{\phi/(\phi-1)}. \tag{57}$$

Eqs. (54) and (55) imply that, in equilibrium, $C_H(t) + C_F(t) = D(t)$. Let $\lambda(t)$ denote the relative weight of agent F in consumption, so that

$$C_H(t) = \frac{1}{1+\lambda(t)} D(t) \qquad \text{and} \qquad C_F(t) = \frac{\lambda(t)}{1+\lambda(t)} D(t). \tag{58}$$

We redefine the state variable δ as $\delta(t) \equiv \frac{p_H(t) D_H(t)}{D(t)}$. In equilibrium, $p_H D_H + p_F D_F = D$. Together with (56), this implies

$$\delta(t) = \frac{1}{1 + (D_F(t)/D_H(t))^{\frac{\phi-1}{\phi}}}. \tag{59}$$

Lemma 3. *In the case of perfect integration with two differentiated goods, asset prices can be expressed as in Cochrane, Longstaff and Santa-Clara (2007):*

$$S_{i0}(t) = D(t) y_i(\delta(t)), \quad i = H, F, \tag{60}$$

for newly defined hypergeometric functions y_H and y_F (see Appendix for the definition of the parameters of y_H and y_F as a function of ϕ).

From Lemma 3, we can derive the stochastic properties of asset returns for $\tau = 0$. Figures 4 and 5 show return volatility and correlation in the symmetric case as a function of the elasticity of substitution ϕ (for orthogonal fundamentals, keeping the same volatility as before). In particular, due to the terms of trade adjustment, stock return correlations can be arbitrarily high for low enough values of ϕ, despite zero fundamental correlation.[31]

[Figures 4 and 5 here]

Proposition 7. *For $\tau > 0$, the approximate expressions of Section 4 for asset prices and portfolios remain valid, for redefined state variables D and δ and pricing functions y_H and y_F.*

Due to the isomorphy with the case of perfect substitutability, portfolios can be described as in Proposition 6. The key difference with the one-good case is that for small positive values of τ a reasonable level of home bias can be obtained without requiring an unrealistically high level of fundamental correlation. Figure 6 shows the level of foreign exposure increasing as a function of the elasticity of substitution, under the assumption that fundamentals are uncorrelated. For reasonable values of ϕ and τ, portfolios exhibit a very substantial degree of home bias.

[31] Standard estimates of ϕ in the international real business cycle literature are between 0.5 and 2.5 (see for instance Backus, Kehoe and Kydland (1994) and Heathcote and Perri (2002))

[Figure 6 here]

6 Conclusion

Thanks to an original application of approximation techniques, we provided a complete description of equilibrium asset prices and holdings in a dynamic model of imperfectly integrated stock markets. We characterized the effect of integration (understood as a decrease in τ) on asset prices. We showed how the CCAPM is modified relative to the fully-integrated case and how the impact of integration on the cost of capital depends on the respective size of opposite effects on the riskless rate and on the risk premium. We exhibited a second-order effect of integration on asset returns second moments, driven by the fact that impediments to cross-border equity holdings prevent risksharing and make pricing kernels more volatile and less synchronized. In Section 5, we showed that higher return correlation could be obtained, for a given level of fundamental correlation, by decreasing the elasticity of substitution between home and foreign goods, leading to a more pronounced home bias for a given level of financial friction. We find our setup appealing as it is all at once parsimonious and able to account for various dimensions of the data. When attempting to assess the degree of integration empirically, one gets a different impression by looking at different variables: domestic biases in portfolios point to segmentation, whereas institutional measures of financial openness and large capital flows point to a high degree of integration. Our model shades light on the different facets of financial integration.

FIGURES AND TABLE

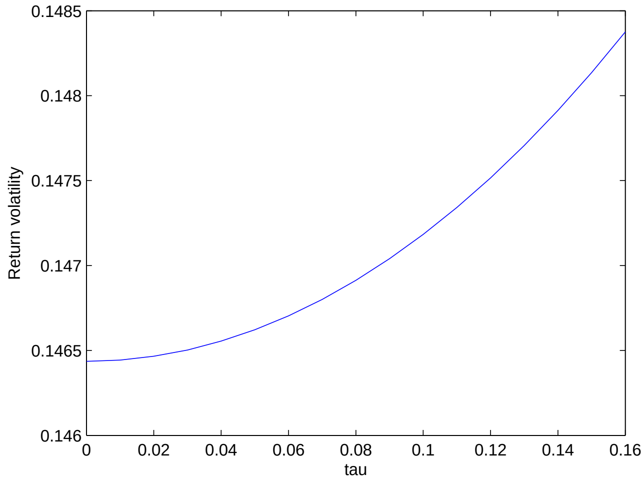

Figure 1: Stock return volatility as a function of τ when $\delta = 0.5$ and $\lambda = 1$ (calibration : $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H,1} = \sigma_{D_F,2} = 0.145$,$\sigma_{D_H,2} = \sigma_{D_F,1} = 0.039$).

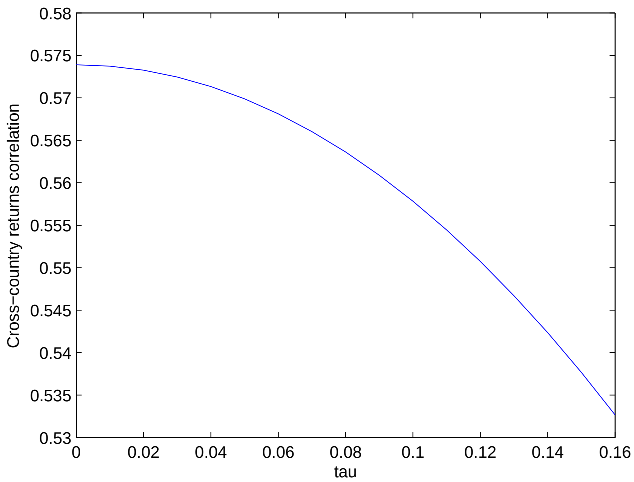

Figure 2: Stock return correlation as a function of τ when $\delta = 0.5$ and $\lambda = 1$ (calibration : $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H,1} = \sigma_{D_F,2} = 0.145$,$\sigma_{D_H,2} = \sigma_{D_F,1} = 0.039$).

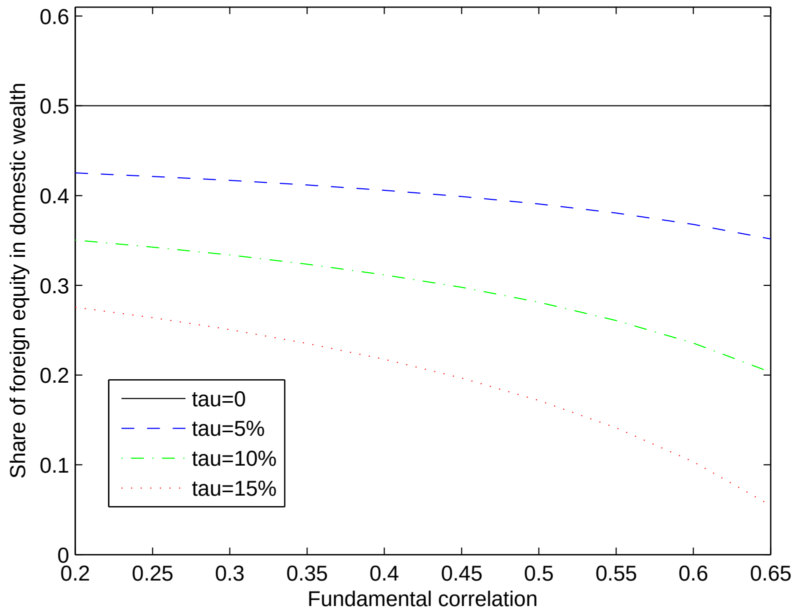

Figure 3: Share of domestic wealth invested abroad as a function of fundamental correlation, for various τ, when $\delta = 0.5$ and $\lambda = 1$ (calibration: $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\sigma_{D_H} = \sigma_{D_F} = 0.15$)

		η_S		
		$\rho = 0.1$	$\rho = 0.05$	$\rho = 0.01$
$\eta = 0$	$\tau = 0$	0.086	0.147	0.394
	$\tau = 5\%$	0.080	0.143	0.393
	$\tau = 10\%$	0.060	0.130	0.391
$\eta = 0.25$	$\tau = 0$	0.313	0.358	0.544
	$\tau = 5\%$	0.305	0.353	0.542
	$\tau = 10\%$	0.282	0.340	0.538
$\eta = 0.5$	$\tau = 0$	0.535	0.562	0.679
	$\tau = 5\%$	0.526	0.557	0.677
	$\tau = 10\%$	0.501	0.543	0.672

Table 1: Stock return correlation η_S as a function of the fundamental correlation η and τ when $\delta = 0.5$ and $\lambda = 1$ (fundamental volatility $= 0.15$).

