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Agency, Firm Growth, and Managerial Turnover

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ABSTRACT

We study managerial incentive provision under moral hazard when growth opportunities arrive stochastically and pursuing them requires a change in management. A trade-off arises between the benefit of always having the "right" manager and the cost of incentive provision. The prospect of growth-induced turnover limits the firm's ability to rely on deferred pay, resulting in more front-loaded compensation. The optimal contract may insulate managers from the risk of growth-induced dismissal after periods of good performance. The evidence for the United States broadly supports the model's predictions: Firms with better growth prospects experience higher CEO turnover and use more front-loaded compensation.

WHEN OWNERSHIP AND CONTROL ARE SEPARATED, firm performance depends crucially on having the right managers at the helm and incentivizing them properly. Over time, changes in business conditions may call for a change in top management for the firm to seize new opportunities or overcome challenges. But this may complicate the task of incentivizing incumbent managers. For instance, if managers anticipate that their tenure at the firm will be short, they will be reluctant to accept any form of deferred compensation, a standard feature of incentive contracts. The firm may therefore face a dilemma: By changing management to adapt to evolving business conditions, it may increase the costs of incentive provision.

To analyze this tension, this paper introduces the idea of *growth-induced turnover* into a dynamic moral hazard framework. Growth-induced turnover refers to the replacement of top management that is motivated by the need to

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have managers who possess the appropriate skill set and experience to lead the firm in its *current* circumstances. This may involve, for instance, adopting new production techniques, making acquisitions, launching a new product, or expanding into new markets. If the incumbent lacks the vision or skills necessary to implement such transformations, appointing new management is the only way for the firm to successfully pursue its course.[1] At the same time, proper dynamic incentive provision requires a combination of deferred compensation and a threat of dismissal following poor performance, both of which constitute agency costs. By introducing the possibility of managerial turnover for the sake of growth as well as for discipline, we show how these costs are affected. The main insight of the paper is that the prospect of growth-induced dismissal effectively increases managers' impatience, which increases agency costs and generates a tendency to front-load compensation. In fact, the firm may actually be better off ex ante by committing to pass up otherwise attractive growth opportunities in some circumstances. More generally, our analysis delivers empirical predictions on the effects of a firm's growth prospects on managerial turnover and compensation, predictions that we show are broadly supported in the data.

Although our analysis is set up in a continuous-time stationary environment, we first develop the theory in the context of a two-period model. The simplicity of the framework enables us to distill most of the economics of the paper in a transparent way. In particular, the trade-off faced by the firm between the benefit of having a manager who is able to seize new opportunities and the cost of incentive provision appears starkly in this setting. Moreover, the key empirical implications of the theory are derived analytically.

In the continuous-time model, a long-lived firm is run by a sequence of risk-neutral managers protected by limited liability. A moral hazard problem arises because, while they are in charge, managers can divert cash flows for their own private benefit. The firm can fire the incumbent manager at any time and replace him at a cost. Fleeting growth opportunities arrive stochastically over time, and a change in management is needed to seize them. If the firm decides to pursue an opportunity, it pays the costs associated with replacing the manager and its size (or profitability) increases. A long-term incentive contract is signed between the firm and its successive managers at the time they are hired.

As in previous dynamic contracting studies, we show that optimal compensation and turnover policies in this environment can be described in terms of a state variable that coincides with the agent's expected discounted compensation, referred to as his contractual *promise*. The manager receives

[1] In some circumstances, a change in management may be required to avoid decay, rather than to pursue growth—for example, when by sticking with the status quo, the firm would fail to face up to a disruptive competitive threat. For instance, in his narrative of the battle waged in Canada around 1820 between the long-established Hudson Bay Company (HB) and its upstart rival North West Company (NW), Roberts (2004, p. 5) recounts that "HB did respond to the threat, essentially by copying NW's new approach. It did so, however, *only after the leaders of the firm had been replaced by new ones who understood the nature of the threat and were not tied to the old ways that had worked so well for so long*." (emphasis added)

cash compensation only when his promise reaches an endogenous *bonus threshold*. When the manager's promise lies below this threshold, cash compensation is deferred, and the promise is increased at a contractually specified rate plus a positive or negative adjustment based on the firm's current performance. If the firm suffers a sustained period of poor performance, the manager's promise can be lowered sufficiently to reach zero, the *firing threshold*, at which point the incumbent is replaced by a new manager who receives an initial promise that is no less than his exogenous reservation value.

In contrast with other studies, the manager's contract in our framework is also contingent on the presence or not of a growth opportunity. If no growth opportunity becomes available, the manager continues his tenure so long as his promise stays above the firing threshold, and he is compensated with bonuses and performance-related changes in his promise as described above. If a growth opportunity arises and the firm takes it, the manager is replaced. However, not all growth opportunities are seized by all firms—even though they would be under first-best. Specifically, we show that, depending on the characteristics of the firm and its environment, the optimal growth policy can be one of two types. For some firms, it is optimal to pursue all growth opportunities as they come. For other firms, it is optimal to forgo opportunities that arise after periods of good performance, that is, when the incumbent manager's promise is above a certain *growth threshold*. We refer to these two types of firms as *high-growth firms* and *low-growth firms*, respectively. In effect, optimal incentive provision in low-growth firms calls for some degree of job protection against the risk of growth-induced termination. Intuitively, the reason job protection is granted after a spell of good cash flows is that losses due to agency problems are reduced after good performance, thus increasing the value of continuing with the incumbent manager net of the forgone benefit of growth. In high-growth firms, the benefit of growth always dominates.

Under the optimal contract, managerial compensation is affected by the possibility of growth-induced turnover through the drift of the manager's promise during his tenure. In the absence of growth opportunities, this drift would simply be equal to the manager's discount rate. The key novelty in our setup is that, whenever the firm stands ready to seize an opportunity that might become available, the drift rate needs to be augmented to compensate the manager for the risk of growth-induced termination, with the drift modification depending on the arrival intensity of growth opportunities. This upward adjustment of the drift when the firm stands ready to pursue a growth opportunity explains why firms with better growth prospects tend to have more front-loaded compensation. It also sheds light on why low-growth firms grant job protection when past performance has been good but not if it has been bad. A higher drift is indeed less costly to the firm after poor performance, when the manager's promise is close to the firing threshold, as it reduces the likelihood of a subsequent inefficient disciplinary turnover.

Our analysis explicitly allows for the possibility of lump-sum payments, and we show that severance pay is suboptimal in our setting even in the case of growth-induced turnover. Indeed, it is always better for the firm to increase the incumbent's future promise conditional on him being retained, thereby making inefficient termination less likely in the future, than to give cash to a departing manager. However, we establish that an incoming manager may be given a "signing bonus" when his reservation value is sufficiently high.

To derive these results for the second-best incentive contract, our approach roughly follows the same logic as in previous continuous-time analyses of dynamic moral hazard. First, we establish a state-space representation of long-term incentive contracts, where the state process coincides with the manager's promise as described above. Similar to other studies, no stealing is incentive compatible under a dynamic contract if the sensitivity of the manager's promise to reported cash flows is large enough. We next formulate the firm's contracting problem recursively to characterize the optimal incentive-compatible dynamic contract in the presence of stochastic growth opportunities. We show that the firm's size-adjusted value function can be characterized as the solution to a Hamilton-Jacobi-Bellman (HJB) equation that incorporates the possibility of growth-induced turnover in an intuitive way. This crucial step in the analysis is established through a verification theorem from which the main properties of optimal compensation and turnover policies follow. Based on the HJB, we also provide a characterization of the determinants of a firm's growth type. In particular, we show that low-growth firms tend to be those plagued with more severe agency problems. This finding suggests that better governance can work as an effective tool to promote economic growth.

Having characterized the optimal contract, we take full advantage of the dynamic nature of our model and provide a suggestive analysis of its quantitative implications for the distribution of tenure length and for the timing of managerial compensation over tenure. In the model, these are partly determined by the firm's type as well as the compensation and growth thresholds, all of which are endogenous. We discuss the impact of a firm's growth prospects on turnover and compensation under the optimal contract through a numerical example. The simulation outcome illustrates the fact that firms with better growth prospects, in particular, those with more attractive opportunities (i.e., holding their arrival intensity fixed), tend to have shorter tenure length and more front-loaded compensation.

Finally, we examine the data in light of the theory. Merging data from CRSP, Compustat, and ExecuComp for U.S. public companies over the period 1992 to 2014, we investigate empirically the links between firms' growth prospects, CEO turnover, and CEO compensation. Following an extensive literature in empirical corporate finance, we use average Q to capture firms' growth prospects. Specifically, we proxy for the ex ante growth prospects of a firm at the time a new CEO is appointed by the value of the firm's Q in the year before the CEO's appointment. We first sort CEO episodes along this proxy and compare the distributions of tenure length and compensation duration across the highest and lowest quantiles of growth prospects. In line with the model predictions, we find that the CEOs of firms with better prospects tend to have shorter tenure and more front-loaded compensation. We confirm these findings using regression analysis. In a probit model, our proxy for firms' growth prospects is positively related to the likelihood of turnover, controlling for past performance. An increase in initial Q of one standard deviation leads to an increase in the probability of turnover of 85 basis points. Since the unconditional frequency of CEO turnover in our sample is 8.4%, the effect is economically significant. We also find that the arrival of an opportunity, proxied by an increase in the firm's average Q since the beginning of a CEO's tenure, increases the probability of a turnover event, consistent with the notion of growth-induced turnover. Furthermore, the likelihood of turnover is less sensitive to the arrival of an opportunity when ex ante growth prospects were poor, in line with the prediction that firms with more modest growth prospects are more likely to insulate their managers from the risk of growth-induced turnover. Finally, we find that managerial pay tends to be lower in firms with worse growth prospects, and that the slope of the compensation profile over tenure years tends to be higher in such firms, which can be viewed as a manifestation of their greater reliance on compensation back-loading.

The idea that the pursuit of valuable growth opportunities may rely on a change in management can be found in early contributions to the management literature, going back to Penrose (1959). More recently, Roberts (2004) studies a number of business cases in which managerial limitations to firm growth play a prominent role and a change in management is instrumental in unlocking the growth potential of a firm.[2] Bertrand and Schoar (2003) provide compelling evidence that managers do indeed matter for firm performance and that they differ in their management styles. Bennedsen, Pérez-González, and Wolfenson (2012) further report that CEO effects are particularly important in rapidly growing environments. Building on the idea that firm productivity is determined by the quality of the match between the skill set of the manager and the current circumstances of the firm, Eisfeldt and Kuhnen (2013) analyze a competitive assignment model of CEO turnover where the skills demanded by the firm are subject to random shocks. In a similar vein, Jenter and Lewellen (2014) extend the standard Bayesian learning model of CEO turnover (e.g., Harris and Holmström (1982)) by allowing the quality of the firm-CEO match to vary over time. In contrast with our work, these papers abstract from agency issues and incentive considerations, which occupy center stage in our analysis.

Our paper relates to a large body of work that applies the tools of dynamic contracting to the study of the firm in the presence of agency conflicts.[3] In particular, Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and

[2] See in particular his discussion of the British Petroleum (BP) and General Motors (GM) cases. Both firms achieved major increases in value by undertaking discrete changes in organizational structure implemented by new CEOs with a different vision (John Browne at BP and "Jack" Smith at GM) and only after a sustained period of poor performance. This can be viewed as evidence of the type of behavior that characterizes low-growth firms in our analysis. Cheng and Hambrick (2012) document that, in turnaround situations, companies substantially improve performance when they replace incumbent CEOs who are poorly suited to the conditions at hand with new ones who are well matched to those conditions.

[3] For seminal contributions to the literature on dynamic moral hazard, see Rogerson (1985) and Spear and Srivastava (1987) in discrete time, as well as Holmström and Milgrom (1987) and Sannikov (2008) in continuous time. Recent applications to the study of CEO turnover and

Fishman (2007a), Philippon and Sannikov (2007), He (2008), Biais et al. (2010), and DeMarzo et al. (2012) investigate the link between moral hazard and firm growth when the firm can grow with the incumbent. Our main theoretical contribution is to focus instead on growth-induced turnover and its interactions with incentive provision. To the extent that the optimal contract in our setting is contingent on the realization of observable shocks, our work also bears some similarity to Piskorski and Tchistyi (2010) and Li (2015). More specifically, our continuous-time framework builds on the cash diversion model of DeMarzo and Sannikov (2006),[4] which we extend to incorporate the stochastic arrival of growth opportunities. From a technical point of view, our contributions are as follows. First, we introduce an additional source of uncertainty beyond the Brownian cash flow shocks. Consequently, our analysis borrows techniques from the credit risk literature to derive the state-space representation of the contract and develop the proof of the verification theorem. Second, we consider a stationary environment where the firm's continuation value at the time a manager is fired is fully endogenous. Third, we endogenize the initial promise that the firm offers to each manager. In particular, we derive a necessary and sufficient condition for the managers' participation constraint to be binding in high-growth firms. Fourth, we explicitly allow for jumps in the cumulative compensation process, which allows us to assess the optimality of severance pay. Finally, our extensive analysis of the HJB equation and associated free-boundary problems allows us to derive explicit existence and uniqueness results, as well as comparative statics that are new to our setting.

The implications of our model and the evidence we provide are connected to a vast empirical literature on the determinants of turnover and compensation for top management.[5] The literature on CEO turnover has mostly focused on the link between turnover and performance. Empirical studies in this vein include the work of Coughlan and Schmidt (1985), Warner, Watts, and Wruck (1988), Weisbach (1988), Kim (1996), and Denis, Denis, and Sarin (1997), as well as more recent contributions by Jenter and Lewellen (2014) and Jenter and Kanaan (2015). We find that, controlling for performance, firms' growth prospects also help explain the likelihood of CEO turnover. In terms of managerial compensation, the model predictions are in line with Murphy (1999), who points out that pay packages often include a bonus system based on the firm's reported earnings in excess of a performance target. They also echo Kaplan and Minton (2012), who discuss the coincidence of shorter CEO tenures and higher CEO pay in the time series. The degree of reliance on deferred compensation has received relatively little attention in the literature so far. An exception is the analysis by Clementi and Cooley (2010), who exploit information on CEOs' holdings of stocks and stock options to construct a measure of

compensation include, among others, Spear and Wang (2005), Hoffman and Pfeil (2010), He (2012), Edmans et al. (2012), and Garret and Pavan (2012, 2015).

[4] See DeMarzo and Fishman (2007b) for a discrete-time version, and Biais et al. (2007) for an analysis of convergence from discrete to continuous time.

[5] For surveys of the literature on CEO compensation and on managerial incentive packages more generally, see, for instance, Murphy (1999, 2013).

deferred compensation. Gopalan et al. (2014) focus on the duration of a CEO's total compensation award in a given year based on information about the vesting periods of separate components in the package. Instead, we measure the duration of compensation received over the entire tenure of a CEO, and we document that this measure varies negatively with the firm's growth prospects at the time the CEO is hired. More broadly, we add to existing empirical studies on CEO compensation by investigating how the profile of CEO pay over tenure relates to firms' growth prospects.

