Increasing Skill Premium and Skill Supply
– Steady State Effects or Transition?

Zsófia L. Bárány*

This version: July 23, 2011

Abstract: I challenge the existing literature that claims that strongly biased technology is necessary to observe a simultaneous increases in the skill supply and the skill premium. I highlight the importance of the joint determination of the direction of technical change and skill formation, as there is a positive feedback between them. Technological progress is driven by profit oriented R&D firms, where profits are increasing in the amount of labour that is able to use these technologies. Therefore, when the supply of high-skilled labour increases, technology endogenously becomes more skill-biased. A more skill-biased technology leads to a higher skill premium, which increases the incentives to acquire education, and the supply of high-skilled labour rises. During the transition to the steady state, both quantities increase simultaneously. I map the dependence of the transition path of the economy on the initial skill supply and relative technology between the high- and the low-skilled sector. I find that, contrary to the previous literature, the skill premium and the skill supply can increase jointly even if the bias of technology is weak.

Keywords: Education, endogenous technology, skill premium

JEL-Classification: E24, E23.

---

* I am grateful to Francesco Caselli, Aytek Malkhozov and Christian Siegel for many comments and suggestions that have improved this paper.

Economics Department, London School of Economics and Political Science, Houghton Street, WC2A 2AE, London, UK. email: barany@lse.ac.uk

1 Introduction

In this paper I challenge the existing literature that claims that strongly biased technology is necessary to observe a simultaneous increase in the skill supply and in the skill premium. Their joint increase throughout the past few decades is well-documented[1] and extensively researched. Theoretical explanations for this phenomenon either treat the increase in the supply of high-skilled labour or the increase in the skill-bias of technology as exogenous. When both are treated as endogenous, the skill bias of technology and the skill supply depend positively on each other. This positive dependence is crucial in understanding that the joint increase of these two variables can emerge during the transition to the steady state, independent of the strength of the bias in technology.

I present a model where both the relative technology and the relative supply of high-skilled labour is endogenous. I show that in such a framework the supply of high-skilled workers and the relative quality in the high-skilled sector change in the same direction during the transition to the steady state. I also characterise conditions under which the transition path to the steady state features an increase in the supply of skills *and* a parallel increase in the relative wages of high-skilled workers.

In the model technological progress is driven by profit oriented R&D firms, where profits are increasing in the amount of labour that is able to use these technologies. Hence when the relative supply of labour in one sector increases, the relative profitability of investing into that sector increases as well, thereby increasing the relative technology in that sector. This is referred to as the *bias of technology*: when a factor becomes more abundant, technology endogenously becomes more biased towards that factor. If this bias is large enough, then the increase in the relative technology more than offsets the negative effect of the higher relative supply, and the relative factor price rises in the long-run. This is termed *strong bias* of technology. On the other hand, if the effect of the increase in the relative technology is not large enough, then the relative factor price decreases, and technology displays a *weak bias*.

The supply of labour is determined by individual choices: everyone whose cost of education does not exceed the lifetime gains from working as high-skilled rather than low-skilled, acquires education, and becomes high-skilled. In such a setup, if the relative technology increases in the high-skilled sector, then the skill premium increases, thereby increasing the incentives to acquire education, and educational attainment increases.

It is a well-known fact that the supply of college graduates has been continuously increasing over the past few decades.[2] Human capital accumulation takes several generations, even if technology is fixed. However, in an environment where technology and human capital are evolving jointly, the transition process potentially takes longer, as both the skill supply and the technology adjusts more slowly. In this model,

[1] For example **?**, **?**, **?**, **?**, **? ?**, **?**, **?**, **?**, **?**, Goldin and Katz (2007 and 2008).
[2] See for example **?**.

since I allow the supply of the different types of labour to be endogenous, it is natural to consider an economy that is on the transition path towards its steady state. I numerically map the dependence of this path on the initial values of relative quality and relative supply. I find that there is a set of initial values from which the transition features a continuously increasing supply of high-skilled workers, increasing relative quality and increasing skill premium. This feature persists even if the elasticity of substitution is low between the two sectors, although the set of such initial values shrinks.

There are two main strands of literature that relate to this paper. The first strand is based on an exogenous technological progress, and the supply of high- and low-skilled labour adjusts endogenously. **?** models the effects of skill-biased technological revolutions, where learning to use new machines is more costly than old ones. In such a scenario, new technologies are adopted slowly, there is a gradual shift of skills to new technologies and the skill premium increases. **?** allows for endogenous skill formation in an overlapping generations model, with worker heterogeneity in ability. He explores the effects of an exogenous skill-biased technological shock on educational attainment, and finds that the slow adjustment in the supply of educated labour can result in a nonmonotonic pattern of the skill premium. His model, similarly to mine, features a slow adjustment in human capital, which is driven by the optimal educational choices of consecutive cohorts. **?** develop a model where an increase in the rate of technological progress increases the returns to education. In such a context, similarly to the paper here, a feedback mechanism arises: with a higher supply of human capital, the rate of technological progress increases, and a higher rate of technological progress induces more human capital accumulation. The feedback works through a very different channel: it works through easier R&D and not more profitable. **?** develop a general equilibrium model with endogenous skill formation, physical capital accumulation, and heterogeneous human capital to explain rising wage inequality. In this framework they find that skill-biased technical change explains the patterns of skill premium and overall inequality rather well.