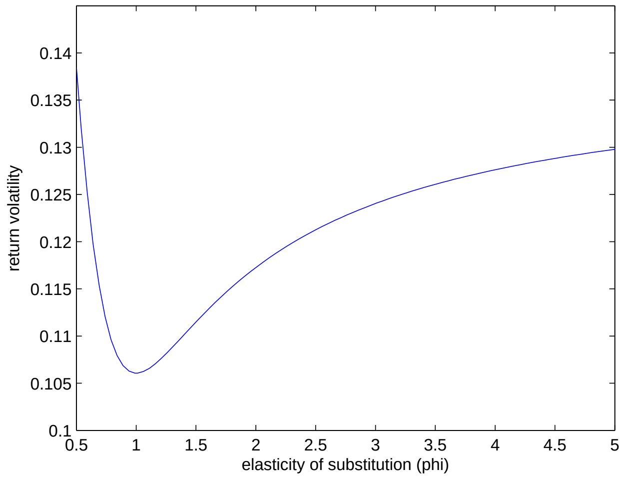

Figure 4: Stock return volatility as a function of the elasticity of substitution ϕ when $\tau = 0$ and $\delta = 0.5$ (calibration : $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\eta = 0$, fundamental volatility $= 0.15$).

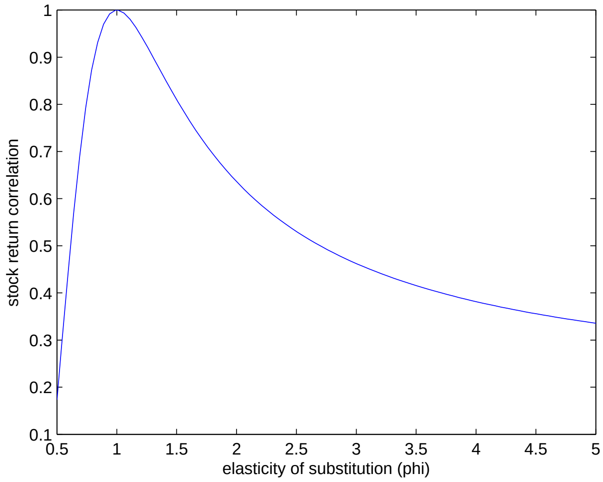

Figure 5: Stock return correlation as a function of the elasticity of substitution ϕ when $\tau = 0$ and $\delta = 0.5$ (calibration : $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\eta = 0$, fundamental volatility $= 0.15$).

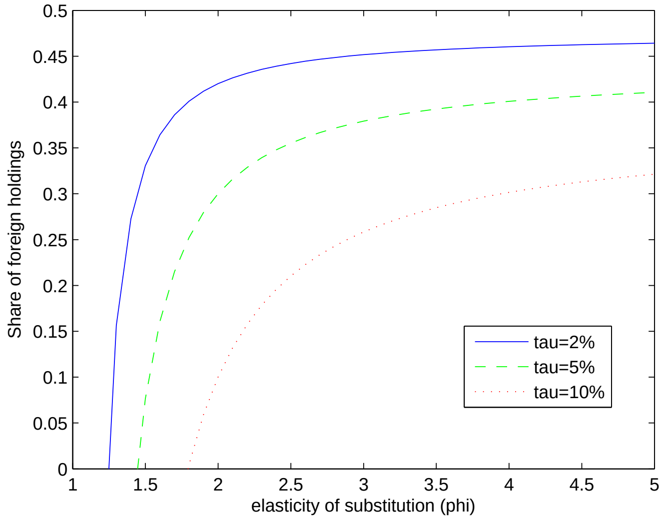

Figure 6: Share of domestic wealth invested abroad as a function of the goods elasticity of substitution, for three values of τ, when $\delta = 0.5$ and $\lambda = 1$ (calibration: $\rho = 0.04$, $\mu_{D_H} = \mu_{D_F} = 0.025$, $\eta = 0$, fundamental volatility 0.15)

APPENDIX

A Hypergeometric function

Cochrane et al. (2007) show that

$$\begin{aligned} y_H(\delta) &\equiv \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\delta(s)ds\bigg|\,\delta(t)=\delta\right] \\ &= \frac{1}{\psi(1-\gamma)}\left(\frac{\delta}{1-\delta}\right)F\left(1,1-\gamma;2-\gamma;\frac{\delta}{\delta-1}\right)+\frac{1}{\psi\theta}F\left(1,\theta;1+\theta;\frac{\delta-1}{\delta}\right), \end{aligned}$$

with F the standard (2,1)-hypergeometric function and

$$\begin{aligned} \psi &= \sqrt{\nu^2+2\rho\chi^2}, \\ \gamma &= \frac{\nu-\psi}{\chi^2}, \\ \theta &= \frac{\nu+\psi}{\chi^2}, \end{aligned}$$

where

$$\nu = \mu_{D_F}-\mu_{D_H}-\frac{\sigma^2_{D_{F,1}}+\sigma^2_{D_{F,2}}}{2}+\frac{\sigma^2_{D_{H,1}}+\sigma^2_{D_{H,2}}}{2}, \tag{A-1}$$

$$\chi^2 = \left(\sigma^2_{D_{H,1}}+\sigma^2_{D_{H,2}}\right)+\left(\sigma^2_{D_{F,1}}+\sigma^2_{D_{F,2}}\right)-2(\sigma_{D_{H,1}}\sigma_{D_{F,1}}+\sigma_{D_{H,2}}\sigma_{D_{F,2}}). \tag{A-2}$$

In the same way

$$\begin{aligned} y_F(\delta) &\equiv \mathbb{E}_t\left[\int_0^\infty e^{-\rho(s-t)}\left(1-\delta(s)\right)ds\bigg|\,\delta(t)=\delta\right] \\ &= \frac{1}{\psi(1+\theta)}\left(\frac{1-\delta}{\delta}\right)F\left(1,1+\theta;2+\theta;\frac{\delta-1}{\delta}\right)-\frac{1}{\psi\gamma}F\left(1,-\gamma;1-\gamma;\frac{\delta}{\delta-1}\right). \end{aligned}$$

B Two useful results

Lemma B-1. *The functions h and f defined in Section 3.3 are solutions of the following PDE's*

$$\rho h = (1+\lambda)\,\delta+\delta\mu_\delta h_\delta+\lambda\mu_\lambda h_\lambda+\tfrac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)h_{\delta\delta}+\tfrac{1}{2}\lambda^2(\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda)h_{\lambda\lambda}+\delta\lambda(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\lambda)h_{\delta\lambda}, \tag{B-1}$$

$$\rho f = \tfrac{1+\lambda}{\lambda}(1-\delta)+\delta\mu_\delta f_\delta+\lambda\mu_\lambda f_\lambda+\tfrac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)f_{\delta\delta}+\tfrac{1}{2}\lambda^2(\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda)f_{\lambda\lambda}+\delta\lambda(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\lambda)f_{\delta\lambda}, \tag{B-2}$$

with μ_δ, σ_δ, μ_λ and σ_λ defined in (5)-(6) and (27)-(28).