The rest of the paper proceeds as follows. Section I develops the theory in a simple two-period framework and analytically derives its empirical implications for managerial turnover and compensation. Section II describes the continuous-time modeling setup and derives the state-space representation of long-term incentive contracts. Section III characterizes the optimal dynamic contract for high-growth and low-growth firms, as well as the determinants of firm type, and illustrates the model's implications through simulations. Section IV presents the empirical evidence. Section V concludes.

I. A Two-Period Model

We consider a firm that hires a manager at time $t = 0$ to run its operations for at most two periods. The firm and its manager(s) are risk-neutral and have discount rates r and ϱ, respectively, with $\varrho > r$.[6] The random cash flows generated by the firm's operations at $t = 1$ and $t = 2$ are independently distributed. The first-period cash flow Y_1 is equal to either $y > 0$ with probability $p \in (0, 1)$ or zero with probability $1 - p$. At the end of the first period, a growth opportunity may arrive with probability $q \in (0, 1)$, independently of Y_1. The arrival of a growth opportunity is publicly observable. Crucially, we assume that, in order to pursue an available opportunity, the firm must dismiss the incumbent manager and appoint a new one. If a growth opportunity arises and the firm hires a new manager to seize this opportunity, the second-period cash flow Y_2 is either $(1 + \gamma)y$ with probability p, where $\gamma > 0$, or zero with probability $1 - p$. If no growth opportunity arises or if the firm forgoes an available opportunity, the firm may either continue with the incumbent manager or dismiss him and hire a new one, in which case the distribution of Y_2 is the same as that of Y_1.

The assumption that the firm cannot grow without a change in management occupies center stage in our analysis. This assumption captures circumstances in which value creation requires specific managerial skills to carry out radical transformations of the firm and the incumbent does not have the ability to realize the firm's growth potential. We let $\kappa > 0$ denote the exogenous cost of managerial replacement at $t = 1$, which may include search fees as well as

[6] The assumption that the agent is more impatient than the principal is standard in the dynamic contracting literature and allows us to derive sharper predictions on managerial compensation (see also footnote 20).

indirect costs such as the disruption of ongoing business, and we assume that the net present value of taking a growth opportunity is positive, that is,

$$\frac{p\gamma y}{1+r} - \kappa > 0. \tag{1}$$

Our analysis focuses on a second-best environment in which cash flows are not observable by the firm. In the case of a high cash flow realization, the manager can underreport, steal the entire cash flow, and get private benefit λ per unit of stealing, where $0 < \lambda \leq 1$ captures the severity of the moral hazard problem. We let $\hat{Y}_t \leq Y_t$ denote the level of reported cash flow in period t, where $\hat{Y}_t$ has the same support as the actual cash flow Y_t. Managers are protected by limited liability and have zero reservation value.[7] Thus, if the firm hires a new manager at $t = 1$, it offers him a one-period incentive contract with compensation $\lambda\hat{Y}_2$ at $t = 2$. This is the optimal one-period contract that supports no stealing (see Lemma IA1 in Internet Appendix Section I).[8]

At $t = 0$, the firm offers a two-period incentive contract to the initial manager. Such a contract specifies the firm's dismissal policy along with a compensation policy, and both parties fully commit to the terms of the contract. Specifically, we let $G(\hat{Y}_1) \in [0, 1]$ denote the probability of taking a growth opportunity if one arises, thereby replacing management, conditional on first-period reported cash flow. We let $F(\hat{Y}_1) \in [0, 1]$ denote the probability of the manager being dismissed at the end of the first period if no growth opportunity arises, conditional on first-period reported cash flow. Hence, G and F determine the occurrence of *growth-induced dismissal* and *disciplinary dismissal*, respectively. Furthermore, we let $C_1(\hat{Y}_1)$ and $C_2(\hat{Y}_1, \hat{Y}_2)$ denote the compensation received by the manager in the first and second period, respectively, contingent on reported cash flows, and we denote by $C_g(\hat{Y}_1)$ the amount of severance pay upon growth-induced turnover. Limited liability requires

$$C_1(\hat{Y}_1) \geq 0, \quad C_{\mathrm{g}}(\hat{Y}_1) \geq 0, \quad \text{and} \quad C_2(\hat{Y}_1, \hat{Y}_2) \geq 0. \tag{2}$$

After the adoption of a two-period contract at $t = 0$, the timing is as follows. At $t = 1$, the first-period cash flow realizes. The manager reports $\hat{Y}_1$ and receives $C_1(\hat{Y}_1)$. The uncertainty about the availability of a growth opportunity is resolved. The manager is either retained or dismissed—with dismissal probability $G(\hat{Y}_1)$ or $F(\hat{Y}_1)$ depending on whether a growth opportunity is available or not. In the case of a growth-induced dismissal, the departing manager receives $C_g(\hat{Y}_1)$ upon leaving office. For simplicity, we assume that the continuation value of a dismissed manager is zero. At $t = 2$, the second-period cash flow realizes and the initial or newly hired manager reports $\hat{Y}_2$. If the initial manager is still in office, he receives $C_2(\hat{Y}_1, \hat{Y}_2)$. Otherwise, the newly hired manager receives $\lambda\hat{Y}_2$.

[7] With zero reservation value, limited liability ensures that the manager's participation constraint is satisfied. We allow for a positive reservation value in the continuous-time model that we study in Sections II and III.

[8] The Internet Appendix may be found in the online version of this article.

A. The Optimal Two-Period Contract

We look for a two-period contract that maximizes the firm's expected discounted profit while inducing truthful reporting by the manager. Under such a contract, reported cash flows $\hat{Y}_t$ coincide with actual cash flows Y_t, hence we dispense with the notational distinction.

In the second period, when the realized cash flow is high ($Y_2 = y$), the manager has the choice between truthfully reporting good performance or reporting poor performance and stealing the cash flow. The incentive compatibility (IC) condition requires that the manager prefer to report truthfully. This will be the case provided that the difference in compensation upon good and bad reported performance, $C_2(Y_1, y) - C_2(Y_1, 0)$, is sufficiently large, that is,

$$C_2(Y_1, y) \geq \lambda y + C_2(Y_1, 0). \tag{3}$$

Likewise, in the first period, the manager needs to be incentivized to report truthfully when the realized cash flow is high ($Y_1 = y$). At this early stage, incentives are determined by the *total* expected discounted payoff that the manager receives upon reports of either good or bad performance. For a given report Y_1, his intertemporal payoff includes first-period compensation, $C_1(Y_1) + qG(Y_1)C_g(Y_1)$, and expected second-period pay. The latter depends on both the probability of being retained, $1 - [qG(Y_1) + (1-q)F(Y_1)]$, and the expected pay received at $t = 2$ conditional on being retained, $pC_2(Y_1, y) + (1-p)C_2(Y_1, 0)$. The first-period IC constraint thus requires that

$$\begin{aligned} &C_1(y) + qG(y)C_g(y) + (1 - [qG(y) + (1-q)F(y)])\frac{pC_2(y, y) + (1-p)C_2(y, 0)}{1+\varrho} \\ &\geq \lambda y + C_1(0) + qG(0)C_g(0) \\ &\quad + (1 - [qG(0) + (1-q)F(0)])\frac{pC_2(0, y) + (1-p)C_2(0, 0)}{1+\varrho}, \end{aligned} \tag{4}$$

that is, the difference in the manager's intertemporal payoffs upon good and bad performance needs to be sufficiently large. Importantly, (4) captures the fact that first-period incentives are shaped by both the compensation scheme and the firm's dismissal policy. In particular, proper incentive provision requires that good reported performance at $t = 1$ be associated with higher contemporaneous levels of pay, higher future levels of pay, or a lower likelihood of dismissal.

Our first lemma characterizes the optimal compensation scheme. The proof of this result, along with the proofs of all results derived in this section, can be found in Internet Appendix Section I.

LEMMA 1: *The compensation policy that maximizes the firm's expected discounted profit while respecting the limited liability constraint* (2) *and the IC constraints* (3) *and* (4) *is such that*

$$C_1(0) = C_g(0) = C_2(0, 0) = C_2(y, 0) = 0, \tag{5}$$

$$C_2(0, y) = C_2(y, y) = \lambda y, \tag{6}$$

$$C_1(y) + qG(y)C_g(y) = \lambda y - ([qG(0) + (1-q)F(0)] - [qG(y) + (1-q)F(y)])\frac{p\lambda y}{1+\varrho}. \tag{7}$$

Equation (5) shows that, under the optimal contract, the manager receives zero compensation in any given period upon a report of poor performance in that period. Limited liability precludes a tougher penalty, that is, negative compensation. Equation (6) establishes that second-period compensation conditional on good reported cash flow at $t = 2$ is equal to the agency rent λy, independent of first-period performance.[9] Indeed, it is optimal to set $C_2(0, y)$ to the minimum level that satisfies the second-period IC constraint (3) after poor performance, so as to relax the first-period IC constraint (4). On the other hand, the second-period IC constraint after good performance is also binding because, when $\varrho > r$, deferring compensation is costly for the firm. Hence, the optimal two-period contract involves the minimum amount of deferred compensation that is compatible with proper incentive provision.

Equation (7), which follows directly from the binding first-period IC constraint (4), determines the level of first-period compensation upon good performance, $C_1(y) + qG(y)C_g(y)$, and establishes a crucial link between the compensation and dismissal policies. While the first term on the right-hand side of the equation, λy, is the rent that the manager would be given to report performance truthfully at $t = 1$ under a one-period contract, the second term is a distinct feature of the two-period contract. To interpret this term, recall that if continued into the second period, the manager receives an agency rent λy at $t = 2$ conditional on good second-period performance, independent of his report in the first period. That is, a manager who is retained at $t = 1$ contemplates an expected discounted payoff equal to $p\lambda y/(1 + \varrho)$, regardless of whether reported performance in the first period was good or not. Hence, a key determinant of first-period incentives is the wedge between dismissal probabilities after poor performance, $qG(0) + (1 - q)F(0)$, and after good performance, $qG(y) + (1 - q)F(y)$. As (7) reveals, a larger wedge incentivizes the manager not to steal and reduces the need to use first-period compensation upon good performance to do so—especially when the expected discounted value of second-period compensation is large.

It is worth noting that zero severance pay upon growth-induced turnover, $C_g(y) = C_g(0) = 0$, is weakly optimal in this setup. In particular, the firm is indifferent between granting positive severance pay $C_g(y)$ upon growth-induced dismissal after good performance or instead increasing regular compensation $C_1(y)$ by the amount $qG(y)C_g(y)$, taking into account the effective

[9] Note that (5) and (6) imply that $C_2(Y_1, Y_2) = \lambda Y_2$, that is, the compensation scheme in the second period does not depend on whether the firm is run by the initial manager or by a new manager.

probability $qG(y)$ of growth-induced turnover conditional on good first-period performance.[10]

Our next result characterizes the optimal turnover and growth policies. These are captured by $F(Y_1)$ and $G(Y_1)$, which determine the conditional probabilities of disciplinary dismissal and growth-induced dismissal, respectively.

LEMMA 2: *The optimal contract is such that $F(y) = 0$. Furthermore, the following statements hold true:*

i. Setting $F(0) = 1$ is optimal if and only if

$$\kappa \leq \frac{p}{1-p}\frac{p\lambda y}{1+\varrho} =: \hat{\kappa}_{F(0)}. \tag{8}$$

ii. Setting $G(0) = 1$ is optimal if and only if

$$\kappa \leq \frac{p(1-\lambda)\gamma y}{1+r} + \frac{p}{1-p}\frac{p\lambda y}{1+\varrho} =: \hat{\kappa}_{G(0)}. \tag{9}$$

iii. Setting $G(y) = 1$ is optimal if and only if

$$\kappa \leq \frac{p(1-\lambda)\gamma y}{1+r} - \frac{p\lambda y}{1+\varrho} =: \hat{\kappa}_{G(y)}. \tag{10}$$

If any of the inequalities in (8), (9), or (10) is violated, it is optimal to set the corresponding dismissal probability equal to zero.

The first part of Lemma 2 characterizes the firm's optimal dismissal policy in the absence of a growth opportunity, $F(Y_1)$. When no growth opportunity is available, resorting to dismissal after good performance (i.e., $F(y) > 0$) is clearly suboptimal. Indeed, this would bring no benefit and would be costly because of the replacement cost $\kappa > 0$ and because of a deterioration in incentives in the first period, which would have to be compensated by an increase in first-period compensation upon good performance—as per (7). By contrast, the optimal choice of $F(0)$ involves a trade-off between incurring the replacement cost κ and improving first-period incentives by increasing the wedge in dismissal probabilities. Equation (8) shows that disciplinary dismissal after poor performance is optimal when κ is low enough or when the impact of the wedge on the firm's net profit is sufficiently large (i.e., high λ, high p, or low ϱ). Furthermore, a higher probability of first-period success makes it less likely that the replacement cost κ will have to be paid while increasing the expected gain that the firm obtains from a reduction in $C_1(y) + qG(y)C_g(y)$, which explains why the ratio $p/(1-p)$ appears in the expression for the cutoff value $\hat{\kappa}_{F(0)}$.

[10] In Section III, we show that zero severance pay is *strictly* optimal in the continuous-time version of our model (Property 3). The reason only a weak version of this "no-severance" result holds in the two-period setup considered in this section is that the firm's continuation value at the end of the first period is linear in the manager's continuation payoff.

Statements (ii) and (iii) in Lemma 2 characterize the firm's optimal growth policy, $G(Y_1)$. The direct gain from taking an available growth opportunity is to improve the distribution of the second-period cash flow. The benefit for the firm, captured by the term $p(1-\lambda)\gamma y/(1+r)$ in (9) and (10), is increasing in p and γ and is decreasing in λ and r. The firm's growth policy also affects profit indirectly via its impact on the dismissal wedge and first-period incentives. On the one hand, systematically taking growth opportunities after *poor* performance facilitates incentive provision. Thus, as revealed by (9), only very high values of the replacement cost κ would make it optimal for the firm to forgo an available growth opportunity after poor performance. On the other hand, replacing management to take a growth opportunity after good performance is detrimental to incentive provision and is therefore less attractive, that is, $\hat{\kappa}_{G(y)} < \hat{\kappa}_{G(0)}$.[11]

In the remainder of this section, we assume that

$$\frac{p}{1-p} \geq \frac{1+\varrho}{1+r}\gamma. \tag{11}$$

Combined with (1), this restriction implies that $\kappa < \hat{\kappa}_{G(0)}$ and therefore ensures that the firm systematically pursues growth opportunities after poor performance.[12] However, it may or may not be optimal for the firm to also pursue an available growth opportunity after good performance, as our next result shows. In what follows, we distinguish between two types of configurations: we refer to the case $G(y) = 1$ as the *high-growth regime* and to the case $G(y) = 0$ as the *low-growth regime*. In the former configuration the firm pursues any growth opportunity that arises, while in the latter it pursues an available growth opportunity only upon poor performance.