The second strand takes the path of high-and low-skilled labour as given, while technological progress is endogenous. The most closely related papers are Acemoglu (1998 and 2002) and **?**, which study a model similar to the one presented here, and consider an exogenous increase in the supply of high-skilled labour. If the elasticity of substitution between the output of the different types of labour is sufficiently high, then the skill premium increases in the long run. **?** studies the equilibrium bias of technology in a more general context and shows that if technologies are factor-augmenting, then the increase in the supply of a factor induces technical change to be relatively biased towards that factor. The condition under which this relative bias is strong enough to offset the price effect of increased supply is a sufficiently high elasticity of substitution between the different factors of production.

2 The model

The model is along the lines of the model in paper 1. There are two differences: there is no minimum wage, and individuals are only heterogeneous in their cost of acquiring education. The structure of this section follows the structure in the previous paper. I begin by describing the model's production technologies, the R&D sector, the demographic structure and educational choices. Next I define the decentralised equilibrium, I analyse the balanced growth path, and finally, I analyse the transitional dynamics.

2.1 Overview

Time is infinite and discrete, indexed by $t = 0, 1, 2...$ The economy is populated by a continuum of individuals who survive from one period to the next with probability λ, and in every period a new generation of measure $1 - \lambda$ is born. Individuals are heterogeneous in their cost of acquiring education.

In the first period of his life every individual has to decide whether to acquire education or not, with the cost of education varying across individuals. Those who acquire education become high-skilled. Those who opt out from education remain low-skilled. Workers with high and low skills perform different tasks, are employed in different occupations, and produce different goods. The high-skilled sector includes skill-intensive occupations and production using high-skilled labour, while the low-skilled sector includes labour-intensive occupations and production using low-skilled labour. In equilibrium working in the high-skilled sector provides higher wages.

There is a unique final good in this economy, which is used for consumption, the production of machines, and as an investment in R&D. It is produced by combining the two types of intermediate goods: one produced by the low- and the other by the high-skilled workers. Intermediate goods are produced in a perfectly competitive environment by the relevant labour and the machines developed for them.

Technological progress takes the form of quality improvements of machines that complement a specific type of labour, either high- or low-skilled. R&D firms can invest in developing new, higher quality machines. Innovators own a patent for machines and enjoy monopoly profits until it is replaced by a higher quality machine. There is free entry into the R&D sector, and more investment will be allocated to developing machines that are complementary with the more abundant labour type.

The economy is in a decentralised equilibrium at all times: all firms maximise their profits – either in perfect competition or as a monopoly – and individuals make educational decisions to maximise their lifetime income. I analyse how the distribution of costs and the characteristics of the production function and the R&D process affect the steady state and the transitional dynamics within this equilibrium framework.

2.2 Production

The production side of the model is exactly the same as in paper 1. It is a two-sector endogenous growth model, where technological advances feature a *market size* effect, by which more R&D investment is allocated to develop machines complementary to the more abundant factor.

2.2.1 Final and intermediate goods

There is a unique final good, which is produced in perfect competition by combining the two intermediate goods:

$$Y = \left((Y^l)^\rho + \gamma(Y^h)^\rho\right)^{\frac{1}{\rho}},$$

where Y_l and Y^h is the intermediate good produced by the low- and high-skilled workers respectively. The elasticity of substitution between the two intermediate goods is $1/(1-\rho)$, where $\rho \leq 1$. In perfect competition the relative price of the two intermediates is:

$$p \equiv \frac{p^h}{p^l} = \gamma \left(\frac{Y^l}{Y^h}\right)^{1-\rho}. \tag{1}$$

I normalise the price of the final good to one, hence the price of intermediate goods is:

$$p^l = \left(1 + \gamma p^{\frac{\rho}{\rho-1}}\right)^{\frac{1-\rho}{\rho}}, \tag{2}$$

$$p^h = \left(p^{\frac{\rho}{1-\rho}} + \gamma\right)^{\frac{1-\rho}{\rho}}. \tag{3}$$

In both sectors intermediate good production is perfectly competitive. To simplify notation I allow a representative firm:

$$Y^s = A^s(N^s)^\beta \quad \text{for} \quad s = \{l, h\}, \tag{4}$$

where $\beta \in (0,1)$, N^s is the amount of labour employed and A^s is the level of technology in sector s. Each machine is sector specific in the sense that exclusively high- or low-skilled workers can operate it respectively. Firms decide the quantity, $x^{s,j}$ of the machine to use given the supply of labour, N^s, and the quality of a machine, $q^{s,j}$. Sector s productivity is given by:

$$A^s = \tfrac{1}{1-\beta} \textstyle\int_0^1 q^{s,j}(x^{s,j})^{1-\beta} dj \quad \text{for} \quad s \in \{l, h\}.$$