Proof: We apply Feynmac-Kac formula to h and f. ∎

Lemma B-2. *$\boldsymbol{\sigma}_H$ and $\boldsymbol{\sigma}_F$ must verify*

$$h\boldsymbol{\sigma}_H = h\boldsymbol{\sigma}_D + \lambda\left(h_\lambda - \frac{h}{1+\lambda}\right)\boldsymbol{\sigma}_\lambda + \delta h_\delta \boldsymbol{\sigma}_\delta, \tag{B-3}$$

$$f\boldsymbol{\sigma}_F = f\boldsymbol{\sigma}_D + \lambda\left(f_\lambda + \frac{f}{\lambda(1+\lambda)}\right)\boldsymbol{\sigma}_\lambda + \delta f_\delta \boldsymbol{\sigma}_\delta. \tag{B-4}$$

Proof: We apply Itô's lemma to $S_H(t) = \frac{D(t)}{1+\lambda(t)} h(\delta(t), \lambda(t))$ and $S_F(t) = D(t)\frac{\lambda(t)}{1+\lambda(t)} f(\delta(t), \lambda(t))$, and we identify the diffusion terms. ∎

C Proofs

Proof of Lemma 1: We use the stochastic discount factor of investor H to price asset H and respectively for asset F. We obtain:

$$\begin{aligned} S_H(t) &= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{c_H(t)}{c_H(s)}D_H(s)ds\right] = \frac{D(t)}{1+\lambda(t)}\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\left[1+\lambda(s)\right]\delta(s)ds\right], \\ S_F(t) &= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{c_F(t)}{c_F(s)}D_F(s)ds\right] = \frac{\lambda(t)D(t)}{1+\lambda(t)}\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{1+\lambda(s)}{\lambda(s)}(1-\delta(s))ds\right]. \end{aligned}$$

The conditional expectations that appear in these two equations can be written as two functions h and f of $\delta(t)$ and $\lambda(t)$. ∎

Proof of Lemma 2: The outline of the proof is as follows: start from first-order condition (17) and apply Itô's lemma to identify individual consumption volatility. Then use market clearing for goods to identify aggregate consumption volatility and derive an equilibrium condition on the market prices of risk.

Applying Itô's lemma on both sides of (17) implies

$$-\rho e^{-\rho t}\frac{1}{c_i(t)}dt - e^{-\rho t}\frac{1}{c_i(t)^2}dc_i + e^{-\rho t}\frac{1}{c_i(t)^3}dc_i^2 = -\Psi_i\xi_i(t)[r(t)dt + \boldsymbol{\theta}_i^T(t)d\boldsymbol{W}(t)]. \tag{C-1}$$

We define μ_{c_i} and $\boldsymbol{\sigma}_{c_i}$ such that

$$\frac{dc_i}{c_i} = \mu_{c_i}dt + \boldsymbol{\sigma}_{c_i}^T d\boldsymbol{W}, \qquad i = H, F. \tag{C-2}$$

Identifying diffusion terms in (C-1) implies

$$-e^{-\rho t}\frac{1}{c_i(t)}\boldsymbol{\sigma}_{c_i}(t) = -\Psi_H \xi_i(t)\boldsymbol{\theta}_i(t) \tag{C-3}$$

$$\Rightarrow -e^{-\rho t}\frac{1}{c_i(t)}\boldsymbol{\sigma}_{c_i}(t) = -e^{-\rho t}\frac{1}{c_i(t)}\boldsymbol{\theta}_i(t) \quad \text{(using Eq. (17))} \tag{C-4}$$

$$\Rightarrow \boldsymbol{\sigma}_{c_i}(t) = \boldsymbol{\theta}_i(t), \qquad i = H, F. \tag{C-5}$$

Market clearing ($c_H + c_F = D$) implies

$$c_H \boldsymbol{\sigma}_{c_H} + c_F \boldsymbol{\sigma}_{c_F} = D\boldsymbol{\sigma}_D \tag{C-6}$$

$$\Rightarrow c_H(t)\boldsymbol{\theta}_H(t) + c_F(t)\boldsymbol{\theta}_F(t) = D(t)\left[\delta(t)\boldsymbol{\sigma}_{D_H} + (1-\delta(t))\boldsymbol{\sigma}_{D_F}\right]. \tag{C-7}$$

Solving Eqs. (15) and (C-7) for $\boldsymbol{\theta}_H$ and $\boldsymbol{\theta}_F$, we obtain the expressions given in Lemma 2. ∎

Proof of Proposition 1: From $d\lambda/\lambda = \mu_\lambda dt + \boldsymbol{\sigma}_\lambda^T d\boldsymbol{W}$, we obtain for $s > t$

$$\lambda(s) = \lambda(t)\exp\left\{\int_t^s \left[\mu_\lambda - \frac{1}{2}\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda\right]du + \int_t^s \boldsymbol{\sigma}_\lambda^T d\boldsymbol{W}_u\right\}.$$

Let $\boldsymbol{\Gamma}(\delta) \equiv (\boldsymbol{\sigma}_0^T)^{-1}\left[-\frac{D_H}{S_{H0}} \quad \frac{D_F}{S_{F0}}\right]^T$. This two-dimensional vector can be obtained as a function of δ using the hypergeometric function. We can rewrite (29) and (30) as

$$\begin{aligned}\mu_\lambda &= \tau\boldsymbol{\Gamma}(\delta).\boldsymbol{\sigma}_D(\delta) + o(\tau),\\ \boldsymbol{\sigma}_\lambda &= \tau\boldsymbol{\Gamma}(\delta) + o(\tau).\end{aligned}$$

Lemma C-1. *The first-order approximation of μ_λ can be written as $\mu_\lambda = \tau\rho(1-2\delta) + o(\tau)$.*

Proof: From the definition of $\boldsymbol{\Gamma}$, we have

$$\boldsymbol{\Gamma}.\boldsymbol{\sigma}_D = \left[-\left(\frac{D_H}{S_H}\right)_0 \quad \left(\frac{D_F}{S_F}\right)_0\right]\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_D.$$

The vector $\boldsymbol{\sigma}_0^{-1}\boldsymbol{\sigma}_D$ is exactly the vector of stock holdings of a representative agent in an equilibrium without frictions, which in turn must be equal to the market portfolio. Therefore:

$$\boldsymbol{\Gamma}.\boldsymbol{\sigma}_D = \left[-\left(\frac{D_H}{S_H}\right)_0 \quad \left(\frac{D_F}{S_F}\right)_0\right]\left[\begin{array}{c}\left(\frac{S_H}{S_H+S_F}\right)_0\\ \left(\frac{S_F}{S_H+S_F}\right)_0\end{array}\right].$$

Then, using $(S_H + S_F)_0 = (X_H + X_F)_0 = \frac{D}{\rho}$, we get $\mathbf{\Gamma}.\boldsymbol{\sigma}_D = \rho(1-2\delta)$. $\square$

Therefore, introducing $g(\delta) = \rho(1-2\delta)$, we can write

$$
\begin{aligned}
\lambda(s) &= \lambda(t)\exp\left\{\tau\left[\int_t^s g(\delta_u)du + \int_t^s \mathbf{\Gamma}(\delta_u).d\boldsymbol{W}_u\right] + o(\tau)\right\} \\
\Rightarrow \lambda(s) &= \lambda(t)\left[1 + \tau\int_t^s g(\delta_u)du + \tau\int_t^s \mathbf{\Gamma}(\delta_u).d\boldsymbol{W}_u\right] + o(\tau) \\
\Rightarrow h(\delta(t),\lambda(t)) &= \mathbb{E}_t\left\{\int_t^\infty e^{-\rho(s-t)}\left[1 + \lambda(t) + \tau\lambda(t)\int_t^s g(\delta_u)du + \tau\lambda(t)\int_t^s \mathbf{\Gamma}(\delta_u).d\boldsymbol{W}_u + o(\tau)\right]\delta_s ds\right\} \\
&= (1+\lambda(t))\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\delta_s ds\right] - \tau\lambda(t)H(\delta(t)) + o(\tau) \\
&= (1+\lambda(t))y_H(\delta(t)) - \tau\lambda(t)H(\delta(t)) + o(\tau), \qquad \text{(C-8)}
\end{aligned}
$$

with $H(t) \equiv -\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\left[\int_t^s g(\delta_u)du + \int_t^s \mathbf{\Gamma}(\delta_u).d\boldsymbol{W}_u\right]\delta_s ds\right]$. Therefore,

$$
S_H(D_t,\delta_t,\lambda_t;\tau) = D_t\left[y_H(\delta_t) - \tau\frac{\lambda_t}{1+\lambda_t}H(\delta_t)\right] + o(\tau).
$$

In the same way, we get S_F as

$$
S_F(D_t,\delta_t,\lambda_t;\tau) = D_t\left[y_F(\delta_t) - \tau\frac{1}{1+\lambda_t}F(\delta_t)\right] + o(\tau).
$$