LEMMA 3: *High-growth and low-growth regimes can both arise.*

The main insight delivered by Lemma 3 is that some firms—namely, low-growth firms—may find it preferable to forgo a growth opportunity following good performance even though they would undertake any such opportunity under first-best, as implied by (1). Indeed, under second-best, it can be optimal for a firm to commit ex ante to forgo growth opportunities after good performance to reduce the cost of incentive provision.

The following lemma characterizes the determinants of a firm's growth regime, thus shedding light on the circumstances under which it is optimal for the firm to grant partial job protection to the initial manager.

[11] Note that $\hat{\kappa}_{G(0)} > \hat{\kappa}_{F(0)}$, so if the firm stands ready to fire the manager after poor performance absent a growth opportunity, then a fortiori it will fire him after poor performance for the sake of growth. If growing the firm involved a specific cost $\chi > 0$, then $\hat{\kappa}_{G(y)}$ and $\hat{\kappa}_{G(0)}$ would both be translated to the left by χ, making it less likely that growth opportunities are undertaken, but none of the results below would be affected.

[12] This assumption, which effectively rules out the possibility that the firm never grows, only serves to simplify the exposition and shorten some of the proofs. All of the empirical implications stated in Propositions 1 and 2 remain true if condition (11) is relaxed.

LEMMA 4: *An increase in γ, y, ϱ, or p or a drop in λ, r, or κ can induce a switch from a low-growth to a high-growth regime. A change in q has no impact on the optimal growth policy.*

Lemma 4 establishes that firms with better opportunities (i.e., high γ) tend to fully expose their managers to the risk of growth-induced turnover. Firms with smaller discount rates (i.e., low r) tend to do the same as they give more weight to the future benefit from growth relative to the compensating increase in first-period compensation. By contrast, firms facing larger turnover costs (i.e., high κ) have a natural tendency to grant partial job protection to their managers, thus forgoing growth opportunities after good performance. Low-growth firms also tend to be plagued by severe agency issues (i.e., high λ). Indeed, as reflected in the expression for the threshold $\hat{\kappa}_{G(y)}$ in (10), a high dismissal probability $G(y)$ is less appealing when moral hazard is more severe, because both the fraction of enhanced second-period cash flows accruing to the firm $(1-\lambda)$ is small and the second-period agency rent is large—implying that any increase in the risk of growth-induced dismissal needs to be matched by a larger increase in $C_1(y) + qG(y)C_g(y)$ to keep the agent incentivized.

B. Empirical Implications

In this section, we derive some of the empirical implications that arise in this simple two-period framework combining growth-induced turnover and moral hazard. The following two propositions summarize the theoretical predictions for managerial turnover and managerial compensation, respectively.

PROPOSITION 1: *The following statements hold true:*

i. The likelihood of turnover is decreasing in performance, that is,

$$qG(y) + (1-q)F(y) \leq qG(0) + (1-q)F(0).$$

ii. The likelihood of turnover, $qG(Y_1) + (1-q)F(Y_1)$, is increasing in the quality of growth opportunities, γ, and in their arrival probability, q.

iii. The probability of turnover is higher when a growth opportunity arises, that is,

$$G(Y_1) \geq F(Y_1).$$

Moreover, the impact of the arrival of a growth opportunity on the probability of turnover is stronger in firms with better opportunities, that is, $G(Y_1) - F(Y_1)$ is increasing in γ.

The results derived in Proposition 1 follow immediately from Lemmas 2 and 4. Statement (i) establishes a negative relationship between firm performance and turnover. This is a standard prediction of dynamic moral hazard models. In particular, the result would hold equally true under second-best in the absence of growth-induced turnover (i.e., in the limit as q goes to zero), when dismissal upon poor performance is used purely as an incentive device. The prediction carries over to our setup because growth-induced dismissal is also less likely to occur after good performance. Statements (ii) and (iii) characterize the impact of ex ante growth prospects and the effect of a growth opportunity realization on the likelihood of turnover, respectively. These predictions are driven primarily by the possibility of growth-induced dismissal introduced in our setup.[13]

The second set of empirical implications emphasizes important features of the compensation scheme under the optimal two-period contract. In particular, when thinking about taking the model predictions to the data, it is useful to consider the average compensation profile $(\bar{C}_1, \bar{C}_2)$, where $\bar{C}_t$ denotes the expected level of compensation that the initial manager receives at time t conditional on running operations in period t:

$$\bar{C}_1 = p[C_1(y) + qG(y)C_g(y)] \quad \text{and} \quad \bar{C}_2 = p\lambda y.$$

PROPOSITION 2: *The following statements hold true:*

i. *Compensation is increasing in performance, that is,*

$$C_1(y) + qG(y)C_g(y) > C_1(0) + qG(0)C_g(0) \quad \textit{and} \quad C_2(Y_1, y) > C_2(Y_1, 0).$$

ii. *The average compensation profile is increasing over tenure, that is, $\bar{C}_1 \leq \bar{C}_2$.*

iii. *Average first-period compensation $\bar{C}_1$ is increasing in the quality of growth opportunities, γ. Hence, the slope of the compensation profile, $\bar{C}_2 - \bar{C}_1$, is decreasing in γ.*

The first statement in Proposition 2 establishes a positive relationship between firm performance and managerial compensation, which immediately follows from Lemma 1.[14] The other two results in the proposition characterize the shape of the average compensation profile $(\bar{C}_1, \bar{C}_2)$. While statement (ii) shows that the compensation profile is back-loaded, statement (iii) emphasizes that the extent of compensation back-loading depends on the firm's ex ante growth prospects.[15] Specifically, the model predicts that firms with better growth prospects tend to have more front-loaded pay. Indeed, as Lemma 4 shows, an improvement in the quality of growth opportunities makes it more likely that the firm finds it optimal to fully expose the initial manager to the risk of growth-induced turnover, setting $G(y) = 1$. In turn, the associated decrease in the dismissal wedge needs to be compensated by an increase in $C_1(y) + qG(y)C_g(y)$ to satisfy the first-period IC constraint (see equation (7)),

[13] It is worth noting, however, that, under the assumption that growth is efficient (see condition (1)), the comparative statics with respect to γ in statements (ii) and (iii) would not hold true under first-best.

[14] Compensation increases with *recent* performance, not with the entire history of performance. Indeed, $C_2(Y_1, Y_2)$ is independent of Y_1, that is, second-period pay does not depend on first-period performance.

[15] The inequality in statement (ii), like that in Proposition 1(i), is strict unless $F(0) = 0$ and $G(y) = 1$. Necessary and sufficient conditions for this case to arise are provided in the proof of Proposition 2 in the Internet Appendix.

which translates into an increase in the average first-period compensation level $\bar{C}_1$. It is worth noting that, whereas statements (i) and (ii) are driven by moral hazard and would also hold true under second-best in the absence of growth-induced dismissal, the last prediction is specific to our setup and is driven by the interaction between moral hazard and the possibility of growth-induced turnover.

II. The Continuous-Time Model

Having previewed the basic economics of growth-induced managerial turnover and its interaction with moral hazard, we now turn to the main focus of our analysis and consider the continuous-time stationary version of the environment introduced in Section I. This modeling setup is better able to fully capture the dynamic nature of the agency relationship between a firm and any of its successive managers.

We consider a firm run by a sequence of managers protected by limited liability. The firm and its managers are risk-neutral, with discount rates r and ϱ, respectively. The firm's operations generate a stream of instantaneous cash flows $\Phi_t\, dY_t$, where Φ_t denotes the size of the firm at time t, and the cumulative size-adjusted cash flow process $Y = \{Y_t\}$ follows

$$dY_t = \mu\, dt + \sigma\, dZ_t, \qquad \mu, \sigma > 0,$$

where $Z = \{Z_t\}$ denotes a standard one-dimensional Brownian motion. The firm starts with unit size ($\Phi_0 = 1$) and can later expand. At any point in time, two conditions must be met for the firm to expand: (i) it must have a growth opportunity, and (ii) it must hire a new manager to take up the opportunity. Growth opportunities arrive sequentially, independent of cash flow shocks, and the waiting time for the arrival of the next opportunity is exponentially distributed with parameter q. If not taken immediately, an opportunity is lost and no further growth is possible until a new one arrives.

As in the context of the two-period framework studied in the previous section, the assumption that value creation entails a change in management is central to our analysis. For convenience, we model value enhancement as a discrete change in firm size that scales up the distribution of cash flows. We assume that, when the firm expands, the size of the firm increases by a factor $1 + \gamma > 1$. Firm growth, when it occurs, is the result of bringing in a new manager able to take advantage of newly available opportunities—thereby achieving a permanent increase in expected cash flows.[16]

The second main feature of the model is a standard agency problem arising from the fact that, while running the firm's operations, managers can divert cash flows. The residual cash flow received by the firm is $\Phi_t\,(dY_t - dA_t)$, where

[16] This may or may not involve an increase in the fixed assets of the firm. If it does, future scaled cash flows should be thought of as net of the financing cost of capital investments.

$A = \{A_t\}$ denotes the cumulative size-adjusted amount of "stealing."[17] Managers enjoy a private benefit $\lambda \in (0, 1]$ for each unit of diverted cash flow, so that λ measures the severity of moral hazard.

The firm has deep pockets and can cover negative cash flows as well as the costs associated with managerial compensation and turnover. As a result, the firm's decisions are not driven by financing constraints. A manager hired to run the firm at size Φ_t has reservation value $\bar{w}\Phi_t$, and the cost of replacing him is $\kappa\Phi_t$, where $\bar{w}, \kappa > 0$ are given constants. While it is natural to assume that the manager's reservation value (which can be interpreted as a nonpecuniary cost of running the firm) and the cost of managerial replacement (which may include disruption costs) are increasing in firm size; the stronger assumption of proportionality is made to ensure size homogeneity and preserve tractability.[18] The continuation value of a departing manager is equal to zero.[19]

We further assume that

$$\varrho > r, \tag{12}$$

$$r > q\gamma, \tag{13}$$

$$\frac{\gamma\mu}{r} > \kappa + (1+\gamma)\bar{w}, \tag{14}$$

and we refer to parameter values that satisfy these conditions, along with those previously imposed in this section, as *permissible*. Condition (12) implies that managers are more impatient than the firm.[20] Condition (13) implies that the average growth rate when the firm takes all growth opportunities is smaller than the firm's discount rate, which ensures finite valuation. Finally, together with (13), condition (14) implies that, in the absence of moral hazard, it would be optimal for the firm to take all growth opportunities—as we next establish.

A. First-Best Policy

The first-best policy can be characterized as follows. First, the optimal compensation policy involves giving to a manager a size-adjusted transfer $\bar{w}$ *at the*

[17] The stealing strategy A chosen by the manager is adapted to the Brownian filtration and has continuous sample paths. Given that Y is continuous, any jump in A would be immediately detected by the firm.

[18] Empirically, executive pay is positively correlated with firm size both over time and across firms, as documented by Kostiuk (1990), Murphy (1999), and Gabaix and Landier (2008). On the other hand, estimates of the various costs associated with CEO transitions for mid-cap companies are roughly twice as large as those borne by small-cap companies, and less than half the costs borne by large-cap companies (Nat Stoddard and Claire Wyckoff, "The Costs of CEO Failure," *Chief Executive Magazine*, Nov/Dec, 2008). Biais et al. (2010) and DeMarzo et al. (2012) make proportionality assumptions similar to ours.

[19] Allowing for a nonzero continuation value would alter the details of our analysis to the extent that this would affect the dynamics of the manager's promise (20) as well as the HJB equation (28).

[20] This assumption is standard in the dynamic contracting literature (e.g., DeMarzo and Sannikov (2006), Biais et al. (2007), Biais et al. (2010), and DeMarzo et al. (2012)). The wedge in discount rates rules out indefinitely postponing payments to managers.

outset of his tenure. Indeed, since managers are more impatient than the firm, deferring compensation would affect firm value negatively. Second, to reduce replacement and hiring costs, managerial turnover only occurs for the sake of pursuing a growth opportunity. Third, the optimal growth policy involves either pursuing all growth opportunities or never pursuing any. If the firm pursues all opportunities, its expected discounted profit V^* satisfies

$$V^* = -\bar{w} + \mathbb{E}\left[\int_0^\tau e^{-rt} dY_t + e^{-r\tau}[(1+\gamma)V^* - \kappa]\right],$$

where τ is the random arrival time of the first growth opportunity. Solving for V^* under the assumption that τ is exponentially distributed with parameter q yields

$$V^* = \frac{\mu - q\kappa}{r - q\gamma} - \frac{r+q}{r-q\gamma}\bar{w}.$$

If instead the firm forgoes all opportunities, its expected discounted profit is given by

$$-\bar{w} + \mathbb{E}\left[\int_0^\infty e^{-rt} dY_t\right] = -\bar{w} + \frac{\mu}{r}.$$

It is straightforward to see that conditions (13) and (14) are sufficient for the inequality

$$V^* > \max\left\{-\bar{w} + \frac{\mu}{r},\ 0\right\}$$

to hold. Therefore, our assumptions ensure that it would be optimal for the firm to pursue all growth opportunities under first-best.

B. Long-Term Incentive Contract

We now turn to the case in which managers can divert cash flows and stealing is not observable by the firm. The firm enters into a long-term contract with each manager at the time he is hired, and both parties fully commit to the terms of the contract. A contract specifies circumstances under which the manager will be dismissed, including when the firm pursues a growth opportunity, as well as the manager's pay over the course of his tenure based on the information that becomes available to the firm over time. The arrival of a growth opportunity is assumed to be perfectly observable and contractible. To fix ideas and simplify the exposition, we initially restrict our attention to the contract with the first manager. Readers interested in the technical aspects of the sequential contracting environment are referred to Internet Appendix Section II.

First, we discuss how dismissal and compensation are determined for a given stealing strategy A. The information available to the firm comes from observing the cumulative reported cash flows $\hat{Y} = Y - A$, as well as the arrival of growth opportunities. We denote by $\mathcal{F}_t$ the information gathered by the firm up to time t, which includes information about the occurrence of growth opportunities. We denote by $\hat{\mathcal{F}}_t \subseteq \mathcal{F}_t$ the information that comes only from the history of reported cash flows up to time t.

The manager can be dismissed for two distinct reasons in our setting. First, the manager can be sacked for poor reported performance. By committing ex ante to fire the incumbent after a history of poor reported cash flows, the firm can incentivize the manager not to steal. Second, the firm can replace the manager to pursue a growth opportunity that becomes available. Hence, turnover is governed in part by the firm's *growth policy*, which determines the firm's response to the potential arrival of a growth opportunity. This policy is modeled by an $(\hat{\mathcal{F}}_t)$-progressively measurable process $G = \{G_t\}$ that takes values in $\{0, 1\}$, with $G_t = 1$ indicating that the firm stands ready to pursue a growth opportunity at time t and $G_t = 0$ indicating that it does not.[21] By controlling G, the firm effectively determines the instantaneous intensity of growth-induced dismissal, which is equal to qG_t at time t.