Industry demand for machine line j of quality $q^{s,j}$ and price $\chi^{s,j}$ by the perfectly competitive intermediate good production is:

$$X^{s,j} = \left(\frac{p^s q^{s,j}}{\chi^{s,j}}\right)^{\frac{1}{\beta}} N^s \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1]. \tag{5}$$

2.2.2 R&D firms

Investment in R&D stochastically produces innovations. Innovations improve the quality of an existing line of machine by a fixed factor, $\overline{q} > 1$. Innovations follow a Poisson process, with an arrival rate for firm k that invested z_k^j on line j is ηz_k^j. If total investments on line j is $\overline{z}^j \equiv \sum_k z_k^j$, the economy wide arrival rate of innovations in line j is $\eta \overline{z}^j$. The probability of an innovation in line j in one period is $(1 - e^{-\eta \overline{z}^j})$. The probability that the innovation is performed by firm k is $(1 - e^{-\eta \overline{z}^j}) z_k^j / \overline{z}^j$. Investing z_k^j units in R&D costs Bqz_k^j in terms of final good, therefore a lower B implies less expensive innovation. There are two important things to note: one is that the probability of success is increasing and concave in total investment, $\overline{z}^j$, the other is that the cost of investment is increasing in the quality of the machine line. Due to the first feature there exists an interior solution, while due to the second one a steady state exists. There is free entry into the R&D sector.

Successful R&D firms become the monopolist owners of the machine they patented. As in paper 1, if quality improvements are sufficiently large, then in equilibrium only the best quality of any machine is sold at its monopoly price. I assume that this condition applies, hence the price of the leading vintage in line j and sector s with quality q is:

$$\chi^{s,j} = \frac{q}{1-\beta} \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1].$$

The per period profit of the owner of the leading vintage using monopoly pricing and industry demand (5) can be expressed as:

$$\pi^{s,j} = q^{s,j} \beta (1-\beta)^{\frac{1-\beta}{\beta}} (p^s)^{\frac{1}{\beta}} N^s \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1]. \tag{6}$$

The profit in each period is increasing in the price of the intermediate good, p^s, in the quality of the machine, $q^{s,j}$, and in the amount of labour that can use the machine, N^s. The value of owning the leading vintage is the expected discounted value of all future profits, and can be expressed recursively as:

$$V_t^{j,s}(q) = \pi_t^{j,s}(q) + \frac{1}{1+r}(e^{-\eta \overline{z}_t^{j,s}(q)}) V_{t+1}^{j,s}(q) \quad \text{for} \quad s = \{l, h\} \quad \text{and} \quad j \in [0, 1]. \tag{7}$$

Where $\overline{z}_t^{j,s}(q)$ is the total R&D spending on line j in sector s of current quality q at time t, and $\frac{1}{1+r} V_{t+1}^{j,s}(q)$ is the present value of owning the leading vintage of quality q in line j and sector s in period $t+1$. The probability that quality q remains the leading vintage in line j in period $t+1$ is $e^{-\eta \overline{z}_t^{j,s}(q)}$.

Due to free entry into the R&D sector all profit opportunities are exhausted. Therefore for each firm the expected return from R&D investment has to equal its cost:

$$\frac{E_t(V_{t+1}^{s,j}(q_t^{s,j}))}{1+r}(1-e^{-\eta \overline{z}_t^{j,s}(q_t^{s,j})})\frac{z_k^{j,s}}{\overline{z}_t^{j,s}(q_t^{s,j})} = Bq_t^{s,j}z_k^{j,s} \quad \text{for} \quad s=\{l,h\} \quad \text{and} \quad j \in [0,1]. \tag{8}$$

In equilibrium, only the total amount of R&D spending targeted at improving line j in sector s is determined, since both the expected return and the costs are proportional to the R&D investment of firm k.

2.2.3 Technology and prices

The equilibrium production of intermediate goods given monopoly pricing is:

$$Y_t^s = (1-\beta)^{\frac{1-2\beta}{\beta}}(p_t^s)^{\frac{1-\beta}{\beta}}N_t^s Q_t^s \quad \text{for} \quad s=\{l,h\}, \tag{9}$$

where $Q_t^s = \int_0^1 q_t^{j,s}dj$ is the average quality of the leading vintages in sector s. This evolves according to the R&D investments targeted at improving the machine in sector s:

$$Q_{t+1}^s = \int_0^1 q_t^{j,s}\left((1-e^{-\eta \overline{z}_t^{j,s}(q_t^{j,s})})\overline{q} + \left(e^{-\eta \overline{z}_t^{j,s}(q_t^{j,s})}\right)\right)dj \quad \text{for} \quad s=\{l,h\}. \tag{10}$$