Lemma C-2. *The functions H and F must satisfy the following boundary value problems*

$$
\begin{aligned}
\rho H - \delta\mu_\delta H' - \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)H'' &= \delta \\
H(0) &= 0 \\
H(1) &= 1/\rho
\end{aligned}
$$

$$
\begin{aligned}
\rho F - \delta\mu_\delta F' - \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)F'' &= 1-\delta \\
F(0) &= 1/\rho \\
F(1) &= 0.
\end{aligned}
$$

Proof: We can substitute (C-8) in Eq. (B-1). At the same time, Feynmac-Kac formula applied to y_H implies $\rho y_H = \delta + \delta\mu_\delta y_H' + \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)y_H''$. By difference, we obtain:

$$
\rho H(\delta) - \delta\mu_\delta H'(\delta) - \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)H''(\delta) = -\rho(1-2\delta)y_H(\delta) - \delta\left(\boldsymbol{\sigma}_\delta(\delta).\mathbf{\Gamma}(\delta)\right)y_H'(\delta).
$$

The first boundary condition follows from the fact that given the nature of the dividend process

$$S_H(D, 0, \lambda) = 0.$$

The second boundary condition follows from the fact that

$$\lim_{\lambda \to \infty} S_H(D, 1, \lambda) = \frac{(1-\tau)D}{\rho}.$$

Indeed, when δ goes to 1 and λ goes to infinity, the economy tends to an economy with one tree only ($D = D_H$) and one investor located in the foreign country, thus facing an after-tax dividend stream $(1-\tau)D$. In the same way, we characterize the foreign asset price through a function F solution of the PDE

$$\rho F - \delta \mu_\delta F' - \frac{1}{2}\delta^2 (\boldsymbol{\sigma}_\delta . \boldsymbol{\sigma}_\delta) F'' = \rho(1-2\delta) y_F + (\boldsymbol{\sigma}_\delta . \mathbf{\Gamma})\, y_F',$$

with analogous boundary conditions.

We now prove that the non-homogenous terms in the above PDEs can be rewritten as

$$\begin{aligned} -\rho(1-2\delta) y_H - \delta\,(\boldsymbol{\sigma}_\delta .\, \mathbf{\Gamma})\, y_H' &= \delta, \\ \rho(1-2\delta) y_F + \delta\,(\boldsymbol{\sigma}_\delta .\, \mathbf{\Gamma})\, y_F' &= 1-\delta. \end{aligned}$$

We start from (31)-(32) to write

$$\begin{aligned} \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_{H0} &= \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_D + \delta \frac{y_H'}{y_H} \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_\delta \\ &= \begin{bmatrix} \left(\frac{S_H}{S_H+S_F}\right)_0 \\ \left(\frac{S_F}{S_H+S_F}\right)_0 \end{bmatrix} + \delta \frac{y_H'}{y_H} \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_\delta, \end{aligned}$$

where the second equality follows from the fact that in the equilibrium without frictions $\boldsymbol{\sigma}_0{}^{-1} \boldsymbol{\sigma}_D$ is exactly the vector of stock holdings of a representative agent, which must be equal to the market portfolio. Symmetrically,

$$\boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_{F0} = \begin{bmatrix} \left(\frac{S_H}{S_H+S_F}\right)_0 \\ \left(\frac{S_F}{S_H+S_F}\right)_0 \end{bmatrix} + \delta \frac{y_F'}{y_F} \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_\delta.$$

Then, since $\boldsymbol{\sigma}_0 = [\boldsymbol{\sigma}_{H0}\ \boldsymbol{\sigma}_{F0}]$, we have

$$\boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_0 = \begin{bmatrix} \left(\frac{S_H}{S_H+S_F}\right)_0 & \left(\frac{S_H}{S_H+S_F}\right)_0 \\ \left(\frac{S_F}{S_H+S_F}\right)_0 & \left(\frac{S_F}{S_H+S_F}\right)_0 \end{bmatrix} + \begin{bmatrix} \delta \frac{y_H'}{y_H} \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_\delta & \delta \frac{y_F'}{y_F} \boldsymbol{\sigma}_0^{-1} \boldsymbol{\sigma}_\delta \end{bmatrix} = \boldsymbol{I}_2.$$

Pre-multiplying both sides by $\begin{bmatrix} -\frac{D_H}{S_{H0}} & \frac{D_F}{S_{F0}} \end{bmatrix}$ and using the definition of $\mathbf{\Gamma}$, we get

$$\begin{aligned}
\begin{bmatrix} \rho(1-2\delta) & \rho(1-2\delta) \end{bmatrix} + \begin{bmatrix} \delta\left(\boldsymbol{\sigma}_\delta.\mathbf{\Gamma}\right)\frac{y'_H}{y_H} & \delta\left(\boldsymbol{\sigma}_\delta.\mathbf{\Gamma}\right)\frac{y'_F}{y_F} \end{bmatrix} &= \begin{bmatrix} -\frac{D_H}{S_{H0}} & \frac{D_F}{S_{F0}} \end{bmatrix} \\
\Rightarrow \begin{bmatrix} \rho(1-2\delta)y_H + \delta\left(\boldsymbol{\sigma}_\delta.\mathbf{\Gamma}\right)y'_H & \rho(1-2\delta)y_F + \delta\left(\boldsymbol{\sigma}_\delta.\mathbf{\Gamma}\right)y'_F \end{bmatrix} &= \begin{bmatrix} -\frac{D_H}{S_{H0}}y_H & \frac{D_F}{S_{F0}}y_F \end{bmatrix} \\
&= \begin{bmatrix} -\delta & 1-\delta \end{bmatrix}.
\end{aligned}$$

□

It is immediate that y_H and y_F are solutions of the boundary value problems stated in Lemma C-2. Therefore,

$$\begin{aligned}
S_H(D,\delta,\lambda;\tau) &= D_t\left[1-\tau\frac{\lambda}{1+\lambda}\right]y_H(\delta)+o(\tau), \\
S_F(D,\delta,\lambda;\tau) &= D_t\left[1-\tau\frac{1}{1+\lambda}\right]y_F(\delta)+o(\tau).
\end{aligned}$$

∎

Proof of Proposition 2: Let $\mathbf{\Omega} \equiv \frac{1}{1+\lambda}\left(\boldsymbol{\sigma}_0^T\right)^{-1}\begin{bmatrix} -\frac{D_H}{S_{H0}} & \frac{D_F}{S_{F0}} \end{bmatrix}^T$. We first show that

$$\mu_\lambda = \tau\rho(1-2\delta)+\tau^2\left[\frac{\rho}{1+\lambda}-\rho\delta+\frac{1}{1+\lambda}\left(\mathbf{\Gamma}.\mathbf{\Gamma}\right)\right]+o(\tau^2), \tag{C-9}$$

$$\boldsymbol{\sigma}_\lambda = \tau\mathbf{\Gamma}+\tau^2\mathbf{\Omega}+o(\tau^2). \tag{C-10}$$

First-order approximations for dividend yields are

$$\begin{aligned}
\frac{D_H}{S_H}=\frac{D_H}{D}\frac{D}{S_H} &= \frac{D_H}{D}\frac{1}{\left(1-\tau\frac{\lambda}{1+\lambda}\right)y_H(\delta)+o(\tau)} \\
&= \frac{D_H}{D}\frac{1}{y_H(\delta)}\frac{1}{1-\tau\frac{\lambda}{1+\lambda}+o(\tau)} \\
&= \frac{D_H}{Dy_H(\delta)}\left(1+\tau\frac{\lambda}{1+\lambda}+o(\tau)\right) \\
&= \frac{D_H}{S_{H0}}\left(1+\tau\frac{\lambda}{1+\lambda}\right)+o(\tau),
\end{aligned}$$

and in the same way

$$\frac{D_F}{S_F} = \frac{D_F}{S_{F0}}\left(1+\frac{\tau}{1+\lambda}\right)+o(\tau).$$

We therefore obtain the second-order approximation of $\boldsymbol{\sigma}_\lambda$, Eq. (C-10):

$$\begin{aligned}\boldsymbol{\sigma}_\lambda &= \tau\left(\boldsymbol{\sigma}^T\right)^{-1}\begin{pmatrix} -\frac{D_H}{S_H} \\ \frac{D_F}{S_F}\end{pmatrix} \\ &= \tau\boldsymbol{\Gamma} + \tau^2\boldsymbol{\Omega} + o(\tau^2),\end{aligned}$$

as well as the second-order approximation of μ_λ:

$$\begin{aligned}\mu_\lambda &\equiv \boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_D + \frac{1}{1+\lambda}\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda \\ &= \tau(\boldsymbol{\Gamma}.\boldsymbol{\sigma}_D) + \tau^2\left(\ \boldsymbol{\Omega}.\boldsymbol{\sigma}_D\right) + \tau^2\frac{1}{1+\lambda}\left(\boldsymbol{\Gamma}.\boldsymbol{\Gamma}\right) + o(\tau^2).\end{aligned}$$

(C-9) follows from the last equation by substituting in $\boldsymbol{\Gamma}.\boldsymbol{\sigma}_D = \rho(1-2\delta)$ and $\boldsymbol{\Omega}.\boldsymbol{\sigma}_D = \frac{\rho}{1+\lambda} - \rho\delta$.