Using the notation $x \wedge y$ (respectively, $x \vee y$) to denote the minimum (maximum) of x and y, the random time τ at which the manager is fired can be represented as

$$\tau = \tau_d \wedge \tau_g,$$

where τ_d denotes an $(\hat{\mathcal{F}}_t)$-stopping time and the random time τ_g satisfies[22]

$$\mathbb{P}(\tau_g > t \,|\, \hat{\mathcal{F}}_t) = \exp\left(-\int_0^t qG_s\, ds\right). \tag{15}$$

In the event that $\tau = \tau_d$, the manager is replaced for the sake of incentive provision, which we refer to as *disciplinary turnover*. When instead $\tau = \tau_g$, the manager is dismissed for the sake of growth, which we refer to as *growth-induced turnover*.

Compensation to the manager over the course of his tenure is captured by an $(\hat{\mathcal{F}}_t)$-adapted cumulative compensation process $C = \{C_t\}$. Limited liability implies that C is increasing. A positive jump ΔC_t represents a lump-sum payment at time t.[23] In particular, ΔC_0 and ΔC_{τ_d} denote a signing bonus and severance pay upon disciplinary dismissal, respectively. To capture severance pay upon growth-induced turnover, we introduce a separate $(\hat{\mathcal{F}}_t)$-progressively

[21] In the Internet Appendix, we allow G to take values in $[0, 1]$ and show that randomization of the growth decision is suboptimal.

[22] The left-hand side of (15) denotes the probability that the manager has not been dismissed for the sake of growth by time t, conditional on the history of reported cash flows up to time t. The right-hand side captures the fact that the instantaneous intensity of growth-induced dismissal at time $s \leq t$ is qG_s. When the firm stands ready to pursue all growth opportunities, in which case $G \equiv 1$, the probability that the manager survives the threat of growth-induced termination up to time t is given by $\exp(-qt)$, reflecting the fact that the arrival of opportunities is exponentially distributed with parameter q.

[23] We assume that C is right continuous with left limits and $C_{0-} = 0$, and hence $\Delta C_t = C_t - C_{t-}$ and $\Delta C_0 = C_0$. Further details on the modeling of long-term incentive contracts are given in Internet Appendix Section II.A.

measurable process $S = \{S_t\}$. The amount of severance received by a manager dismissed for the sake of growth is given by S_{τ_g}.

Now consider the set $\mathcal{A}$ of all possible stealing strategies. A contract can be viewed as a function mapping each stealing strategy $A \in \mathcal{A}$ to a collection

$$C = C(A), \quad S = S(A), \quad G = G(A), \quad \text{and} \quad \tau_d = \tau_d(A),$$

as just described. Such mapping should be consistent across stealing strategies in the sense that any given history of reported cash flows should result in the same compensation and termination outcomes, independent of the underlying combination of true cash flows and stealing that gave rise to that observed history. The contract space $\mathcal{G}$ identifies with the set of all functionals $\Gamma : A \mapsto (C(A), S(A), G(A), \tau_d(A))$ on $\mathcal{A}$ that satisfy this requirement.

C. The Firm's Problem

Given a contract Γ and a stealing strategy A, the manager's expected discounted payoff at the time he is hired is given by

$$M(\Gamma, A) = \mathbb{E}\left[\int_{[0,\tau[} e^{-\varrho t}(dC_t + \lambda\, dA_t) + e^{-\varrho\tau}(\Delta C_{\tau_d}\mathbf{1}_{\{\tau=\tau_d\}} + S_{\tau_g}\mathbf{1}_{\{\tau=\tau_g\}})\right].$$

For a given contract Γ, a stealing strategy A is said to be *incentive compatible* if it maximizes the manager's payoff. We refer to a contract as *admissible* if it is such that (i) no stealing is incentive compatible and (ii) the manager's expected discounted payoff under no stealing is greater than or equal to his reservation value $\bar{w}$. Formally, the subset $\mathcal{G}_{\mathrm{a}}$ of admissible contracts includes all contracts $\Gamma \in \mathcal{G}$ such that

$$M(\Gamma, 0) = \sup_{A\in\mathcal{A}} M(\Gamma, A) \qquad \text{and} \qquad M(\Gamma, 0) \geq \bar{w}.$$

Given an admissible contract Γ, the firm's expected discounted profit at $t = 0$ is

$$\begin{aligned} F(\Gamma) = \mathbb{E}\Bigg[\int_{[0,\tau]} & e^{-rt}(\mu\, dt - dC_t) + e^{-r\tau}([V_d - \Delta C_{\tau_d} - \kappa]\mathbf{1}_{\{\tau=\tau_d\}} \\ & + [V_g - S_{\tau_g} - \kappa]\mathbf{1}_{\{\tau=\tau_g\}})\Bigg], \end{aligned} \tag{16}$$

where V_d and V_g denote the firm's continuation values after dismissal of the first manager (for disciplinary reasons or to pursue a growth opportunity, respectively), which we endogenize below in Section III.[24] The firm's problem is

[24] In Internet Appendix Section II.B, we provide an expression for the firm's value at $t = 0$ for a given sequence of admissible contracts. In particular, when the same admissible contract Γ is offered to all managers, we show that the firm's size-adjusted expected discounted profit $F(\Gamma)$ satisfies (16) with $V_d = F(\Gamma)$ and $V_g = (1+\gamma)F(\Gamma)$. We restrict our attention to contracts that

to find an admissible contract that maximizes its expected discounted profit. Formally, the firm's objective is to find Γ^* such that

$$F(\Gamma^*) = \sup_{\Gamma \in \mathcal{G}_a} F(\Gamma).$$

D. Admissible Dynamic Contracts

As in previous work on dynamic moral hazard, the challenge in analyzing this type of environment comes from the complexity of the contract space and from the difficulty one faces in evaluating agents' incentives in a tractable way. In this section, we build on the approach of DeMarzo and Sannikov (2006), Sannikov (2008), Biais et al. (2007), and Biais et al. (2010) and consider a state-space representation of incentive contracts. Under no stealing, the state variable in this representation should coincide with the manager's expected payoff. As a preliminary step, we characterize the dynamics of

$$M_t = \mathbb{E}\left[\int_{]t,\tau[} e^{-\varrho(s-t)} dC_s + e^{-\varrho(\tau-t)}(\Delta C_{\tau_d}, \mathbf{1}_{\{\tau=\tau_d\}} + S_{\tau_g}\mathbf{1}_{\{\tau=\tau_g\}}) \,\middle|\, \mathcal{F}_t\right], \quad t < \tau,$$

which represents the manager's expected future payoff at time $t < \tau$ when he refrains from stealing.[25]

LEMMA 5: *For any given contract $\Gamma \in \mathcal{G}$, there exists a process $\beta = \{\beta_t\}$ such that*

$$dM_t = [\varrho M_t + qG_t(M_t - S_t)]dt - dC_t + \sigma\beta_t\, dZ_t, \quad \textit{for } t < \tau. \tag{17}$$

PROOF: See Internet Appendix Section III.A. □

The presence of the diffusion term in the dynamics of the agent's expected payoff is natural. Since compensation and dismissal policies are contingent on the history of reported cash flows, the evolution of the manager's expected future payoff under a long-term incentive contract is sensitive to currently reported cash flows. The process β can be interpreted as the sensitivity induced by a long-term contract. Since reported cash flows coincide with true cash flows when the manager refrains from stealing, the stochastic evolution of the manager's expected payoff under no stealing M_t is driven directly by the true cash flow shocks dZ_t.

implement no stealing, which is standard in the literature when moral hazard is modeled as a cash diversion problem (e.g., see DeMarzo and Sannikov (2006)).

[25] It is worth noting that, in our model, uncertainty is driven not only by the Brownian cash flow shock but also by the stochastic arrival of growth opportunities. As a result, the derivation of (17) does not rely simply on the martingale representation theorem, as in the standard martingale approach developed by Sannikov (2008), but rather also on a "change in filtration" formula and other techniques borrowed from the credit risk literature.

In light of Lemma 5, we consider *dynamic contracts* whose implementation is driven by a state process $W = \{W_t\}$ that evolves according to

$$dW_t = [\varrho W_t + qG_t(W_t - S_t)]\,dt - dC_t + \beta_t\,(d\hat{Y}_t - \mu\,dt). \tag{18}$$

Along with compensation and growth policies, a dynamic contract specifies the sensitivity β of the state variable W to the reported cash flows. Importantly, since the dynamics of the state variable are driven by processes that are either observed or controlled by the firm, its evolution over time can be tracked by the firm. While growth-induced turnover is jointly determined by the growth policy and the random arrival of opportunities, disciplinary dismissal occurs when the state process W hits zero, that is,

$$\tau_d = \inf\{t \geq 0 :\ W_t = 0\}. \tag{19}$$

Noting that $d\hat{Y}_t - \mu\,dt = -dA_t + \sigma\,dZ_t$, it is straightforward to see that, when the manager refrains from stealing, the dynamics of the state process become

$$dW_t = [\varrho W_t + qG_t(W_t - S_t)]\,dt - dC_t + \sigma\beta_t\,dZ_t \tag{20}$$

and hence mirror (17). Indeed, when the manager refrains from stealing, the value taken by the state variable at any time during his tenure coincides with his expected future compensation under the contract, as stated in the following lemma.

LEMMA 6: *Consider a dynamic contract with termination occurring at time $\tau = \tau_g \wedge \tau_d$, where τ_g satisfies (15), τ_d is defined by (19), and W follows (18) for some initial condition $W_{0-} = w_{\text{init}} > 0$. Then the manager's expected future payoff at time $t < \tau$ if he refrains from stealing is equal to W_t, that is,*

$$W_t = \mathbb{E}\left[\int_{]t,\tau[} e^{-\varrho(s-t)}dC_s + e^{-\varrho(\tau-t)}(\Delta C_{\tau_d}\mathbf{1}_{\{\tau=\tau_d\}} + S_{\tau_g}\mathbf{1}_{\{\tau=\tau_g\}})\,\middle|\,\mathcal{F}_t\right], \tag{21}$$

on the event $\{t < \tau\}$. Moreover, if $\beta \geq \lambda$, it is optimal for the manager not to steal.

PROOF: See Internet Appendix Section III.C. □

Equation (21) confirms that the state process W under a dynamic contract can be interpreted as the manager's expected future payoff if he refrains from stealing, which we refer to as the manager's promise. Lemma 6 also establishes an incentive compatibility condition, extending the one derived by DeMarzo and Sannikov (2006) to our environment with growth-induced turnover. This condition is intuitive: Since the manager enjoys a private benefit λ per unit of diverted cash, incentivizing the manager not to steal requires that his promise increases by at least λ for each extra unit of reported cash flow, that is, $\beta \geq \lambda$. A dynamic contract is admissible if it satisfies this condition as well as the initial promise condition $W_{0-} \geq \bar{w}$. Since $\beta \geq \lambda > 0$ under an admissible dynamic contract, (18) and (19) imply that inefficient disciplinary turnover occurs as a result of *poor* reported cash flows.

Before proceeding further, it is important to observe that, relative to the environment considered in DeMarzo and Sannikov (2006), the introduction of growth-induced turnover affects the dynamics of the agent's promise in a substantial way. The key difference lies in the drift of the promise, which in our setup is equal to $\varrho W_t + qG_t(W_t - S_t)$ instead of simply ϱW_t. The reason for this difference is that, whenever the manager is at risk of being fired for the sake of growth (i.e., whenever $G_t = 1$), he needs to be "compensated" for the loss that he would incur if a growth opportunity arises. The potential loss corresponds to the difference $(W_t - S_t)$ between the manager's current promise and the severance pay he would receive if replaced for the sake of growth, while the chances of incurring such a loss are determined by the instantaneous intensity of growth-induced dismissal qG_t. Compensation for the risk of growth-induced termination comes in the form of an augmented drift, which translates into a faster increase in the manager's promise in states of the world in which no growth opportunity materializes. In other words, in our setup the law of motion for the agent's promise is modified in such a way that the *promise-keeping* condition remains satisfied.

III. Optimal Dynamic Contract

Having characterized the set of admissible dynamic contracts, we reformulate the firm's optimization problem as a stochastic control problem. We denote by $V(\phi, w)$ the firm's value function, which gives the firm's expected discounted profit at a given current size ϕ and for a given size-adjusted promise w to the incumbent manager. The firm's value function satisfies the recursive dynamic programming equation

$$V(\phi, w) = \sup_{C,S,G,\beta} \mathbb{E}\Bigg[\phi \int_{[0,\tau[} e^{-rt}(\mu dt - dC_t) - \phi e^{-r\tau}\left(\Delta C_{\tau_d}\mathbf{1}_{\{\tau=\tau_d\}} + S_{\tau_g}\mathbf{1}_{\{\tau=\tau_g\}}\right) + e^{-r\tau}(-\kappa\phi + V_d\mathbf{1}_{\{\tau=\tau_d\}} + V_g\mathbf{1}_{\{\tau=\tau_g\}})\Bigg], \tag{22}$$

where

$$\tau = \tau_d \wedge \tau_g, \quad V_d = V\left(\phi, w_h^{\phi}\right), \quad \text{and} \quad V_g = V\left((1+\gamma)\phi, w_h^{(1+\gamma)\phi}\right), \tag{23}$$

subject to the incentive compatibility constraint $\beta \geq \lambda$ and subject to (15), (19), and (20) with initial condition $W_{0-} = w$. In this formulation, C and S denote the manager's size-adjusted cumulative compensation and size-adjusted severance upon growth, respectively, and the continuation values V_d and V_g involve *hiring promises*

$$w_h^{\phi} = \bar{w} \vee \arg\max_{w>0} V(\phi, w) \quad \text{and} \quad w_h^{(1+\gamma)\phi} = \bar{w} \vee \arg\max_{w>0} V((1+\gamma)\phi, w), \tag{24}$$

respectively. Expression (24) captures the possibility that a new manager's participation constraint may not be binding, as it may be optimal for the firm to give him a rent in excess of his reservation value. Since cash flows, turnover costs, and reservation values are all proportional to firm size, it follows that firm value itself is homogeneous in size, that is,

$$V(\phi, w) = \phi V(1, w) =: \phi v(w). \tag{25}$$

In particular, stationarity and size homogeneity imply that the firm offers the same dynamic contract to all successive managers. Using (22) to (25), the size-adjusted value function $v(w)$ is determined along with the optimal contract by

$$\begin{aligned} v(w) = \sup_{C,S,G,\beta} \mathbb{E}\Bigg[& \int_{[0,\tau]} e^{-rt}(\mu dt - dC_t) - e^{-r\tau}\left(\Delta C_{\tau_d}\mathbf{1}_{\{\tau=\tau_d\}} + S_{\tau_g}\mathbf{1}_{\{\tau=\tau_g\}}\right) \\ & + e^{-r\tau}(-\kappa + v(w_h)\mathbf{1}_{\{\tau=\tau_d\}} + (1+\gamma)v(w_h)\mathbf{1}_{\{\tau=\tau_g\}})\Bigg] \end{aligned} \tag{26}$$

subject to the same constraints as above, where the size-adjusted hiring promise w_h satisfies

$$w_h = \bar{w} \vee \arg\max_{w>0} v(w). \tag{27}$$

The following proposition is central to our characterization of the optimal dynamic contract.