The average quality in sector s grows at rate:

$$g_{t+1}^s = \frac{Q_{t+1}^s}{Q_t^s} \quad \text{for} \quad s=\{l,h\}.$$

I denote the relative average quality or relative technology by $Q_t \equiv \frac{Q_t^h}{Q_t^l}$. This evolves according to:

$$Q_{t+1} = \frac{g_{t+1}^h}{g_{t+1}^l}\frac{Q_t^h}{Q_t^l} = \frac{g_{t+1}^h}{g_{t+1}^l}Q_t. \tag{11}$$

The relative prices of intermediates can be expressed by combining (9) with (1):

$$p_t = \gamma^{\frac{\beta}{(1-(1-\beta)\rho)}}\left(\frac{Q_t^h}{Q_t^l}\frac{N_t^h}{N_t^l}\right)^{-\frac{(1-\rho)\beta}{(1-(1-\beta)\rho)}}. \tag{12}$$

The relative price is decreasing in the relative supply of high-skilled labour and in the relative quality of the machines used by high-skilled workers. This is because if the relative share of the high-skilled or the relative quality of the machines that complement them increases, then the production of the high-skilled sector increases compared to the production of the low-skilled sector. This leads to a fall in the relative price.

2.3 Labour supply

Individuals are heterogeneous in their cost of acquiring education, c. The total cost of acquiring education is cw_t^h, where w_t^h is the high-skilled wage in period t, and c is the idiosyncratic cost drawn from the time invariant distribution of education costs, $f(c)$. The crucial part of the assumption is that the cost is proportional to one of the wage rates in the economy, without this assumption the economy would not have a steady state.[3] This assumption is reasonable: the cost of education is partly a time cost, thereby involving foregone earnings, moreover the tuition fees and other expenses incurred while studying are likely to depend on the wage rates in the economy as well.

The demographics follow the perpetual youth model: every period a new generation of mass $1-\lambda$ is born, while the probability of surviving from period t to $t+1$ is λ. Hence both the size of the population and the distribution of costs are constant over time.

In the first period of his life each individual decides whether to acquire education, those born in period t can enrol to study in and only in period t. Acquiring education involves a cost cw_t^h, where c is idiosyncratic, determined at birth and the total cost is paid upon enrollment into education. Individuals who complete education become high-skilled, work in the high-skilled sector and earn wage w_t^h in period t. Those who choose not to acquire education, work as low-skilled for wage w_t^l in period t.

Monopoly pricing and the industry demand for machines implies a wage:

$$w_t^s = \beta(1-\beta)^{\frac{1-2\beta}{\beta}}(p_t^s)^{\frac{1}{\beta}}Q_t^s \quad \text{for} \quad s = \{l,h\}. \tag{13}$$

The wage in sector s is increasing in the price of intermediate good s and the average quality in sector s. Individuals choose their education level to maximise the present value of their expected lifetime utility:

$$\max_{e(c)_t} E_t \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+r}\right)^j u_{t+j},$$

where u_{t+j} is their consumption of the final good, λ is the probability of staying alive until the next period, r is the discount rate, which is also the interest rate due to linear utility. Since utility is linear, lifetime utility is increasing in lifetime earnings. Therefore individuals make educational decisions to maximise the expected present value of lifetime income.

Let $\mathbf{W}_t^h(c)$ denote the expected present value of lifetime income of an individual with cost c born in

[3] If the steady state features growth, wages grow, hence if the costs of education remain the same, more and more people would have an incentive to acquire education.

period t if high-skilled, and $\mathbf{W}_t^l(c)$ denote the same if low-skilled. The optimal decision is:

$$e(c)_t = \begin{cases} 1 & \text{if} \quad \mathbf{W}_t^h(c) \geq \mathbf{W}_t^l(c), \\ 0 & \text{if} \quad \mathbf{W}_t^h(c) < \mathbf{W}_t^l(c), \end{cases} \tag{14}$$

where $e(c)_t = 1$ if the individual acquires education and $e(c)_t = 0$ otherwise.

The lifetime earnings of an educated individual can be expressed as:

$$\mathbf{W}_t^h(c) = \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^h - w_t^h c. \tag{15}$$

The lifetime earnings of a high-skilled individual are decreasing in his cost of acquiring education c.

Whereas the lifetime earnings of a low-skilled individual are unaffected by the costs of education:

$$\mathbf{W}_t^l(c) = \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^s w_{t+s}^l. \tag{16}$$

Education is worth the investment for an individual with cost c if $W_t^h(c) > W_t^l(c)$. This requires that the wage for high-skilled is higher than for low-skilled workers. Hence the following remark,

Remark 1. *To have high-skilled individuals in a generation born in period t, there has to be at least one period $s \geq t$, such that the wage is higher for the high-skilled than for the low-skilled: $w_s^l < w_s^h$.*