Like in the proof of Proposition 1, we can now use (C-9) and (C-10) in the computation of the conditional expectation $\mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\left[1+\lambda(s)\right]\delta(s)ds\,|\delta(t)=\delta,\lambda(t)=\lambda\right]$. Then, it appears that h can be written as follows, for some function Φ_H to be determined:

$$h(\delta,\lambda;\tau) = (1+\lambda)\,y_H(\delta) - \tau\lambda y_H(\delta) + \tau^2\lambda\Phi_H(\delta,\lambda) + o(\tau^2). \tag{C-11}$$

We can substitute (C-11) in Eq. (B-1). By equalizing terms in τ^2, we obtain a PDE for Φ_H:

$$\rho\Phi_H - \delta\mu_\delta\frac{\partial\Phi_H}{\partial\delta} - \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)\frac{\partial^2\Phi_H}{\partial\delta^2} = \frac{1}{1+\lambda}\delta + \frac{1}{1+\lambda}\left(\boldsymbol{\Gamma}.\boldsymbol{\Gamma}\right)y_H. \tag{C-12}$$

To obtain the non-homogenous term in (C-12), we used:

$$\begin{aligned}-\rho(1-2\delta)y_H - \delta(\boldsymbol{\sigma}_\delta.\boldsymbol{\Gamma})y_H' &= \delta, \\ \left(\frac{\rho}{1+\lambda} - \rho\delta\right)y_H + \delta(\boldsymbol{\sigma}_\delta.\boldsymbol{\Omega})y_H' &= -\frac{\lambda}{1+\lambda}\delta.\end{aligned}$$

From (C-12), using the fact that $\rho y_H = \delta + \delta\mu_\delta y_H' + \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta^T\boldsymbol{\sigma}_\delta)y_H''$, one can infer that there exists a function φ_H such that $\Phi_H(\delta,\lambda) = \frac{1}{1+\lambda}\left[y_H(\delta) + \varphi_H(\delta)\right]$, with the function φ_H verifying:

$$\rho\varphi_H - \delta\mu_\delta\varphi_H' - \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)\varphi_H'' = \left(\boldsymbol{\Gamma}.\boldsymbol{\Gamma}\right)y_H.$$

Respectively for f, we can show that

$$f(\delta,\lambda;\tau) = \frac{1+\lambda}{\lambda}y_F(\delta) - \frac{\tau}{\lambda}y_F(\delta) + \frac{\tau^2}{\lambda}\Phi_F(\delta,\lambda) + o(\tau^2), \tag{C-13}$$

with Φ_F satisfying the following PDE

$$\rho\Phi_F - \delta\mu_\delta\frac{\partial\Phi_F}{\partial\delta} - \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta.\boldsymbol{\sigma}_\delta)\frac{\partial^2\Phi_F}{\partial\delta^2} = (1-\delta)\frac{\lambda}{1+\lambda} + \frac{\lambda}{1+\lambda}\left(\boldsymbol{\Gamma}.\boldsymbol{\Gamma}\right)y_F.$$

Furthermore, we can show that $\Phi_F(\delta,\lambda) = \frac{\lambda}{1+\lambda}\left[y_F(\delta)+\varphi_F(\delta)\right]$, where φ_F is solution of the ODE

$$\rho\varphi_F(\delta) - \delta\mu_\delta(\delta)\varphi_F'(\delta) - \frac{1}{2}\delta^2(\boldsymbol{\sigma}_\delta(\delta).\boldsymbol{\sigma}_\delta(\delta))\varphi_F''(\delta) = (\boldsymbol{\Gamma}(\delta).\boldsymbol{\Gamma}(\delta))y_F(\delta).$$

Therefore, at this stage, we can write

$$\begin{aligned}
S_H(D,\delta,\lambda;\tau) &= D\left[y_H(\delta)\left(1-\tau\frac{\lambda}{1+\lambda}+\tau^2\frac{\lambda}{(1+\lambda)^2}\right)+\tau^2\frac{\lambda}{(1+\lambda)^2}\varphi_H(\delta)\right]+o(\tau^2),\\
S_F(D,\delta,\lambda;\tau) &= D\left[y_F(\delta)\left(1-\tau\frac{1}{1+\lambda}+\tau^2\frac{\lambda}{(1+\lambda)^2}\right)+\tau^2\frac{\lambda}{(1+\lambda)^2}\varphi_F(\delta)\right]+o(\tau^2).
\end{aligned}$$

We now turn to the determination of boundary conditions for φ_H and φ_F. The conditions $\varphi_H(0)=\varphi_F(1)=0$ are required since the price of assets yielding zero payoff must be null. The derivation of the other two boundary conditions ($\varphi_H(1)$ and $\varphi_F(0)$) is more subtle. When $\delta\to 1$, $S_H(t)$ tends to

$$\frac{D_H(t)}{1+\lambda(t)}\mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\left[1+\lambda(s)\right]ds\,\middle|\,\lambda(t)\right].$$

Let $V(\lambda_t;\tau)\equiv\mathbb{E}\left[\int_t^\infty e^{-\rho(s-t)}\left[1+\lambda(s)\right]ds\,|\lambda(t)\right]$, so that $\lim_{\delta\to1}S_H(D,\delta,\lambda;\tau)=\frac{D}{1+\lambda}V(\lambda_t;\tau)$. Applying the Feynman-Kac formula to V gives:

$$\rho V(\lambda) = (1+\lambda)+\lambda\bar{\mu}_\lambda V'(\lambda)+\frac{1}{2}\lambda^2\bar{\boldsymbol{\sigma}}_\lambda.\bar{\boldsymbol{\sigma}}_\lambda V''(\lambda), \tag{C-14}$$

$$\begin{aligned}
\text{where}\qquad \bar{\mu}_\lambda=\lim_{\delta\to1}\mu_\lambda &= -\tau\rho+\tau^2\left[-\frac{\lambda\rho}{1+\lambda}+\frac{1}{1+\lambda}\left(\boldsymbol{\Gamma}(1).\boldsymbol{\Gamma}(1)\right)\right],\\
\bar{\boldsymbol{\sigma}}_\lambda=\lim_{\delta\to1}\ \boldsymbol{\sigma}_\lambda &= \tau\boldsymbol{\Gamma}(1)+\tau^2\boldsymbol{\Omega}(1).
\end{aligned}$$

We know that up to the second order in τ:

$$h(\delta,\lambda)=(1+\lambda)\,y_H(\delta)-\tau\lambda y_H(\delta)+\tau^2\frac{\lambda}{1+\lambda}\left[y_H(\delta)+\varphi_H(\delta)\right]. \tag{C-15}$$

Taking the limit when δ goes to 1, we get

$$\lim_{\delta\to1}h(\delta,\lambda)=V(\lambda)=\frac{1}{\rho}\left[1+\lambda-\tau\lambda+\tau^2\frac{\lambda}{(1+\lambda)}+\tau^2\frac{\lambda}{(1+\lambda)}\rho\varphi_H(1)\right].$$

From this, we can compute $V'(\lambda)$ and $V''(\lambda)$ and plug the expressions for V and its derivatives in (C-14). Then, identifying terms in τ^2, we get:

$$\varphi_H(1) = \frac{1}{\rho^2}(\mathbf{\Gamma}(1).\mathbf{\Gamma}(1)).$$

In the same way, we obtain the boundary condition $\varphi_F(0) = \frac{1}{\rho} + \frac{1}{\rho^2}(\mathbf{\Gamma}(0).\mathbf{\Gamma}(0))$. ∎