PROPOSITION 3: *Let $u : \mathbb{R}_+ \to \mathbb{R}$ be a concave C^2 function that satisfies the HJB equation*

$$\begin{aligned} \max \Bigg\{ & \frac{\sigma^2\lambda^2}{2}u''(w) + \varrho w u'(w) - ru(w) + \mu \\ + \quad & q\left[(1+\gamma)u(w_h) - \kappa + wu'(w) - u(w)\right]^+, -u'(w) - 1\Bigg\} = 0 \end{aligned} \tag{28}$$

with boundary condition

$$u(0) = u(w_h) - \kappa, \tag{29}$$

where $w_h = \bar{w} \vee \arg\max_{w>0} u(w)$. Also, suppose that $\lim_{w\downarrow 0} |u'(w)| < \infty$ and $u'(w) = -1$ for some $w < \infty$. Then the function u identifies with the value function v defined by (26), that is, $v(w) = u(w)$ for all $w \geq 0$. Moreover, the optimal dynamic contract satisfies Properties 1 to 5 below.

PROOF: See Internet Appendix Section IV. □

We rely on Proposition 3 to construct the firm's value function and solve for the optimal dynamic contract. As observed in previous work on dynamic moral hazard, the concavity of the value function is related to the fact that a change in w affects firm value not only directly by increasing the amount of compensation owed to the manager, but also via its impact on the likelihood of disciplinary turnover. Indeed, by reducing the prospect of costly disciplinary turnover, a one-dollar increase in the agent's promise effectively costs the firm less than one dollar. Moreover, since the probability of disciplinary turnover is higher after poor performance, the reduction in agency costs induced by a marginal increase in the agent's promise is larger for low values of w. This is what gives rise to concavity.

A. Optimality Properties

We now turn to the properties satisfied by the optimal dynamic contract, as implied by Proposition 3. These properties impose restrictions on the cash flow sensitivity, the compensation policy, and the growth policy. The first two properties also hold in the absence of growth opportunities and are derived in that context by DeMarzo and Sannikov (2006) and Biais et al. (2007).[26]

PROPERTY 1: *The optimal contract has sensitivity to reported cash flows $\beta = \lambda$.*

The fact that the incentive compatibility constraint should hold as an equality ($\beta = \lambda$) is related to the concavity of the value function. Intuitively, reducing the volatility of the manager's promise as much as possible while satisfying incentive compatibility is optimal for the firm because it lowers the probability that the promise hits zero, which would result in ex post inefficient disciplinary turnover.

PROPERTY 2: *The optimal compensation policy is such that the manager receives transfers only if his current promise w is at least w_c. The compensation threshold w_c satisfies*

$$v'(w_c) = -1.$$

This property can be explained heuristically by observing that, at any instant, the firm has the option to make an immediate transfer to the manager and continue optimally. Hence, the inequality $v(w) \geq -\varepsilon + v(w - \varepsilon)$ holds for any transfer ε, which implies $v'(w) \geq -1$. When the manager's current promise w is such that $v'(w) > -1$, deferring compensation is optimal. By concavity of the value function, this happens when w is below the point w_c that satisfies $v'(w_c) = -1$. In this case, the manager receives no compensation until his promise reaches the compensation threshold. If $\bar{w} > w_c$, the manager receives a signing bonus $\Delta C_0 = \bar{w} - w_c$ when appointed, and his promise later remains in the interval $[0, w_c]$.[27]

[26] From a broader perspective, the smooth pasting condition in Property 2 is a standard feature of the solution to singular stochastic control problems such as that given by (26). See Beneš, Shepp, and Witsenhausen (1980) and Karatzas (1983) for early references.

[27] Technically, the agent's promise W is *reflected* at w_c by the cumulative compensation process. A rigorous construction of this process is provided in Internet Appendix Section IV (see Theorem

PROPERTY 3: *The optimal compensation policy involves no severance payment, that is,* $\Delta C_{\tau_d} = 0$ *and* $S = 0$*.*

Property 3 establishes that severance pay is *strictly* suboptimal in the continuous-time setting, even in the case of growth-induced termination. The reason is that, rather than give cash to a departing manager, the firm is always better off increasing the promise of the incumbent conditional on him being retained, which has the benefit of reducing the likelihood of inefficient turnover later.[28] This result is in contrast to the indifference result that holds in the two-period model where disciplinary dismissal can occur only at the end of the first period (see the discussion following Lemma 1 and footnote 10). It is also worth noting that the no-severance result upon growth-induced dismissal relies crucially on the assumption that the arrival of a growth opportunity is contractible.[29]

PROPERTY 4: *It is optimal for the firm to stand ready to pursue a growth opportunity if and only if the manager's current promise* w *is such that*

$$(1+\gamma)v(w_h) - \kappa + wv'(w) \geq v(w). \tag{30}$$

Condition (30), which we refer to as the *growth optimality condition*, determines the circumstances under which growth-induced turnover can occur. The inequality implies that the optimal growth policy relies not only on a comparison between the status quo continuation value $v(w)$ and the continuation value upon growth $(1+\gamma)v(w_h) - \kappa$. Rather, the extra term $wv'(w)$ accounts for the fact that putting the manager at risk of being fired if a growth opportunity arrives requires that he be compensated in the form of an augmented drift, as discussed in Section II.D. When the firm's value function is decreasing at the current value of the agent's promise (i.e., $v'(w) < 0$), this higher drift constitutes a cost. If this cost is high relative to the potential gains from growth, so that $(1+\gamma)v(w_{\mathrm{h}}) - \kappa + wv'(w) - v(w) < 0$, it is optimal for the firm to insulate the incumbent manager from the risk of being replaced and thus forgo growth opportunities when they become available. We refer to this possibility as contractual *job protection*.

IA2). One way in which the introduction of stochastic growth opportunities and growth-induced turnover modifies the firm's compensation policy is by affecting the value of the optimal threshold w_c.

[28] By the same logic, severance pay would be suboptimal in a simpler setting with exogenous random exit of the manager. We are grateful to an anonymous referee for making this observation.

[29] Positive severance upon growth-induced dismissal would arise as part of the optimal contract if the availability of a growth opportunity were privately observed by the incumbent manager (see Anderson, Bustamante, and Guibaud (2012)). This result is reminiscent of Eisfeldt and Rampini (2008) and Inderst and Mueller (2010), although severance pay serves to incentivize the incubent to reveal *bad* news in their setups. Severance pay upon *disciplinary dismissal* arises in the model analyzed by He (2012) with a risk-averse agent and private savings.

PROPERTY 5: *If partial job protection arises as part of the optimal contract, the firm forgoes growth opportunities if the manager's promise w is above w_g, where the growth threshold w_g satisfies*

$$w_g = \sup\{w \geq 0 : (1+\gamma)v(w_h) - \kappa + wv'(w) - v(w) \geq 0\} < w_c.$$

This property indicates that if some degree of job protection arises as part of the optimal incentive contract, managers are shielded from the risk of growth-induced turnover after good performance. Intuitively, the benefit of retaining the incumbent, net of the forgone gains from growth, is increasing in w because losses due to moral hazard under the incumbent are lower after good performance.[30]

B. Two Types of Firms

In light of our discussion of Properties 4 and 5, two configurations can arise—echoing the two growth regimes encountered in the analysis of the two-period model. In the first configuration, the growth optimality condition (30) holds for all values of the manager's promise $w \in [0, w_c]$. We refer to firms falling into this configuration as *high-growth firms*. In such firms, managers are fully exposed to the risk of being fired for the sake of growth, and the instantaneous rate of growth-induced turnover is always equal to q. Over the course of the manager's tenure, the firm keeps track of the evolution of

$$dW_t = (\varrho + q)W_t\,dt - dC_t + \lambda\,(d\hat{Y}_t - \mu\,dt), \quad W_{0-} = w_h,$$

where transfers dC reflect the manager's promise W at the endogenous compensation threshold w_c. Transfers to the manager can be interpreted as *bonuses* indexed on reported earnings in excess of a performance target, as documented in Murphy (1999). The manager is dismissed when a growth opportunity arises or when W hits zero, whichever comes first.

By contrast, in the second possible configuration, the growth optimality condition does not hold everywhere on the interval $[0, w_c]$, and some degree of job protection is part of the optimal contract. We refer to firms falling into the latter configuration as *low-growth firms*. The contract offered by a low-growth firm specifies, along with a compensation threshold w_c, a growth threshold $w_g < w_c$. Over the course of the manager's tenure, the firm keeps track of

$$dW_t = [\varrho + q\mathbf{1}_{[0,w_g]}(W_t)]W_t\,dt - dC_t + \lambda\,(d\hat{Y}_t - \mu\,dt), \quad W_{0-} = w_h,$$

where bonus transfers dC reflect W at w_c. The manager is dismissed if a growth opportunity arises when $W_t \leq w_g$ or when W hits zero, whichever comes first. Consistent with our discussion of Property 5, the optimal contract in low-growth firms requires that, whenever the manager's promise is above the

[30] In other words, the net benefit of exposing a manager to the risk of growth-induced termination is decreasing in the manager's promise, as revealed by the fact that $(1+\gamma)v(w_h) - \kappa + wv'(w) - v(w)$ is decreasing in w. This is in line with the observation that $\hat{\kappa}_{G(y)} < \hat{\kappa}_{G(0)}$ in the two-period model.

growth threshold w_g, the firm forgoes any growth opportunity that becomes available.[31]

The insights obtained from our analysis of the two-period model in Section I are useful to understanding why the low-growth configuration may sometimes be optimal even though condition (14) guarantees that forgoing growth opportunities is inefficient under first-best. The reason is that exposing managers to early termination risk increases the cost of incentivizing them. In the continuous-time model, this is manifested by the fact that putting a manager at risk of being replaced for the sake of growth effectively makes him more impatient, as revealed by the augmentation of the drift of the contractual promise (which reflects the manager's "effective" discount rate) from ϱ to $\varrho + q$. In the presence of moral hazard, a firm thus faces an ex ante trade-off: A policy of always standing ready to pursue growth by appointing a new, more suitable manager has the advantage of producing higher expected cash flows, but it also entails increased early termination risk for incumbent managers and a higher cost of incentive provision during their tenure. In low-growth firms, the resolution of the trade-off between efficient turnover and the cost of incentive provision gives rise to an interior solution whereby the optimal contract allows for job protection after good performance.[32]

C. High-Growth Firms

In this section, we further characterize the optimal contract offered by a high-growth firm. To this end, we consider the free-boundary problem that consists of finding a free-boundary point w_c and a function u that satisfies the ordinary differential equation (ODE)

$$\frac{\sigma^2\lambda^2}{2}u''(w) + (\varrho + q)wu'(w) - (r+q)u(w) + \mu + q\left[(1+\gamma)u(w_h) - \kappa\right] = 0 \tag{31}$$

in the interval $(0, w_c)$, is given by

$$u(w) = u(w_c) - (w - w_c), \quad \text{if } w > w_c, \tag{32}$$

and satisfies the boundary conditions

$$u(0) = u(w_h) - \kappa, \quad u'(w_c) = -1, \quad \text{and} \quad u''(w_c) = 0, \tag{33}$$

[31] The manager being partially shielded from the risk of growth-induced turnover might be described as an endogenous form of "entrenchment." We do not use this terminology because it more commonly refers to actions taken by a manager to make his replacement costly. A number of recent papers explore frameworks very different from ours where they establish conditions under which managers are protected from termination (see, for example, Atkeson and Cole (2008), Casamatta and Guembel (2010), and Garrett and Pavan (2012)).

[32] A third possible configuration involves fully isolating managers from the risk of growth-induced termination, which corresponds to (30) being violated for all values of $w \in [0, w_c]$. However, we show in Internet Appendix Section VI.E that this no-growth policy can be optimal only if $v(w_h) < 0$, so that the firm would rather not operate. We do not expand on this case further in the remainder of our analysis.

where

$$w_h = \bar{w} \vee \underset{w>0}{\arg\max}\, u(w). \tag{34}$$

PROPOSITION 4: *Given any permissible values of* $(r, \varrho, \mu, \sigma, q, \gamma, \lambda, \kappa, \bar{w})$ *in* $\mathbb{R}^9$*, there exists a unique solution* (u, w_c) *to the free-boundary problem defined by (31) to (34). The function u is* C^2 *and concave, and satisfies the HJB equation*

$$\max\left\{\frac{\sigma^2\lambda^2}{2}u''(w) + (\varrho + q)wu'(w) - (r+q)u(w) + \mu + q[(1+\gamma)u(w_h) - \kappa],\right.$$
$$\left. -u'(w) - 1\right\} = 0.$$

Furthermore, the following statements hold:

i. *The set of permissible parameter values over which u satisfies (30) for all* $w \in [0, w_c]$*, and therefore the HJB equation (28), has nonempty interior in* $\mathbb{R}^9$.

ii. *There exists a unique point* $\bar{w}_\dagger = \bar{w}_\dagger(\frac{\varrho+q}{\sigma^2\lambda^2}, \frac{r+q}{\sigma^2\lambda^2}, \kappa)$ *such that*

$$w_\circ := \arg\max_{w>0} u(w) \geq \bar{w} \;\Leftrightarrow\; \bar{w} \leq \bar{w}_\dagger \quad \textit{and} \quad w_\circ = \bar{w} \;\Leftrightarrow\; \bar{w} = \bar{w}_\dagger.$$

iii. *There exists a unique point* $\bar{w}_\ddagger = \bar{w}_\ddagger(\frac{\varrho+q}{\sigma^2\lambda^2}, \frac{r+q}{\sigma^2\lambda^2}, \kappa) > \bar{w}_\dagger$ *such that*

$$w_c \geq w_h \;\Leftrightarrow\; \bar{w} \leq \bar{w}_\ddagger \quad \textit{and} \quad w_c = w_h = \bar{w} \;\Leftrightarrow\; \bar{w} = \bar{w}_\ddagger.$$

iv. *If* $\bar{w} = \bar{w}_\ddagger$*, u satisfies (30) for all* $w \in [0, w_c]$ *and the HJB equation (28) if and only if*

$$\gamma\mu \geq r\kappa + (r + \gamma\varrho)\bar{w}. \tag{35}$$

PROOF: See Internet Appendix Section VI.B. □

In view of Proposition 3 and the general properties of the solution to the free-boundary problem established in Proposition 4, statement (i) implies that, for a large set of permissible parameter values, the firm is of the high-growth type. For such parameter values, the firm's size-adjusted value function and the optimal compensation threshold are given, along with the hiring promise, by the solution to the free-boundary problem in (31) to (34). Figure 1 illustrates the firm's value function and the optimal compensation threshold in the high-growth configuration for particular parameter values.