The only reason for acquiring education is the *skill premium*, a higher wage in the high- than in the low-skilled sector. Using the relative price of intermediates, (12) and the wage, (13), the skill premium can be expressed as:

$$\frac{w_t^h}{w_t^l} = \gamma^{\frac{1}{1-(1-\beta)\rho}} \left(\frac{Q_t^h}{Q_t^l} \right)^{1-\frac{1-\rho}{1-(1-\beta)\rho}} \left(\frac{N_t^h}{N_t^l} \right)^{-\frac{1-\rho}{1-(1-\beta)\rho}}. \tag{17}$$

Education increases workers' wages potentially through three channels: γ, Q_t^h/Q_t^l and N_t^h/N_t^l. The first two increases the skill premium, as they imply either a higher contribution of high-skilled intermediates to the final good (γ), or better quality machines in the high-skilled sector (Q_t^h/Q_t^l). The last term decreases the skill premium, as there are decreasing returns in production.

The labour supply aggregates N_t^h and N_t^l are the total amount of high- and low-skilled labour available in period t:

$$N_t^h = (1-\lambda) \sum_{j=0}^{\infty} \lambda^j \int_c f(c) e(c)_{(t-j)} dc, \tag{18}$$

$$N_t^l = (1-\lambda) \sum_{j=0}^{\infty} \lambda^j \int_c f(c)(1 - e(c)_{(t-j)}) dc = 1 - N_t^h. \tag{19}$$

3 Equilibrium

All firms maximise profits and all individuals maximise their lifetime utility at all times, that is the economy is in a decentralised equilibrium.

Definition 1. *A* ***decentralised equilibrium*** *is a sequence of optimal education decisions* $\{e(c)_t\}_{t=0}^{\infty}$*, labour supplies* $\{N_t^h, N_t^l\}_{t=0}^{\infty}$*, discounted present values of expected lifetime income* $\{\mathbf{W}_t^h, \mathbf{W}_t^l\}_{t=0}^{\infty}$*, intermediate good prices* $\{p_t^h, p_t^l\}_{t=0}^{\infty}$*, average qualities* $\{Q_t^h, Q_t^l\}_{t=0}^{\infty}$*, investments into R&D* $\{\overline{z}_t^{j,h}, \overline{z}_t^{j,l}\}_{t=0}^{\infty}$ *and values of owning the leading vintage* $\{V_t^{j,h}, V_t^{j,l}\}_{t=0}^{\infty}$ *for all lines* $j \in [0,1]$*, where* $\{Q_0^h, Q_0^l, N_0^h, N_0^l\}$*, such that the following conditions are satisfied:*

1. *the labour supplies satisfy* (18) *and* (19)*;*
2. *lifetime earnings are as in* (15) *and* (16)*;*
3. *the average quality in sector* s *evolves according to* (10)*;*
4. *total R&D investment* $\overline{z}_t^{j,s}$ *satisfies* (8) *for all* $t \geq 0$ *and all* $j \in [0,1]$*;*
5. *the sequence* $\{V_t^{j,s}\}_{t=0}^{\infty}$ *satisfies* (7)*;*
6. *the price sequence* $\{p_t^h, p_t^l\}_{t=0}^{\infty}$ *satisfies* (2) *and the relative price,* p_t *satisfies* (12)*;*
7. *the optimal education decisions,* $\{e(c)_t\}_{t=0}^{\infty}$ *are as in* (14).

3.1 Steady state

In this section I identify the steady states or balanced growth paths (BGP) of this economy, which are decentralised equilibria, where all variables are constant or grow at a constant rate. The solution of the steady state follows that in paper 1, here I present a more informal discussion.

In the steady state the total R&D spending on all lines within a sector are equal, $\overline{z}^{j,s*} = \overline{z}^{s*}$ for $j \in [0,1]$ and $\overline{z}^{s*}$ is given by:

$$\beta(1-\beta)^{\frac{1-\beta}{\beta}}(p^{s*})^{\frac{1}{\beta}}N^{s*} = B\overline{z}^{s*}\frac{(1+r-e^{-\eta\overline{z}^{s*}})}{1-e^{-\eta\overline{z}^{s*}}} \quad \text{for} \quad s = \{l, h\}. \tag{20}$$

Hence R&D effort in a sector is increasing in the period profit from machine sales. As discussed earlier, these profits are increasing in the price of the intermediate, p^{s*}, and in the amount of labour, N^{s*}, which uses this technology.

Relative quality, Q^*, has to be constant along the BGP, which requires equal R&D spending in the two sectors: $\overline{z}^{h*} = \overline{z}^{l*} = \overline{z}^*$. From (20) this holds if:

$$p^* = \frac{p^{h*}}{p^{l*}} = \left(\frac{N^{h*}}{N^{l*}}\right)^{-\beta}. \tag{21}$$

The relative quality in the steady state can be expressed by combining the relative price (1),(21) with the intermediate output (9):

$$Q^* = \frac{Q^{h*}}{Q^{l*}} = \gamma^{\frac{1}{1-\rho}} \left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{\beta\rho}{1-\rho}}. \tag{22}$$

Since the skill premium depends on the relative quality and the relative price, the above two equations are key in understanding the dynamics of the skill premium. These ratios both depend on the relative supply of skills, therefore their interaction determines the response of the skill premium to relative skill supply.