Proof of Proposition 3: We start from the expression for $\boldsymbol{\sigma}_H$ given in Lemma B-2

$$\boldsymbol{\sigma}_H = \boldsymbol{\sigma}_D + \delta \frac{h_\delta}{h} \boldsymbol{\sigma}_\delta + \lambda \left(\frac{h_\lambda}{h} - \frac{1}{1+\lambda} \right) \boldsymbol{\sigma}_\lambda.$$

From (C-15), we get the following second-order approximations:

$$\begin{aligned}
\frac{1}{h} &= \frac{1}{(1+\lambda)y_H} \left[1 + \tau \frac{\lambda}{1+\lambda} - \tau^2 \frac{\lambda}{(1+\lambda)^2} \left(1 + \frac{\varphi_H}{y_H} \right) \right] + o(\tau^2), \\
h_\delta &= (1+\lambda)\, y'_H - \tau \lambda y'_H + \tau^2 \frac{\lambda}{1+\lambda} \left(y'_H + \varphi'_H \right) + o(\tau^2), \\
h_\lambda &= y_H - \tau y_H + \tau^2 \frac{\varphi_H + y_H}{(1+\lambda)^2} + o(\tau^2).
\end{aligned}$$

Using (C-10), we obtain the second-order approximation for $\boldsymbol{\sigma}_H$:

$$\boldsymbol{\sigma}_H = \boldsymbol{\sigma}_{H0} + \tau^2 \frac{\lambda}{(1+\lambda)^2} \left\{ -\mathbf{\Gamma} + \left[\frac{\varphi'_H}{y_H} - \lambda \frac{y'_H}{y_H} - \frac{\varphi_H y'_H}{(y_H)^2} \right] \delta \boldsymbol{\sigma}_\delta \right\} + o(\tau^2).$$

We derive the second-order approximation for $\boldsymbol{\sigma}_F$ in the same way. ∎

Proof of Proposition 4: Let $\boldsymbol{\mu}_i - \boldsymbol{r}$ denote the vector of after-tax expected returns from the perspective of investor i. By definition of $\boldsymbol{\theta}_i$ in (14), we have $\boldsymbol{\mu}_i - \boldsymbol{r} = \boldsymbol{\sigma}^{\boldsymbol{T}} \boldsymbol{\theta}_i$. Then, Lemma 2 implies:

$$\begin{aligned}
\boldsymbol{\mu}_H - \boldsymbol{r} &= \boldsymbol{\sigma}^{\boldsymbol{T}} \boldsymbol{\sigma}_D + \tau \frac{\lambda}{1+\lambda} \begin{pmatrix} \frac{D_H}{S_H}, \\ -\frac{D_F}{S_F} \end{pmatrix} \\
\boldsymbol{\mu}_F - \boldsymbol{r} &= \boldsymbol{\sigma}^{\boldsymbol{T}} \boldsymbol{\sigma}_D + \tau \frac{1}{1+\lambda} \begin{pmatrix} -\frac{D_H}{S_H} \\ \frac{D_F}{S_F} \end{pmatrix}.
\end{aligned}$$

The before-tax risk premia are given by the upper element of $\boldsymbol{\mu}_H - \boldsymbol{r}$ and by the lower element of $\boldsymbol{\mu}_F - \boldsymbol{r}$. The Taylor expansions follow immediately. ∎

Proof of Proposition 5: We apply Itô's lemma to both sides of (17) and equalize the drift coefficients (the notations μ_{c_i} and $\boldsymbol{\sigma}_{c_i}$ were introduced in (C-2)). We obtain

$$-\rho\frac{1}{c_i(t)} - \frac{1}{c_i(t)}\mu_{c_i}(t) + \frac{1}{c_i(t)}(\boldsymbol{\sigma}_{c_i}(t).\boldsymbol{\sigma}_{c_i}(t)) = -\frac{1}{c_i(t)}r(t).$$

Using $\boldsymbol{\sigma}_{c_i} = \boldsymbol{\theta}_i$, this can be written

$$r(t) = \rho + \mu_{c_i}(t) - \boldsymbol{\theta}_i(t).\boldsymbol{\theta}_i(t), \quad i = H, F. \tag{C-16}$$

Summing over $i = H, F$, we get:

$$r(t) = \rho + \frac{1}{2}\left(\mu_{c_H}(t) + \mu_{c_F}(t)\right) - \frac{1}{2}\left(\boldsymbol{\theta}_H(t).\boldsymbol{\theta}_H(t) + \boldsymbol{\theta}_F(t).\boldsymbol{\theta}_F(t)\right).$$

Next, we obtain an expression for μ_{c_H} by applying Itô's lemma to $c_H = D/(1+\lambda)$ and we use the market clearing condition $\mu_{c_H}c_H + \mu_{c_F}c_F = \mu_D D$ to get:

$$r = \rho + \mu_D + \frac{1}{2}\frac{\lambda-1}{1+\lambda}\left(-\mu_\lambda + \frac{\lambda}{1+\lambda}\boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_\lambda - \boldsymbol{\sigma}_\lambda.\boldsymbol{\sigma}_D\right) - \frac{1}{2}\left[\boldsymbol{\theta}_H.\boldsymbol{\theta}_H + \boldsymbol{\theta}_F.\boldsymbol{\theta}_F\right].$$

The last term can be rewritten using the expressions for $\boldsymbol{\theta}_H$ and $\boldsymbol{\theta}_F$ given in Lemma 2. Finally, using (27) and (28), a bit of algebra yields the following (exact) expression for the riskfree rate:

$$r(t) = \rho + \mu_D - \boldsymbol{\sigma}_D.\boldsymbol{\sigma}_D - \tau^2\frac{\lambda}{(1+\lambda)^2}\begin{bmatrix}\frac{D_H}{S_H} & -\frac{D_F}{S_F}\end{bmatrix}(\boldsymbol{\sigma}^T\boldsymbol{\sigma})^{-1}\begin{bmatrix}\frac{D_H}{S_H} \\ -\frac{D_F}{S_F}\end{bmatrix}.$$

The Taylor expansion follows immediately. ∎

Proof of Proposition 6: We start from the intertemporal budget constraint of agent i at time t

$$\begin{aligned}
\xi_i(t)X_i(t) &= \mathbb{E}_t\left[\int_t^\infty \xi_i(s)(c_i(s) - e_i(s))ds\right] \\
\Rightarrow X_i(t) &= \mathbb{E}_t\left[\int_t^\infty \frac{\xi_i(s)}{\xi_i(t)}(c_i(s) - e_i(s))ds\right] \\
&= \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{c_i(t)}{c_i(s)}(c_i(s) - e_i(s))ds\right] \\
&= c_i(t)\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\left(1 - \frac{e_i(s)}{c_i(s)}\right)ds\right] \\
&= c_i(t)\left[\frac{1}{\rho} - \mathbb{E}_t\int_t^\infty e^{-\rho(s-t)}\frac{e_i(s)}{c_i(s)}ds\right], \quad i = H, F.
\end{aligned} \tag{C-17}$$

Since lump-sum transfers are proportional to τ, we can introduce u_H such that

$$\begin{aligned} X_H(t) &= c_H(t)\left[\frac{1}{\rho} - \tau u_H(t)\right] \\ &= \frac{1}{\rho}\frac{D(t)}{1+\lambda(t)}\left[1 - \tau\rho u_H(t)\right]. \end{aligned}$$

Itô's lemma implies that, if we write $dX_H/X_H = \mu_{X_H}dt + \boldsymbol{\sigma}_{X_H}d\boldsymbol{W}$, we have

$$\boldsymbol{\sigma}_{X_H} = \boldsymbol{\sigma}_D - \frac{\lambda}{1+\lambda}\boldsymbol{\sigma}_\lambda + \tau\boldsymbol{\sigma}_e,$$

where $\boldsymbol{\sigma}_e$ is related to the redistribution term u_H. This allows us to identify the diffusion term in (9) and to deduce the composition of investor H's portfolio:

$$\begin{aligned} \begin{bmatrix} \frac{\alpha_{HH}S_H}{X_H} \\ \frac{\alpha_{HF}S_F}{X_H} \end{bmatrix} &= \boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D - \frac{\lambda}{1+\lambda}\boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_\lambda + \underbrace{\tau\boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_e}_{\equiv\boldsymbol{\epsilon}_H} \\ &= \boldsymbol{\sigma}^{-1}\boldsymbol{\sigma}_D + \tau\frac{\lambda}{1+\lambda}\left(\boldsymbol{\sigma}^T\boldsymbol{\sigma}\right)^{-1}\begin{bmatrix} \frac{D_H}{S_H} \\ -\frac{D_F}{S_F} \end{bmatrix} + \boldsymbol{\epsilon}_H \\ &= \boldsymbol{\sigma}^{-1}\boldsymbol{\theta}_H + \boldsymbol{\epsilon}_H. \end{aligned}$$

We obtain investor F's portfolio in the same way and the Taylor approximations follow immediately.