Statement (ii) establishes the condition under which it is optimal for a high-growth firm to grant a new manager a compensation rent in excess of his reservation value. The hiring promise w_h is optimally set above the manager's reservation promise $\bar{w}$ when the latter is sufficiently low, that is, $\bar{w} \leq \bar{w}_\dagger$. In this case, the initial promise w_h is equal to the level $w_\circ \geq \bar{w}$ that maximizes the

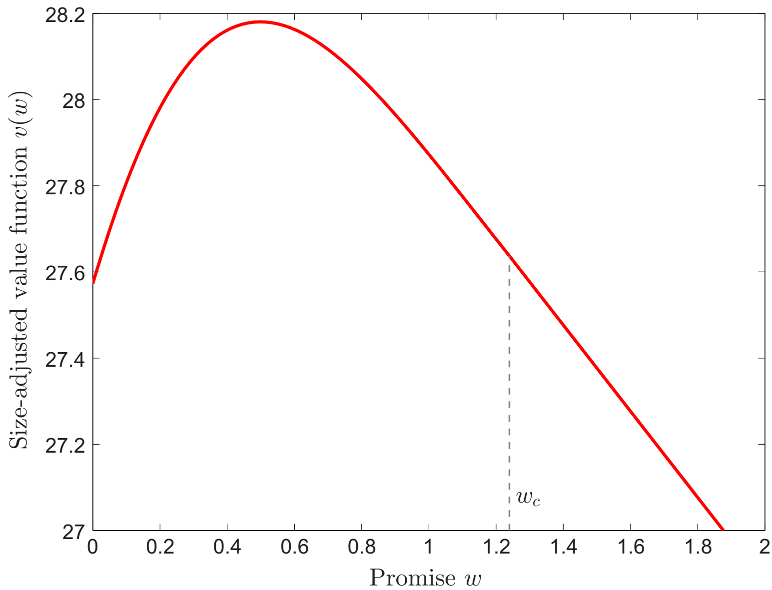

Figure 1. Value function, high-growth firm. The figure depicts the firm's value function and the optimal compensation threshold w_c for parameter values $r = 7\%$, $\varrho = 16\%$, $\mu = 1$, $\sigma = 1$, $q = 0.2$, $\gamma = 0.25$, $\lambda = 0.4$, $\kappa = 0.3$, and $\bar{w} = 1$. The firm's value function and the compensation threshold are determined by solving the free-boundary problem defined by (31) to (34). The growth optimality condition (30) holds for all values of the manager's promise. (Color figure can be viewed at wileyonlinelibrary.com)

firm's value. Otherwise, the manager's participation constraint is binding so that the hiring promise coincides with his reservation value (i.e., $w_h = \bar{w} > w_\circ$).

Statement (iii) sheds light on the optimal compensation policy at the start of a manager's tenure. Depending on the value of the reservation promise $\bar{w}$, three scenarios can arise. When the reservation value is relatively low, in the sense that $\bar{w} < \bar{w}_\ddagger$, the compensation threshold is optimally set above the hiring promise ($w_c > w_h$). In this case, a newly hired manager does not receive any pay until the effect of the positive drift $\varrho + q$, possibly combined with good cash flow realizations, finally raises his promise up to the compensation threshold w_c. In contrast, if the reservation promise is high enough ($\bar{w} > \bar{w}_\ddagger$), a manager receives a signing bonus $\Delta C_0 = \bar{w} - w_c > 0$ when appointed. In the case in which $\bar{w} = \bar{w}_\ddagger$, the hiring promise and compensation threshold are such that $w_c = w_h = \bar{w}$, and the manager starts receiving compensation immediately after taking office.

Statement (iv) provides an explicit condition on exogenous parameter values for the firm to be a high-growth firm. Condition (35) suggests that high-growth firms tend to be the ones that are more productive (high μ) or have better opportunities (high γ). Our next proposition gives further insight into the characteristics of high-growth firms.

PROPOSITION 5: *Consider any permissible values of* $(r, \varrho, \mu, \sigma, q, \gamma, \lambda, \kappa, \bar{w})$ *in* $\mathbb{R}^9$ *such that* $\bar{w} = \bar{w}_{\ddagger}$ *and condition (35) holds with equality. A marginal increase in* λ, σ, *or* κ, *or a marginal decrease in* γ, q, *or* μ, *leads condition (35) to fail.*

PROOF: See Internet Appendix Section VI.C. □

In view of statement (iv), the above proposition suggests that high-growth firms also tend to be characterized by not-too-severe moral hazard and low turnover costs (low λ and κ). These results confirm the insights derived in the two-period framework (see Lemma 4).[33] Proposition 5 further suggests that frequent growth opportunities (high q) and not-too-volatile cash flows (low σ) are other attributes of high-growth firms. In particular, because of the cumulative nature of growth in our stationary environment, more frequent growth opportunities makes it more valuable to pursue any such opportunity.

Finally, we characterize the determinants of the compensation threshold in high-growth firms with the following proposition. At this point, it is worth noting that, holding the dynamics of the manager's promise constant, a lower (higher) compensation threshold results in more front-loaded (back-loaded) compensation.

PROPOSITION 6: *Consider* $(r, \varrho, \mu, \sigma, q, \gamma, \lambda, \kappa, \bar{w})$ *in the interior of the set of permissible parameter values over which the firm is a high-growth firm.*

i. *The optimal compensation threshold* w_c *is increasing in* κ *and is independent of* μ *and* γ.
ii. *If the parameter values are initially such that* $\bar{w} = \bar{w}_{\ddagger}$, *then a marginal increase in* λ *or* σ *leads to an increase in* w_c, *whereas a marginal increase in* q *leads to a reduction in* w_c.

PROOF: See Internet Appendix Section VI.D. □

As the severity of moral hazard, the volatility of cash flows, or the cost of managerial replacement increases, the compensation threshold is raised to reduce the likelihood of inefficient turnover. On the other hand, an increase in the arrival rate of growth opportunities, by increasing the manager's effective discount rate, results in a lower compensation threshold. Furthermore, the quality of growth opportunities, γ, has no impact on the optimal compensation scheme conditional on the firm being of the high-growth type, although it may affect the shape of the compensation profile to the extent that it alters the firm's growth regime (as in the two-period model).[34] We illustrate the implications of our model for the timing of compensation in Section III.E.

[33] The assumption $\bar{w} = \bar{w}_{\ddagger}$, under which Propositions 5 and 6(ii) are derived, implies the identities $w_c = w_h = \bar{w}$, which facilitate the proofs of our results. Establishing such results globally is beyond the scope of this paper.

[34] The result that the threshold w_c is unaffected by the mean size-adjusted cash flow μ differs from the result derived in DeMarzo and Sannikov (2006), where w_c is increasing in μ. This is because the firm's continuation value upon termination is exogenously given in their setup, whereas it is endogenously determined in ours.

D. Low-Growth Firms

We now turn to the low-growth configuration. In light of Proposition 3 and Property 5, we consider the free-boundary problem that consists of finding two free-boundary points w_c and $w_g < w_c$ and a function u that satisfies the ODE

$$\frac{\sigma^2\lambda^2}{2}u''(w) + (\varrho + q)wu'(w) - (r+q)u(w) + \mu + q[(1+\gamma)u(w_h) - \kappa] = 0 \quad (36)$$

in the interval $(0, w_g)$, satisfies the ODE

$$\frac{\sigma^2\lambda^2}{2}u''(w) + \varrho wu'(w) - ru(w) + \mu = 0 \quad (37)$$

in the interval (w_g, w_c), is given by

$$u(w) = u(w_c) - (w - w_c), \quad \text{if } w > w_c, \quad (38)$$

satisfies the boundary conditions given by (33), and satisfies the requirement that

$$u(w_g) - w_g u'(w_g) = (1+\gamma)u(w_h) - \kappa, \quad (39)$$

where w_h is defined as in (34). The analysis of this problem allows us to establish that, despite the assumption that forgoing growth opportunities is suboptimal under first-best, the low-growth configuration can arise, as stated in the following proposition.

PROPOSITION 7: *The set of permissible parameter values over which the solution u to the free-boundary problem defined by (33)–(34) and (36)–(39) is C^2 and concave, and satisfies the HJB equation (28), has nonempty interior in $\mathbb{R}^9$. For such parameter values, the firm is a low-growth firm.*

PROOF: See Internet Appendix Section VII.C. □

In low-growth firms, the optimal contract is as described in Section III.B, with growth and compensation thresholds given by the free-boundary points w_g, w_c of the free-boundary problem defined above and with the hiring promise w_h endogenously determined as part of the problem.[35] Figure 2 depicts the firm's value function along with the optimal thresholds in the low-growth configuration for particular parameter values. The figure also represents what the value of the firm would be if it were constrained to systematically pursue all growth opportunities as they come. The distance between the two curves in the figure illustrates the benefit that a low-growth firm derives from offering partial job protection to its managers, which can ultimately be traced back to a reduction in agency costs.

[35] In Internet Appendix Section VII.A, we derive five possible systems of highly nonlinear equations that should be solved to determine the points w_g, w_c, and w_h; see in particular Problem IA4. Given the complexity of this problem, providing a complete characterization of its solution to derive a suitable solution to the HJB equation (28) with boundary condition (29) is beyond the scope of this paper.

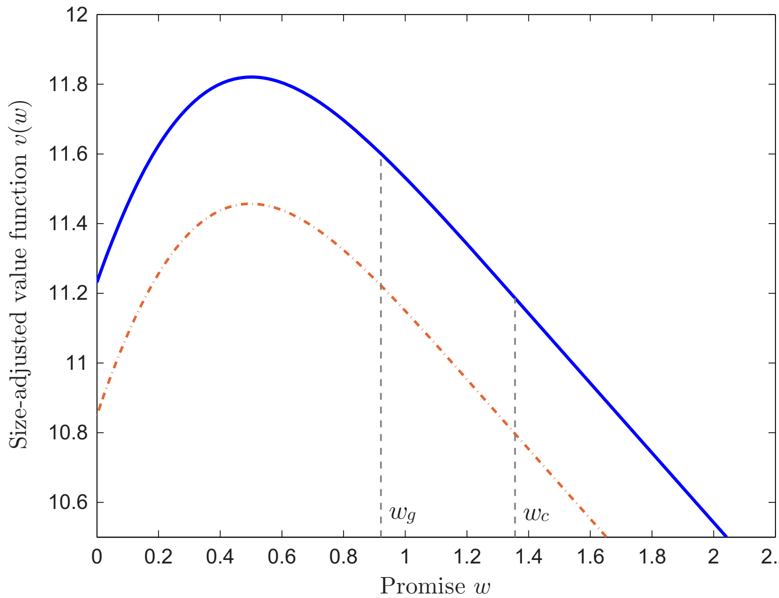

Figure 2. Value function, low-growth firm. The figure depicts the firm's value function (solid line), along with the optimal growth and compensation thresholds (w_g, w_c), for parameter values $r = 7\%$, $\varrho = 16\%$, $\mu = 1$, $\sigma = 1$, $q = 0.2$, $\gamma = 0.1$, $\lambda = 0.4$, $\kappa = 0.3$, and $\bar{w} = 1$. The firm's value function and the thresholds are determined by solving the free-boundary problem defined by (36) to (39) along with (33) and (34). The growth optimality condition (30) holds on $[0, w_g]$ but is violated for $w > w_g$. The figure also plots what firm value would be if the firm were constrained to pursue all growth opportunities (dashed line), with the compensation threshold optimally determined by the solution to the high-growth free-boundary problem. (Color figure can be viewed at wileyonlinelibrary.com)

It is worth noting that, our finding that, in some firms, growth may occur only after poor performance contrasts with the result obtained in setups where the firm can grow through investment with the incumbent (see, e.g., DeMarzo and Fishman (2007a)). In such settings, growth is positively related to past performance because the return on investment is higher after good cash flows, due to a reduction in agency costs. The opposite prediction arises in our setup because the net benefit of exposing a manager to the risk of growth-induced termination is lower after good performance, also due to a reduction in agency costs. In practice, the relevance of these two mechanisms should depend on the extent to which growth is of a "transformative" nature, that is, on whether pursuing a growth opportunity requires a change in management.[36]

[36] The investment–cash flow sensitivity literature points to a positive relationship between investment and past performance. However, these studies do not shed light on the empirical validity of the mechanism at play in our model because they do not account for the fact that firms may grow as a result of marginal changes or of radical transformations requiring a change in top management, nor do they account for the possibility that the firm's value may increase without capital investment.

E. A Numerical Example

We now use numerical simulations to illustrate how a firm's growth prospects may affect managerial turnover and compensation in our setup. The numerical example also provides a sense of the quantitative properties of the continuous-time model.

Frequency of Managerial Turnover. In our setting, the probability of an incumbent manager being dismissed depends on the past performance of the firm under his tenure, as well as on the availability of a growth opportunity and on the ex ante firm characteristics that affect its turnover policy. First, the likelihood of dismissal increases with poor performance—both because a string of bad cash flows can result in disciplinary turnover and because, in some firms, growth-induced turnover occurs only after poor performance. Second, holding performance and firm characteristics constant, the probability of dismissal also increases (at least weakly) upon arrival of a growth opportunity. Finally, the probability of turnover depends on firm characteristics, to the extent that they affect the contract specification and the degree of protection granted to the manager. In particular, firms with better growth prospects should have a higher turnover rate.

To see this last point, consider two firms that are identical in every dimension except for the size (γ) of the growth opportunities they might receive. For the sake of illustration, we take as common parameter values across the two firms $r = 7\%$, $\varrho = 16\%$, $\mu = 1$, $\sigma = 1$, $q = 0.2$, $\lambda = 0.4$, $\kappa = 0.3$, and $\bar{w} = 1$. In the firm with better growth prospects, we set $\gamma = 0.25$; we set $\gamma = 0.10$ in the other firm.[37] The difference in the quality of growth opportunities faced by the two firms makes the former a high-growth firm (i.e., a manager in this firm is never immune to the risk of growth-induced termination), and the latter a low-growth firm (i.e., managers are protected from growth-induced turnover after good performance). The average annualized turnover rate in these two firms is 21.4% and 5.5%, respectively. Changes in growth prospects driven by the arrival rate (q) of growth opportunities have similar effects. To see this, consider variations in q around the high-growth and low-growth baselines. For the high-growth firm, an increase in the frequency of growth opportunities from $q = 0.20$ to $q = 0.22$ causes the average turnover rate to increase to 23.2%, while a decrease to $q = 0.18$ causes the turnover rate to drop to 19.4%. For the low-growth firm, the same variations in q cause the average turnover rate to rise to 5.7% or to drop to 5.3%, respectively.