From (22) the relative quality level depends on the relative abundance of the two types of labour along the balanced growth path. If there are more high-skilled workers, an innovation in the high-skilled sector is more profitable. Hence technology is more skill-biased – Q^* is greater, – if the relative supply of skills is higher.

The relative price of intermediate depends negatively on the relative supply of high-skilled workers (from (21)). Intuitively, more high-skilled workers and better technologies leads to more high-skilled intermediate production, which lowers the relative price of the intermediate. Moreover, since more R&D is directed towards the larger sector (from (22)), more high-skilled workers implies a higher relative quality, Q^*.

Along the steady state, technological *change* is not biased towards either sector, since both sectors are growing at the same rate, implying that the skill-bias of technology is constant.

The skill premium using (17), (22) and (21) can be expressed as:

$$\frac{w_t^{h*}}{w_t^{l*}} = \left(\frac{p^{h*}}{p^{l*}}\right)^{\frac{1}{\beta}} \frac{Q_t^{h*}}{Q_t^{l*}} = \gamma^{\frac{1}{1-\rho}} \left(\frac{N^{h*}}{N^{l*}}\right)^{\frac{\beta\rho}{1-\rho}-1}. \tag{23}$$

The skill premium depends on two components: the relative price and the relative quality. Since the relative price depends negatively, while the relative quality depends positively on the relative supply of skilled workers, the net effect depends on which influences the wages more.

If the two intermediates are highly substitutable (ρ is higher), the price effect is smaller and is dominated by the effect of relative quality. On the other hand, if the elasticity of substitution is low (low ρ), the price effect is stronger than the quality effect in the steady state.

For sufficiently high ρs (if $(\beta\rho)/(1-\rho)-1 > 0$) the skill premium is an increasing function of the relative supply of skills. In this case, the increase in relative quality more than compensates for the decrease in relative price. This is what **?** termed as *strong relative bias* in technology, as increase in the relative supply of skills increases the skill premium. On the other hand if $(\beta\rho)/(1-\rho)-1<0$ then the skill premium is decreasing in the relative supply, and technology displays *weak relative bias*: the technology effect does not compensate for the price effect.

The skill premium is constant in the steady state (from (23)), and from Remark 1 the skill premium has to be greater than one in at least one period. This implies that $w_t^{h*} > w_t^{l*}$ for all $t \geq 0$.

Result 1. *Every individual born in period t acquires education if his cost $c < c^*$, where c^* is the* cutoff cost *implicitly defined by:*

$$c^* = \frac{1 - \frac{w^{l*}}{w^{h*}}}{1 - \frac{g^*\lambda}{1+r}}. \tag{24}$$

Proof. Combining (14) with (15) and (16), implies that the condition for acquiring education is:

$$\sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r}\right)^s w_{t+s}^{h*} - \sum_{s=0}^{\infty} \left(\frac{\lambda}{1+r}\right)^s w_{t+s}^{l*} \geq w_t^{h*} c.$$

This shows that the optimal education decision is equivalent to a threshold time cost, c_t^*. Using the fact that wages in both sectors grow at a constant rate g^*, and that the skill premium, w_t^{h*}/w_t^{l*} is constant, $c_t^* = c^*$ is constant and given by (24). □

The supply of high-skilled workers using the previous result and (18) can be expressed as:

$$N^{h*} = F(c^*) = F\left(\frac{1 - \frac{w^{l*}}{w^{h*}}}{1 - \frac{g^*\lambda}{1+r}}\right), \tag{25}$$

where $F(\cdot)$ is the cumulative distribution function of the cost of acquiring education. The threshold cost for acquiring education and consequently the fraction of high-skilled workers depends positively on the skill premium and on the growth rate of the average qualities. The threshold is increasing in the skill premium, since a higher skill premium implies a greater per period gain from working as high-skilled. The growth rate of wages also increases the threshold cost; if wages grow at a higher rate, then for a given skill premium, future gains are greater.

The growth rate of the economy depends on the amount of R&D spending, z^*, which can be expressed as (using (2) and (21)):

$$B\overline{z}^* \frac{(1 + r - e^{-\eta\overline{z}^*})}{1 - e^{-\eta\overline{z}^*}} = \beta(1-\beta)^{\frac{1-\beta}{\beta}} \left(\gamma N^{h*\frac{\beta\rho}{1-\rho}} + N^{l*\frac{\beta\rho}{1-\rho}}\right)^{\frac{1-\rho}{\beta\rho}}. \tag{26}$$