In the case $e_H = \tau\alpha_{FH}D_H$, we approximate the hedging component $\boldsymbol{\epsilon}_H$ as follows. We write

$$\begin{aligned} e_H = \tau\alpha_{FH}D_H &= \tau\frac{D_H}{S_H}\alpha_{FH}S_H \\ &= \tau\frac{D_H}{S_{H0}}\frac{S_{H0}}{S_{H0}+S_{F0}}X_F + o(\tau) \\ &= \tau\frac{D_H}{S_{H0}+S_{F0}}X_F + o(\tau) \\ &= \tau\frac{D_H}{D}\frac{D}{S_{H0}+S_{F0}}X_F + o(\tau) \\ &= \tau\delta\rho X_F + o(\tau). \end{aligned}$$

Therefore, (C-17) becomes

$$X_H(t) = c_H(t)\left[\frac{1}{\rho} - \tau\mathbb{E}_t\int_t^\infty e^{-\rho(s-t)}\delta(s)\frac{\rho X_F(s)}{c_H(s)}ds\right] + o(\tau)$$

Since $X_F = \lambda X_H + o(1)$ and $c_H = \rho X_H + o(1)$, we have $\frac{\rho X_F}{c_H} = \lambda + o(1)$. Besides, for $s > t$, $\lambda(s) = \lambda(t) + o(1)$. Therefore, we get

$$\begin{aligned} X_H(t) &= c_H(t)\left[\frac{1}{\rho} - \tau\lambda_t \mathbb{E}_t \int_t^\infty e^{-\rho(s-t)}\delta(s)ds\right] + o(\tau) \\ &= \frac{D(t)}{1+\lambda_t}\left[\frac{1}{\rho} - \tau\lambda_t y_H(\delta(t))\right] + o(\tau). \end{aligned}$$

This expression allows us to identify the wealth diffusion and to deduce the portfolio composition from (9). We obtain

$$\boldsymbol{\epsilon}_H = \tau\lambda \frac{y_H y_F}{(y_H + y_F)^2}\begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

Respectively, $\boldsymbol{\epsilon}_F = \tau\frac{1}{\lambda}\frac{y_H y_F}{(y_H+y_F)^2}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$. ∎

Proof of Lemma 3: The domestic asset price is

$$S_H(t) = \mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\frac{D(t)}{D(s)}p_H(s)D_H(s)ds\right] \tag{C-18}$$

$$= D(t)\mathbb{E}_t\left[\int_t^\infty e^{-\rho(s-t)}\delta(s)ds\right]. \tag{C-19}$$

As recalled in Appendix A, the above expectation can be written as $y_H(\delta(t))$, for some hypergeometric function y_H. However, because the drift and diffusion of δ as defined in Section 5 differ from those given in (5)-(6), the parameters of the hypergeometric function have to be modified accordingly. This is done by setting

$$\begin{aligned} \nu &= \frac{\phi-1}{\phi}\left[\mu_{D_F} - \mu_{D_H} - \frac{\sigma^2_{D_{F,1}} + \sigma^2_{D_{F,2}}}{2} + \frac{\sigma^2_{D_{H,1}} + \sigma^2_{D_{H,2}}}{2}\right], \\ \chi^2 &= \left(\frac{\phi-1}{\phi}\right)^2\left[\left(\sigma^2_{D_{H,1}} + \sigma^2_{D_{H,2}}\right) + \left(\sigma^2_{D_{F,1}} + \sigma^2_{D_{F,2}}\right) - 2(\sigma_{D_{H,1}}\sigma_{D_{F,1}} + \sigma_{D_{H,2}}\sigma_{D_{F,2}})\right]. \end{aligned}$$

The price of the foreign asset can be written as $S_F(t) = D(t)y_F(\delta(t))$, where $y_F(\delta) = \frac{1}{\rho} - y_H(\delta)$. ∎

Proof of Proposition 7: Asset prices are

$$
\begin{aligned}
S_H(t) &= \mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(s-t)}\frac{C_H(t)}{C_H(s)}p_H(s)D_H(s)ds\right] \\
&= \frac{D(t)}{1+\lambda(t)}\mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(s-t)}\left(1+\lambda(s)\right)\delta(s)ds\right], \qquad \text{(C-20)} \\
S_F(t) &= \mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(s-t)}\frac{C_F(t)}{C_F(s)}p_F(s)D_F(s)ds\right] \\
&= \frac{\lambda(t)D(t)}{1+\lambda(t)}\mathbb{E}_t\left[\int_t^{\infty} e^{-\rho(s-t)}\left(1+\frac{1}{\lambda(s)}\right)(1-\delta(s))ds\right]. \qquad \text{(C-21)}
\end{aligned}
$$

Therefore, our approximations of the pricing functions h and f (defined in Lemma 1) allow us to characterize the equilibrium of the two-good version of the model. ∎

References

[1] Adler, M., and B. Dumas, 1983, "International Portfolio Choice and Corporation Finance: A Synthesis," *Journal of Finance,* 38, 925-984.

[2] Aviat, A., and N. Coeurdacier, 2007, "The Geography of Trade in Goods and Asset Holdings," *Journal of International Economics,* 71, 22-51.

[3] Basak, S., and D. Cuoco, 1998, "An Equilibrium Model with Restricted Stock Market Participation," *Review of Financial Studies,* 11, 309-341.

[4] Backus, D. K., P. J. Kehoe and F. E. Kydland, 1994, "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?," *American Economic Review*, 84(1), 84–103.

[5] Basak, S., and M. Gallmeyer, 2003, "Capital Market Equilibrium with Differential Taxation," *European Finance Review,* 7, 121-159.

[6] Baxter, M., and U. Jermann, 1997, "The International Portfolio Diversification Puzzle is Worse Than You Think," *American Economic Review*, 87, 170-180.

[7] Baxter, M., U. Jermann and R. King, 1998, "Nontraded Goods, Nontraded Factors, and International Non-Diversifcation," *Journal of International Economics,* 44, 211-29.

[8] Bekaert, G., and C. Harvey, 1995, "Time-Varying World Market Integration," *Journal of Finance,* 50, 403-444.

[9] Bekaert, G., and C. Harvey, 2000, "Foreign Speculators and Emerging Equity Markets," *Journal of Finance,* 55, 564-614.

[10] Bhamra, H.S., 2004, "International Stock Market Integration: A Dynamic General Equilibrium Approach," Unpublished Working Paper, London Business School.

[11] Black, F., 1974, "International Capital Market Equilibrium with Investment Barriers," *Journal of Financial Economics,* 1, 337-352.

[12] Bottazzi, L., P. Pesenti and E. van Wincoop, 1996, "Wages, Profits and the International Portfolio Puzzle," *European Economic Review*, 40, 219-254.

[13] Chan, K., V. Covrig and L. Ng, 2005, "What Determines the Domestic and Foreign Bias? Evidence from Mutual Fund Equity Allocations Worldwide," *Journal of Finance,* 60, 1495-1534.

[14] Chari, K., and P. Henry, 2004, "Risk-Sharing and Asset Prices: Evidence from a Natural Experiment," *Journal of Finance,* 59, 1295-1325.

[15] Cochrane, J., F. Longstaff and P. Santa Clara, 2007, "Two Trees," *Review of Financial Studies,* Forthcoming.

[16] Coeurdacier, N., 2008, "Do Trade Costs in Goods Market Lead to Home Bias in Equities?," *Journal of International Economics,* Forthcoming.

[17] Coeurdacier, N., and S. Guibaud, 2007, "International Portfolio Diversification Is Better Than You Think," Unpublished Working Paper, London Business School.