[37] These parameter values are permissible. In particular, the firms' growth prospects are sufficiently attractive as to make pursuing all growth opportunities optimal in the absence of moral hazard. Discount rates r and ϱ, and intensity rate q, are expressed on an annual basis. Given the normalization $\mu = 1$, parameters σ, κ, and $\bar{w}$ are effectively expressed in terms of annual mean cash flow. For given parameter values, we first determine the firm's type and the optimal contractual threshold(s) by solving numerically the free-boundary problems associated with the HJB equation (28) with boundary condition (29). The average turnover rate is then obtained by simulating the dynamics of the promise W under the optimal contract until dismissal for a very large number of managers.

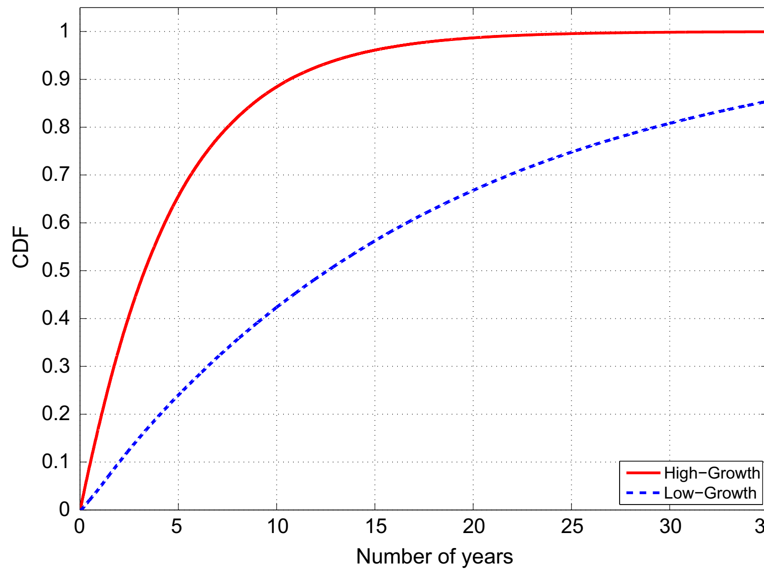

Figure 3. Distribution of tenure length: high-growth vs. low-growth firms. The figure depicts model-implied cumulative distribution functions obtained from simulations for parameter values $r = 7\%$, $\varrho = 16\%$, $\mu = 1$, $\sigma = 1$, $q = 0.2$, $\gamma = 0.25$ (high-growth) or $\gamma = 0.10$ (low-growth), $\lambda = 0.4$, $\kappa = 0.3$, and $\bar{w} = 1$. Optimal contractual thresholds are $w_c = 1.24$ in the high-growth case, and $w_g = 0.92$ and $w_c = 1.36$ in the low-growth case. In both cases, the hiring promise w_h coincides with the reservation promise $\bar{w}$. (Color figure can be viewed at wileyonlinelibrary.com)

Figure 3 depicts the cumulative distribution of tenure length in the two baseline examples. The probability distribution of tenure length for the low-growth firm first-order stochastically dominates the one for the high-growth firm, that is, the probability of a manager reaching any given number of tenure years is higher in the low-growth firm. The median tenure length of a manager is 3.3 years in the firm with better growth prospects ($\gamma = 0.25$), whereas it is 12.6 years in the firm with poorer growth prospects ($\gamma = 0.10$). Changes in the quality of growth prospects driven by the arrival rate of growth opportunities q affect tenure length in a similar way: increasing the frequency of growth opportunities from $q = 0.20$ to $q = 0.22$ causes the median time in office to drop to 3.0 years and 12.1 years, respectively, whereas a decrease to $q = 0.18$ causes the median time in office to rise to 3.6 years and 13.1 years, respectively.

Extent of Compensation Back-Loading. Deferred compensation constitutes an essential feature of the optimal dynamic contract under moral hazard. It is well understood that the degree of compensation back-loading should depend on the severity of moral hazard (λ), cash flow volatility (σ), and the wedge between the manager's and the firm's discount rates ($\varrho - r$). In our setup, the extent of back-loading also depends on the likelihood of growth-induced turnover. Firms' growth prospects may affect the timing of compensation both through the drift of the manager's promise and the level of the compensation threshold.

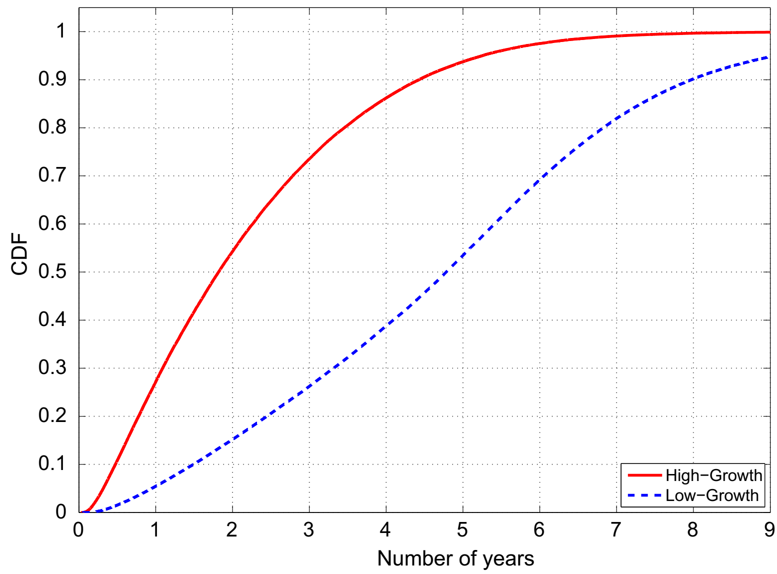

Figure 4. Distribution of realized compensation duration: high-growth vs. low-growth firms. The figure depicts model-implied cumulative distribution functions obtained from simulations for parameter values $r = 7\%$, $\varrho = 16\%$, $\mu = 1$, $\sigma = 1$, $q = 0.2$, $\gamma = 0.25$ (high-growth) or $\gamma = 0.10$ (low-growth), $\lambda = 0.4$, $\kappa = 0.3$, and $\bar{w} = 1$. Optimal contractual thresholds are $w_c = 1.24$ in the high-growth case, and $w_g = 0.92$ and $w_c = 1.36$ in the low-growth case. In both cases, the hiring promise w_h coincides with the reservation promise $\bar{w}$. (Color figure can be viewed at wileyonlinelibrary.com)

To illustrate this aspect of our model, we simulate managerial pay under the optimal contract and characterize the degree of compensation back-loading using the notion of *compensation duration*. In particular, for a given sequence of bonuses received by a manager over his tenure, we compute the weighted average of the points in time when compensation is received—with weights equal to the fraction of the total discounted pay (using the agent's discount rate ϱ) received at each point in time.[38] Figure 4 depicts the cumulative distribution of realized compensation duration in the two baseline configurations introduced earlier in this section. The relative position of the two distributions reflects the fact that, holding the level of expected discounted pay $\bar{w}$ constant across firms, compensation is more front-loaded in firms with better growth prospects. On average, compensation duration is 2.2 years in the high-growth firm, versus 4.8 years in the low-growth firm. Similarly, we find that an increase in the arrival rate of growth opportunities (q) also results in lower compensation duration.

[38] Compensation duration is formally defined as $(\int_0^\tau e^{-\varrho t}\, dC_t)^{-1} \int_0^\tau t e^{-\varrho t}\, dC_t$, in line with the notion of bond duration used in interest rate risk management. In simulations, we use the discretized version of this expression.

IV. Empirical Evidence

In this section, we present evidence consistent with the notion of growth-induced turnover and the empirical implications derived in the context of the two-period framework in Section I.B and illustrated for the continuous-time model in Section III.E. We first investigate the empirical determinants of CEO turnover in light of the theory. We then explore the relation between the timing of CEO compensation and firms' growth prospects.

A. Data

Our empirical analysis relies on information on CEO tenure episodes in U.S. public firms as reported in Standard & Poor's ExecuComp database for the period 1992 to 2014.[39] After merging the ExecuComp sample with accounting information from Compustat and stock return data from CRSP, our sample comprises 4,514 CEO episodes. Of these, 2,510 episodes cover the full tenure of the CEO from beginning to end. The total number of CEO-year observations in our sample is 27,992.[40] The minimum number of firms covered in a given year is 760 in 1992 and the maximum is 1,416 in 2005.

Using information from ExecuComp, we identify the beginning and end years of each completed CEO episode. The variable *TotTenure* is defined as the total number of years the CEO runs the firm. Within an episode, *Turnover* is a dummy variable that equals one in the last year of the CEO's tenure and zero otherwise. We also use Execucomp to construct *TotPay*, which is the total compensation awarded to a CEO in a given year. Table I reports summary statistics for our sample. The average and median CEO tenure lengths are 6.5 years and 5 years, respectively, while the average annual turnover rate is 8.4%.

Our analysis centers on CEO turnover and the timing of CEO compensation in relation to the quality of a firm's growth prospects. Our empirical proxy for a firm's growth prospects during a given CEO episode is based on the "average Q" of the firm. The use of average Q to proxy for a firm's growth opportunities is standard in the empirical corporate finance literature.[41] We construct the average Q for each firm-year in our sample and denote it simply by Q. As a proxy for the quality of growth prospects during a given episode, we use

[39] The ExecuComp database covers all firms included in the S&P 500, MidCap, and SmallCap indexes. It would be interesting to extend the analysis to smaller public firms and private firms. Kaplan, Sensoy, and Stromberg (2008) document high turnover in the management teams of VC-backed private companies before going public.

[40] All CEO-year observations are from 1992 onwards. While the ExecuComp data set starts in 1992, it covers episodes in which the CEO was appointed earlier. Our sample includes observations pertaining to such episodes when the required information (e.g., the firm's average Q at the time the CEO was appointed) is available.

[41] The handbook by Eckbo (2001) surveys multiple studies in which average Q is used as a proxy for growth opportunities. This practice stems from Hayashi (1982), who derives sufficient conditions such that a firm's average Q coincides with its marginal product of capital. See Caballero (1997) and Bond and Van Reenen (2007) for surveys of the empirical literature assessing the links between average Q, marginal q, and investment.

Table I
Summary Statistics

The table reports summary sample statistics for the merged ExecuComp/Compustat/CRSP data set, which covers CEO episodes reported in ExecuComp over the period 1992 to 2014. *TotTenure* is the total number of tenure years for CEO episodes that are completed within the sample period. *Turnover* is a dummy variable that equals one in the last year of a CEO's tenure and zero otherwise. *LnTotPay* is the logarithm of total CEO compensation awarded in a given calendar year. *QInit* is the average Q of the firm in the year before the CEO was appointed; the same value is repeated throughout each CEO episode. *RatioQ* is the ratio of the lagged average Q of a firm in a given year divided by *QInit*. *CAR* is the two-year cumulative abnormal return of the firm (annualized). *ROA* is return on assets. *LnAssets* is the logarithm of the book value of total assets. Variable definitions are provided in the main text and in Internet Appendix Section VIII.

Variable	Mean	*SD*	p25	p50	p75	*N*
TotTenure	6.492	5.001	3.000	5.000	9.000	2,510
Turnover	0.084	0.278	0.000	0.000	0.000	27,992
LnTotPay	7.915	1.053	7.172	7.927	8.663	27,958
QInit	1.788	1.212	1.099	1.371	1.981	27,992
RatioQ	1.069	0.420	0.876	1.000	1.164	27,992
CAR	−0.001	0.244	−0.147	−0.005	0.129	27,992
ROA	0.038	0.078	0.013	0.040	0.075	27,992
LnAssets	7.700	1.699	6.452	7.602	8.887	27,992

the value of Q in the year before the CEO is appointed, which we denote by *QInit*. We interpret a higher value of *QInit* as capturing better ex ante growth prospects at the time a new CEO is hired.

Managerial turnover in our setup is also affected by the availability of growth opportunities. Capturing the arrival of a growth opportunity in any given year during a CEO episode is challenging empirically. As a proxy, we construct *RatioQ* as the ratio of the lagged value of a firm's Q in any given year to *QInit*. A higher value of *RatioQ* is more likely when the firm has new growth opportunities available.

To control for past performance in any given year within a CEO episode, we use the cumulative abnormal return of the firm measured over the previous two years, which we denote by *CAR*.[42] We also consider lagged return on assets (*ROA*) as an additional control for performance. Finally, we use the logarithm of lagged total assets (*LnAssets*) to control for firm size.

B. Determinants of CEO Turnover

We first examine the relation between turnover and the quality of growth prospects. As a first pass, Figure 5 depicts the cumulative distribution of CEO tenure length conditional on ex ante growth prospects as proxied by *QInit*. The solid line plots the kernel estimate of the distribution for the upper 20% of

[42] Our qualitative results are unaffected when we use one-year or three-year cumulative abnormal returns as a measure of past performance, or when we remove the initial years of a CEO's tenure from the sample. Our results are also robust to the use of industry-level measures of Q in the construction of *QInit* and *RatioQ*.

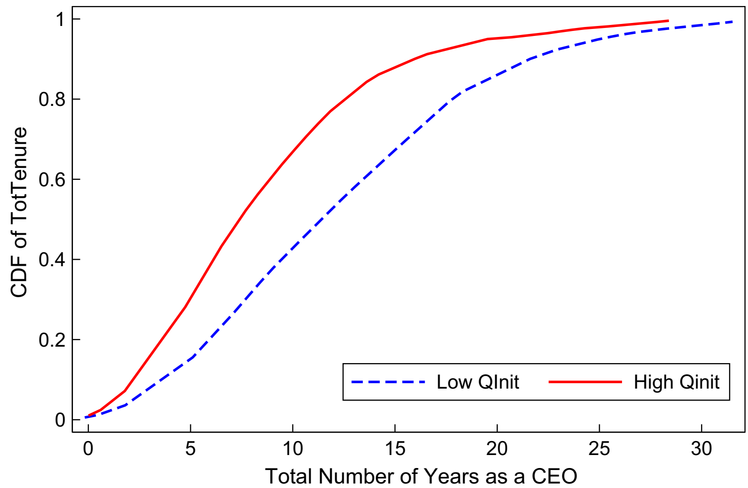

Figure 5. Distribution of CEO tenure length conditional on initial growth prospects. The figure depicts kernel estimates of the empirical cumulative distribution of CEO tenure length (*TotTenure*) for two subsamples. The first subsample ("Low *QInit*") consists of the bottom quintile of CEO episodes sorted by initial Q; the corresponding distribution is plotted as a dashed line. The second subsample ("High *QInit*") consists of the top quintile of episodes sorted by initial Q; the corresponding distribution is plotted as a solid line. Variable definitions are provided in the main text and in Internet Appendix Section VIII. (Color figure can be viewed at wileyonlinelibrary.com)

CEO episodes ranked by *QInit*, whereas the dashed line corresponds to the bottom 20%. The two-sample Wilcoxon-Mann-Whitney test and the Kolmogorov-Smirnov test both confirm the visual impression that the two samples are drawn from different distributions. The cumulative distribution for the upper *QInit* subsample lies significantly above that for the bottom *QInit* subsample. That is, the likelihood that a CEO will not "survive" beyond any number of years is higher for CEOs entering firms with good growth prospects than for CEOs entering firms with poor growth prospects, consistent with the notion of growth-induced turnover. Figure 5 constitutes the empirical counterpart to the simulation results depicted in Figure 3.