The right hand side is the steady state per period profit from owning the leading vintage normalised by the quality of the vintage. The profit is increasing in both N^{h*} and N^{l*}. If the labour supply increases, then any unit of investment into R&D has a higher expected return, since there are more people who are able to use it. The left hand side is increasing in z^*.[4] This implies that the steady state R&D spending, and hence the steady state growth rate is increasing in the labour supplies. The growth rate of the economy is given by:

$$g^* = 1 + (\overline{q} - 1)(1 - e^{-\eta \overline{z}^*}). \tag{27}$$

This completes the identification of the steady state. The cutoff cost for acquiring education determines N^{h*}. In turn, the supply of high-skilled labour, N^{h*}, determines every other variable in the economy in the steady state. From (26) N^{h*} determines the optimal investment into R&D, z^*. This pins down the growth rate, g^*, through (27). The supply of high-skilled workers also determines the skill premium, w^{h*}/w^{l*}, through (23). On the other hand, these variables (w^{h*}/w^{l*} and g^*) pin down the steady state cutoff cost for acquiring education, c^*, which pins down the level of N^{h*} through (25). The possible steady state high-skilled labour supplies of the economy are thus the fixed points of the function $F(h(\cdot))$, and the steady state of the economy is fully characterised by the supply of high-skilled labour, N^{h*}:

$$N^{h*} = F(h(N^{h*})), \quad \text{where} \tag{28}$$

$$h(x) \equiv \frac{1 - \frac{w^{l*}}{w^{h*}}(x)}{1 - \frac{g^*(x)\lambda}{1+r}}. \tag{29}$$

The function $h : (0, 1) \to \mathbb{R}$ is defined as the optimal cutoff value c^* for a given supply of high-skilled workers N^h, where the skill premium is given by (23), and the growth rate is given by (27). The steady state of the economy is the fixed point of function $F(h(N^h))$, as shown in Figure 1.

The panel on the left shows the case of a strongly biased technology, while the panel on the right shows a weakly biased technology. Whether $F(h(N^h))$ is increasing or decreasing in N^h depends on whether $h(N^h)$ increases or decreases in N^h. The optimal c^* depends on N^h through the growth rate, g, and through the skill premium, w^h/w^l. Hence the sign of $h'(N^h)$ depends on the net effect from these two channels.

[4] To see this, take the derivative:

$$\frac{\partial z^* \left(1 + \frac{r}{1-e^{-\eta z^*}}\right)}{\partial z^*} = 1 + \frac{z^*}{1 - e^{-\eta z^*}} \left(1 - \frac{r\eta z^* e^{-\eta z^*}}{1 - e^{-\eta z^*}}\right).$$

A sufficient condition for this derivative to be positive is $1 - \frac{r\eta z^* e^{-\eta z^*}}{1-e^{-\eta z^*}} \geq 0$. This can be rearranged to the following inequality:

$$1 \geq e^{-\eta z^*}(1 + r\eta z^*).$$

For $z^* = 0$ this holds with equality, while the right hand side is decreasing in z^*. QED

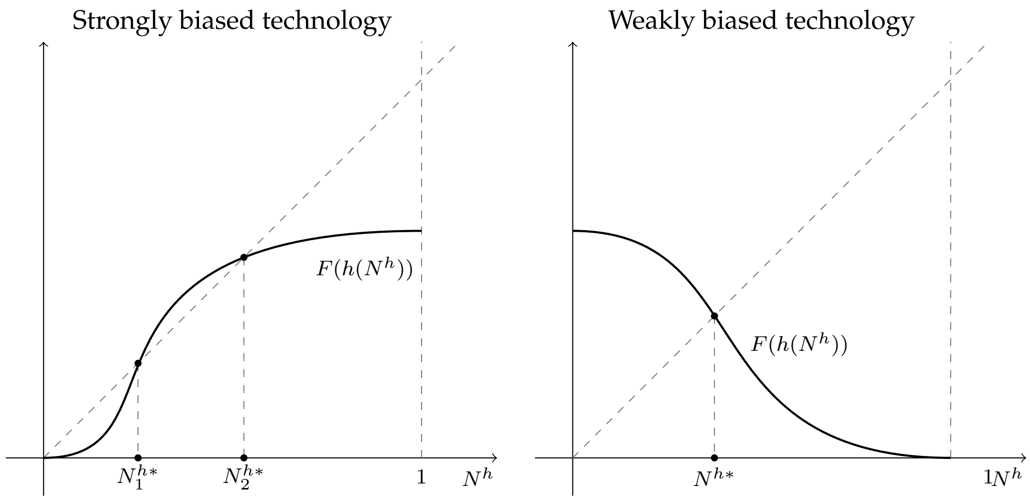

Figure 1: Steady states

The effect of N^h on the growth rate depends on the elasticity of substitution between the two intermediate goods. If the elasticity of substitution is not too high ($\rho < 1/(1+\beta)$), then the growth rate is increasing until it reaches its maximum at $N^h = 1/(1+\gamma^{\frac{1-\rho}{\beta\rho-(1-\rho)}})$, and then it decreases as N^h increases further. The intuition for this result is that when the elasticity is low, then similar amounts are required from the two goods, and hence it is not good to specialise in neither high- nor low-skilled intermediates. On the other hand, if the elasticity of substitution is higher ($\rho > 1/(1+\beta)$), then the two goods can be easily substituted, and the best is to specialise in either high- or low-skilled intermediate production. In this case the growth rate is decreasing until $N^h = 1/(1+\gamma^{\frac{1-\rho}{\beta\rho-(1-\rho)}})$, where the growth rate is the lowest, and then starts increasing as N^h increases further.