[18] Cole, H., and M. Obstfeld, 1991, "Commodity Trade and International Risk Sharing : How Much Do Financial Markets Matter?," *Journal of Monetary Economics,* 28, 3-24.

[19] Constantinides, G., 1986, "Capital Market Equilibrium with Transaction Costs," *Journal of Political Economy,* 94, 842-862.

[20] Cooper, I., and E. Kaplanis, 1994, "Home Bias in Equity Portfolios, Inflation Hedging, and International Capital Market Equilibrium," *Review of Financial Studies,* 7, 45-60.

[21] Cox, J.C., and C.F. Huang, 1989, "Optimal Consumption and Portfolio Policies when Asset Prices Follow a Diffusion Process," *Journal of Economic Theory,* 49, 33-83.

[22] Cuoco, D., and H. He, 1994, "Dynamic Equilibrium in Infinite-Dimensional Economies with Incomplete Financial Markets," Unpublished Working Paper, University of Pennsylvania.

[23] Devereux, M., and A. Sutherland, 2006, "Solving for Country Portfolios in Open Economy Macro Models," Unpublished Working Paper, University of British Columbia.

[24] Duffie, D., 1996, *Dynamic Asset Pricing,* Princeton University Press.

[25] Dumas, B., 1992, "Dynamic Equilibrium and the Real Exchange Rate in a Spatially Separated World," *Review of Financial Studies,* 5, 153-180.

[26] Dumas, B., R. Harvey and P. Ruiz, 2003, "Are Correlation of Stock Returns Justified by Subsequent Changes in National Outputs?," *Journal of International Money and Finance,* 22, 777-811.

[27] Engel, C., and A. Matsumoto, 2006, "Portfolio Choice in a Monetary Open-Economy DSGE Model,"NBER Working Paper 12214.

[28] Errunza, V., and E. Losq, 1985, "International Asset Pricing under Mild Segmentation: Theory and Test," *Journal of Finance,* 40, 105-124.

[29] Errunza, V., and E. Losq, 1989, "Capital Flow Controls, International Asset Pricing, and Investors' Welfare: a Multi-Country Framework," *Journal of Finance,* 44, 1025-1037.

[30] Eun, C., and S. Janakiramanan, 1986, "A Model of International Asset Pricing with a Constraint on the Foreign Equity Ownership," *Journal of Finance,* 41, 1025-1037.

[31] French, K., and J. Poterba, 1991, "Investor Diversification and International Equity Markets," *American Economic Review,* 81, 222-26.

[32] Goetzmann, W., L. Li and K. Rouwenhorst, 2005, "Long-Term Global Market Correlation," *Journal of Business,* 78, 1-38.

[33] Gordon, R.H., and J. Hines, 2002, "International Taxation,"in A. J. Auerbach and M. Feldstein (ed.), Handbook of Public Economics, 4 (28), 1935-1995.

[34] Hau, H., and H. Rey, 2004, "Can Portfolio Rebalancing Explain the Dynamics of Equity Returns, Equity Flows, and Exchange Rates?," *American Economic Review,* 94, 126-133.

[35] Heathcote, J. and F. Perri, 2002, "Financial Autarky and International Business Cycles," *Journal of Monetary Economics,* 49 (3), 601–627.

[36] Heathcote, J., and F. Perri, 2007, "The International Diversification Puzzle is Not as Bad as You Think,"Unpublished Working Paper, Federal Reserve Bank of Minneapolis.

[37] Henry, P., 2000, "Stock Market Liberalization, Economic Reform and Emerging Market Equity Prices," *Journal of Finance,* 55, 529-564.

[38] Hietala, P.T., 1989, "Asset Pricing in Partially Segmented Markets: Evidence from the Finnish Market," *Journal of Finance,* 44, 697-718.

[39] Imbs, J., 2006, "The Real Effects of Financial Integration," *Journal of International Economics,* 68, 296-324.

[40] Judd, K., and S. Guu, 2001, "Asymptotic Methods for Asset market Equilibrium Analysis," *Economic Theory*, 18, 127-157.

[41] Kaminsky, G., and S. Schmukler, 2003, "Short-Run Pain, Long-Run Gain: The Effects of Financial Liberalisation,"NBER Working Paper 9787.

[42] Karatzas, I., Lehoczky, J. and S. Shreve, 1987, "Optimal Portfolio and Consumption Decisions for a "Small" Investor on a Finite Horizon,"*SIAM Journal of Control and Optimization,* 25, 1557-1586.

[43] Kollmann, R., 2006, "A Dynamic Equilibrium Model of International Portfolio Holdings: Comment,"*Econometrica,* 74, 269-273.

[44] Lane, P., and G.M. Milesi-Feretti, 2003, "International Financial Integration,"*IMF Staff Papers*, 50.

[45] Lane, P., and G.M. Milesi-Feretti, 2008, "International Investment Patterns,"*Review of Economics and Statistics*, 90, 538-549.

[46] Lucas, R., 1978, "Asset Prices in an Exchange Economy,"*Econometrica,* 46, 1429-1445.

[47] Ma, J., P. Protter and J. Yong, 1994, "Solving Forward-Backward Stochastic Differential Equations Explicitly − A Four-Step Scheme, "*Probability Theory and Related Fields,* 98, 339-359.

[48] Ma, J., and J. Yong, 1999, *Forward-Backward Stochastic Differential Equations and their Applications,* Lecture Notes in Mathematics, Springer.

[49] Martin, I., 2007, "The Lucas Orchard,"Unpublished Working Paper, Harvard University.

[50] Martin, P., and H. Rey, 2004, "Financial Super-Markets: Size Matters for Asset Trade," *Journal of International Economics,* 64, 335-361.

[51] Menzly, L., T. Santos and P. Veronesi, 2004, "Understanding Predictability," *Journal of Political Economy,* 112, 1-47.

[52] Obstfeld M., 1994, "Risk-Taking, Global Diversification and Growth,"*American Economic Review,* 84, 1310-1329.

[53] Obstfeld M., and K. Rogoff, 2000, "The Six Major Puzzles in International Macroeconomics: Is There a Common Cause?," *NBER Macroeconomics Annual.*

[54] Pavlova, A., and R. Rigobon, 2007, "Asset Prices and Exchange Rates," *Review of Financial Studies,* 20, 1139-1181.

[55] Pavlova, A., and R. Rigobon, 2008, "The Role of Portfolio Constraints in the Propagation of Shocks," *Review of Economic Studies*, Forthcoming.

[56] Portes, R., and H. Rey, 2005, "The Determinants of Cross-Border Equity Flows," *Journal of International Economics,* 65, 269-296.

[57] Quinn, D., 1997, "The Correlates of Change in International Financial Regulation," *American Political Science Review,* 91, 531-551.

[58] Quinn, D., and H.J. Voth, 2008, "A Century of Global Equity Markets Correlations," *American Economic Review,* 98, 535-540.

[59] Santos, T., and P. Veronesi, 2006, "Labor Income and Predictable Stock Returns," *Review of Financial Studies,* 19, 1-44.

[60] Sercu, P., and R. Vanpee, 2007, "Home Bias in International Equity Portfolios: A Review," Unpublished working paper, Leuven School of Business and Economics.

[61] Serrat, A., 2001, "A Dynamic Equilibrium Model of International Portfolio Holdings," *Econometrica,* 69, 1467-1489.

[62] Stockman, A., and H. Dellas, 1989, "International Portfolio Nondiversification and Exchange Rate Variability," *Journal of International Economics,* 26, 271-290.

[63] Stulz, R., 1981, "On the Effects of Barriers to International Investment", *Journal of Finance,* 36, 923-934.

[64] Stulz, R., 1999, "Globalization of Equity Markets and the Cost of Capital,"NBER Working Paper 7021.

[65] Stulz, R., 2005, "The Limits of Financial Globalization," *Journal of Finance,* 60, 1595-1638.

[66] Tille, C., and E. van Wincoop, 2007, "International Capital Flows,"Unpublished working paper, University of Virginia.

[67] Uppal, R., 1993, "A General Equilibrium Model of International Portfolio Choice," *Journal of Finance,* 48, 529-553.

[68] Van Nieuweburgh, S., and L. Veldkamp, 2008, "Information Immobility and the Home Bias Puzzle," *Journal of Finance,* Forthcoming.