Next, we assess the implications of our model for managerial turnover by running a probit regression. The probit specification is

$$\text{Prob}(\mathit{Turnover}_{j,t} = 1) = \Phi[\psi_0 + \psi_1 \mathit{QInit}_j + \psi_2 \mathit{RatioQ}_{j,t-1} + \psi_3 \mathit{CAR}_{j,t-1} + \boldsymbol{\alpha}' \boldsymbol{X}_{j,t-1}],$$

where Φ is the standard normal cumulative distribution function, j denotes a CEO episode, t denotes the calendar year, and $\mathbf{X}$ denotes a vector of control variables. We control for the return on assets and the size of the firm, both lagged one year. Calendar year fixed effects are also included. We do not include firm or industry fixed effects in the probit to avoid the incidental parameters

Table II
Determinants of CEO Turnover

The table reports evidence on the probability of CEO turnover from the probit regression estimated over the merged ExecuComp/Compustat/CRSP data set from 1992 to 2014. The dependent variable is the *Turnover* indicator variable. *QInit* is the average Q of the firm in the year before the CEO was appointed; the same value is repeated throughout each CEO episode. *RatioQ* is the ratio of the lagged average Q of the firm in a given year divided by *QInit*. *CAR* is the two-year cumulative abnormal return of the firm (annualized) over the previous two years. *ROA* is return on assets, lagged one year. *LnAssets* is the logarithm of the lagged book value of total assets. Calendar year fixed effects are included in the regression. Robust standard errors (clustered at the firm level) are shown in parentheses. *** denotes statistical significance at the 1% level.

	(1) Coefficients *b/SE*	(2) Marginal Effects *b/SE*	(3) Coefficients of Variation (in Percentage Points)
QInit	0.047***	0.007***	0.848
	(0.009)	(0.001)	
RatioQ	0.089***	0.014***	0.588
	(0.030)	(0.005)	
CAR	−0.595***	−0.090***	−2.196
	(0.053)	(0.008)	
ROA	−0.806***	−0.122***	−0.952
	(0.131)	(0.020)	
LnAssets	0.040***	0.006***	1.019
	(0.006)	(0.001)	
N	27,992	27,992	27,992
Year fixed effects	Yes	Yes	Yes

problem that arises in the context of nonlinear panel models.[43] In view of the comparative static results derived in Proposition 1, we hypothesize that the coefficients on *QInit* and *RatioQ* should be positive, while the coefficients on *CAR* and *ROA* should be negative.

Table II summarizes the results of the probit regression. Column (1) reports the estimated coefficients of the probit model and their standard errors. All explanatory variables have the expected signs and are highly statistically significant. The coefficient on *QInit* is positive, in line with our model's prediction that turnover is more frequent in firms with better ex ante growth prospects. The coefficient on *RatioQ* is also positive, in line with the idea that turnover is sometimes triggered by the arrival of growth opportunities. Finally, the coefficients on *CAR* and *ROA* are negative, in line with the theoretical prediction that turnover is more likely after poor performance.

Column (2) reports the implied marginal effects, which give the impact on the probability of turnover of a one-unit increase in an explanatory variable when all variables are evaluated at the sample means. In Column (3), the marginal

[43] See Woolridge (2002) for a textbook treatment. Earlier studies of CEO turnover by Kaplan and Minton (2012) and Jenter and Lewellen (2014) employ probit specifications similar to ours. In practice, when controlling for firm- or industry-level fixed effects, our qualitative conclusions are unaffected.

Table III

Initial Growth Prospects and Growth-Induced Turnover

The table reports the marginal effect of *RatioQ* on the likelihood of turnover for different levels of initial average Q, as implied by the probit model estimated over the merged ExecuComp/Compustat/CRSP data set from 1992 to 2014. *QInit* is the average Q of the firm in the year before the CEO was appointed. *RatioQ* is the ratio of lagged Q in a given year divided by *QInit*. The marginal effect of *RatioQ* is evaluated at different quantiles of the distribution of *QInit*. Low, Median, and High quantiles correspond to the 20^{th}, 50^{th}, and 80^{th} percentiles of the distribution of *QInit*, respectively. Robust standard errors (clustered at the firm level) are shown in parentheses. *** denotes statistical significance at the 1% level.

	Marginal Effect of *RatioQ* *b/SE*
Low *QInit*	0.0124***
	(0.0040)
Median *QInit*	0.0126***
	(0.0041)
High *QInit*	0.0133***
	(0.0044)
N	27,992

effects are multiplied by the sample standard deviation of the corresponding explanatory variables. A one-standard-deviation increase in *QInit* is associated with an increase in the probability of turnover of 85 basis points. Similarly, a one-standard-deviation increase in *RatioQ* leads to a 59 basis point increase in the probability of turnover. Since the unconditional frequency of CEO turnover in our sample is 8.4%, these results support the view that the growth-related drivers of managerial turnover emphasized in this paper are economically significant. Of course, this is in addition to the disciplinary role of turnover, which we also find to be important. In our sample, a one-standard-deviation increase in past abnormal returns is associated with a decrease in the probability of turnover of 2.2 percentage points.

An additional implication of our model is a tendency for firms with relatively poor ex ante growth prospects to grant partial job protection to their CEOs. That is, our theory predicts that in such firms, a CEO is less likely to be dismissed for the sake of growth when an opportunity arises calling for his replacement. We explore the empirical validity of this prediction by evaluating the marginal effect of *RatioQ* on the probability of CEO turnover at different levels of *QInit*. According to Proposition 1(iii), the impact of *RatioQ* on turnover should be greater for firms with relatively better ex ante growth prospects, that is, for higher values of *QInit*. Table III reports these differential marginal effects. The marginal effect of *RatioQ* is strictly positive at all levels of *QInit* and is indeed increasing in *QInit*.

C. Growth Prospects and CEO Compensation

A key insight from our theory is that the managers of firms with better and more frequent growth opportunities should have more front-loaded

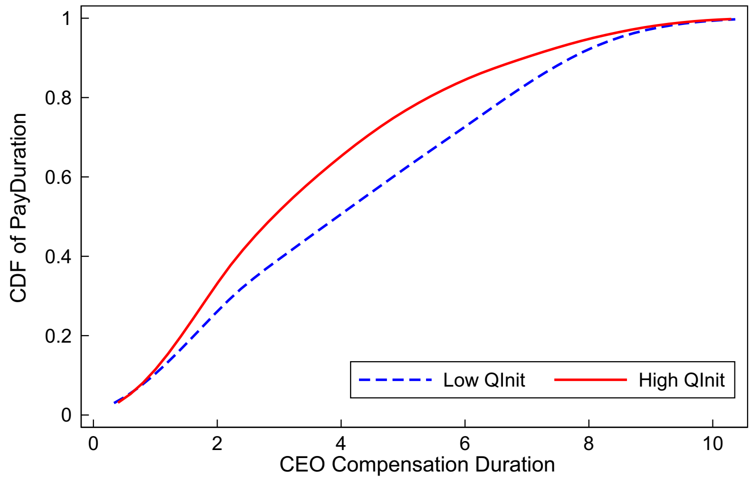

Figure 6. Distribution of CEO pay duration conditional on initial growth prospects. The figure depicts kernel estimates of the empirical cumulative distribution of realized compensation duration (*PayDuration*) for the bottom and top quintiles of CEO episodes sorted by initial Q (*QInit*), denoted by "Low *QInit*" and "High *QInit*," respectively. Variable definitions are provided in the main text and in Internet Appendix Section VIII. (Color figure can be viewed at wileyonlinelibrary.com)

compensation. In this subsection, we provide evidence on the empirical relation between CEO compensation and firms' growth prospects. We explore the data in two ways.

As a first pass, we compute a measure of realized compensation duration for each CEO episode and investigate how it varies across episodes that differ in terms of the firm's growth prospects at the time the CEO was appointed. For a given CEO episode j lasting N_j years, our measure of compensation duration, *PayDuration*, is obtained as

$$PayDuration_j = \sum_{n=1}^{N_j} \frac{DiscPay_{j,n}}{\sum_{k=1}^{N_j} DiscPay_{j,k}} \times n, \tag{40}$$

where $DiscPay_{j,n} = TotPay_{j,n}/(1+\varrho)^n$ corresponds to the present value of the compensation received by the CEO in his n^{th} tenure year. Setting the discount rate ϱ to 10%, we find that, in the subsample of episodes over which this measure is computed, average CEO compensation duration is 3.7 years, while the median is 3.2 years.

Figure 6 provides an empirical counterpart to Figure 4, depicting kernel estimates of the cumulative distribution of *PayDuration* conditional on ex ante growth prospects proxied by *QInit*. The solid line pertains to the upper 20% of *QInit* in our sample, while the dashed line pertains to the bottom 20%. The cumulative distribution for the upper *QInit* subsample lies above that for the

Table IV

Determinants of CEO Compensation

This table reports evidence on the profile of CEO compensation over tenure. *LnTotPay* is the logarithm of total CEO pay awarded in a given year as reported in ExecuComp. *TenureYear* is the number of years in tenure of the CEO in a given calendar year. *QInit* is the average Q of the firm in the year before the CEO was appointed. *CAR* is the two-year cumulative abnormal return of the firm (annualized) over the previous two years. *ROA* is return on assets, lagged one year. *LnAssets* is the logarithm of the lagged book value of total assets. The regression is estimated over all episode-year observations in our sample, some of which pertain to CEO episodes that have not finished by the end of the sample period. Robust standard errors (clustered at the firm level) are shown in parentheses. *, **, and *** denote statistical significance at the 10%, 5%, and 1% level, respectively.

	(1) *LnTotPay*	(2) *LnTotPay*
TenureYear	0.017***	0.008*
	(0.004)	(0.003)
QInit	0.073***	0.054***
	(0.011)	(0.014)
QInit × *TenureYear*	−0.008***	−0.005**
	(0.002)	(0.002)
CAR	0.556***	0.435***
	(0.026)	(0.026)
LnAssets	0.430***	0.199***
	(0.007)	(0.019)
Firm Fixed Effects	No	Yes
Industry Fixed Effects	Yes	No
Year Fixed Effects	Yes	Yes
R^2	0.532	0.663
N	27,615	27,615

bottom *QInit* subsample, which is consistent with our model's insight that firms with better growth prospects should have more front-loaded compensation.[44]

To further investigate how profiles of CEO pay over tenure vary with firms' growth prospects, we consider the regression

$$\begin{aligned}\mathrm{Ln}(\mathit{TotPay}_{j,t}) = \psi_0 + \psi_1\,\mathit{TenureYear}_{j,t} + \psi_2\,\mathit{QInit}_j +\\ + \psi_3\,\mathit{QInit}_j \times \mathit{TenureYear}_{j,t} + \boldsymbol{\alpha}'\mathbf{X}_{j,t-1} + \epsilon_{j,t},\end{aligned}$$

where j denotes a CEO episode, t denotes the calendar year, $\mathit{TenureYear}_{j,t}$ ($\mathit{TotPay}_{j,t}$) denotes the tenure of the CEO in year t (the total compensation received by the CEO in that year), and $\mathbf{X}$ denotes a vector of control variables. We control for past performance and firm size, as well as for calendar year

[44] The results from the two-sample Wilcoxon-Mann-Whitney test and from the Kolmogorov-Smirnov test both confirm that the difference between the two empirical distributions is statistically significant. Furthermore, controlling for year fixed effects, *PayDuration* and *QInit* are significantly negatively correlated across CEO episodes. When also controlling for firm fixed effects, the correlation remains negative but becomes insignificant due to the small number of observations.

fixed effects and industry or firm fixed effects. Our theory predicts (see Proposition 2(iii)) that firms with better ex ante growth prospects are characterized by a higher initial level of pay per period (i.e., ψ_2 positive) and slower growth in compensation over tenure (i.e., ψ_3 negative). Table IV summarizes our empirical findings for two alternative specifications that control for industry and firm fixed effects, respectively. The coefficients of interest are significant with the expected signs, and the results are very similar across both specifications. We also note that the coefficient on past abnormal returns is positive and significant, in line with the theoretical prediction that CEO pay is positively related to past performance.

V. Conclusion

This paper introduces growth-induced turnover in a dynamic moral hazard framework and analyzes the interaction between this type of turnover and managerial incentive provision. In our model, growth opportunities arrive stochastically over time and the firm must appoint a new manager to be able to seize them. Our analysis highlights the trade-off that a firm faces between the benefit of always having a manager who is the right man for the job at hand and the cost of incentive provision. The key new insight is that exposing incumbent managers to the risk of growth-induced dismissal effectively increases their discount rate and thus the cost of incentive provision. As a result, some firms find it optimal to provide managers some degree of job protection, at the cost of forgoing growth opportunities. Across firms, a higher likelihood of growth-induced turnover translates into a greater tendency to front-load compensation. Our empirical findings are consistent with these predictions of the model.

An essential feature of our model is that nondisciplinary managerial turnover can be triggered by the firm contingent on the arrival of exogenous contractible shocks. In our setup, shocks correspond to the arrival of growth opportunities, and it is first-best efficient for the firm to replace the incumbent manager upon arrival of a growth opportunity. Our analysis could be applied to alternative forms of exogenous contractible shocks. First, transformative managerial change may also be important for firms in decline. For instance, a change in management may be required for a firm to respond to increased product market competition or to the threat of a disruptive new technology. Second, the firm may face opportunities to transform—through a change in management—that would bring gains that are too modest to outweigh the cost of implementing them, so that they would not be taken up under first-best. However, in a second-best world, it may be optimal to take these inefficient opportunities when the agency costs associated with the current manager are high. We believe that a number of theoretical insights of the paper would carry through in these alternative settings, although the empirical implications would be quite different.

The existing empirical literature on managerial turnover and compensation has been informed mostly by two paradigms from the contracting literature—

the moral hazard model in which pay and dismissal are used to incentivize the agent, and the learning model in which the principal learns about the unknown quality of the agent over time. In our view, transformative change can be another powerful driver of managerial turnover and compensation. We document that firms with better growth prospects experience higher CEO turnover and rely on more front-loaded compensation schemes. These findings are consistent with the assumption of growth-induced turnover and the predictions of our model. Nonetheless, other theories may be consistent with these findings. Identifying the specific channel through which firms' growth prospects relate to CEO turnover and compensation deserves further empirical work.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.