The elasticity of substitution also determines the effect of N^h on the skill premium. Recall that, with an increase in the (relative) supply of high-skilled labour the steady state relative quality increases (from equation (22)), since when a larger labour force works in a sector, the demand for machines in that sector, and hence profits on machines increases. However, parallel to the increase in the relative quality, the relative price of the intermediate good produced by the high-skilled workers decreases, as the supply of high-skilled intermediates increases. The effect of an increase in N^h on the skill premium depends on the strength of these two responses. As discussed earlier, when the two intermediates are easily substitutable, $\rho > 1/(1+\beta)$, then the effect of the relative quality dominates, and the technology is strongly biased. In this case the skill premium is increasing in N^h. On the other hand, when the two intermediates cannot be substituted that easily, $\rho < 1/(1+\beta)$, then the relative price effect dominates, and technology is weakly biased. The skill premium decreases with N^h in such cases.

For most parameter values, however, the effect of N^h on the growth rate is relatively small, and is dominated by the effect of N^h on the skill premium. This implies that when technology is strongly biased, the skill premium is increasing in the supply of high-skilled workers, and $h'(N^h) > 0$, and hence the $F(h(N^h))$ curve is upward sloping. Conversely, when technology is weakly biased, the skill premium is decreasing in the supply of high-skilled workers, then $h'(N^h) < 0$ and the $F(h(N^h))$ curve is downward sloping.

In the case of a weakly biased technology there is maximum one steady state, depicted in the right panel of Figure 1 by N^{h*}. The graph suggests that this steady state is stable, as for high-skilled labour supplies lower than N^{h*}, a higher fraction of the new cohort would acquire skills than N^{h*}, and the converse is true for high-skilled labour supplies higher than N^{h*}. However, the conditions that govern $F(h(N^h))$ only hold in the steady state, so to fully ascertain the stability of the steady state, an analysis of the transitional dynamics is required.

In the case of a strongly biased technology multiple steady states are possible, as depicted in the left panel of Figure 1 by N_1^{h*} and N_2^{h*}. The graph suggests, that steady states where $F(h(N^h))$ crosses the 45 degree line from below are unstable (like N_1^{h*}), whereas the steady states where it crosses it from above are stable (like N_2^{h*}).

4 Comparative dynamics

In this section I analyse the characteristics of the transition path. I look at two types of transitions. In the first section, I do not assume that the economy is in a steady state, but I analyse the transition path from different initial points to the steady state. In the second, I assume that the economy is in the steady state, introduce a change in of the parameters, and follow the transition path to the new steady state. In both cases, throughout the transition the economy is in a decentralised equilibrium, and the transitional dynamics are governed by the initial value of the state variables and the final steady state. I calculate the transition paths using second order approximation of the decentralised equilibrium around the final steady state.[5]

4.1 Initial values

Available data shows that the supply of high-skilled workers and the educational attainment of consecutive cohorts has been steadily increasing over time, however, the growth rate of educational attainment has significantly slowed down over the last few years.[6] This evidence suggests that the developed economies have been in a transition towards their steady state. Therefore in this section I do not formulate a hypoth-

[5]See Section B.1 of the Appendix for the equations that have to hold during the transition.
[6]See for example OECD Factbook 2005-2010, various editions of OECD Education at a Glance.

esis about a change in steady states. Instead I analyse the dependence of the transition path on the initial values of the economy.

This analysis shows, that an increasing skill supply, increasing relative quality and increasing skill premium can arise during the transition to the steady state regardless of whether the technology is strongly or weakly biased.

I consider two baseline set of parameter values for the steady state, one that features weakly biased technology, and one that features strongly biased technology.[7] I choose the sets of baseline parameters to provide reasonable steady state values: a final skill supply of 45 percent, a final skill premium of around 40 percent, and an annual growth rate of around 2 percent.[8] I analyse the transition path to the steady state from all possible initial skill supply and relative quality pairs. This exercise shows that the steady state can be reached with increasing skill supply and *increasing* skill premium in case of both weakly and strongly biased technologies.

This is an important result, as it implies that observing increasing relative supplies and increasing relative wages can be the result of the economy's normal transition process while building up human capital. On the one hand, the relative quality depends positively on the supply of skills. On the other, the skill premium, which determines the change in the skill supply, depends positively on the relative quality. Therefore, it is not surprising that during the transition the skill supply, the relative quality, and potentially the skill premium increase together. If the economy is not in the steady state, the explanation of this phenomenon does not require exogenously skill biased technological progress nor a strong, endogenous bias in technology. Only two conditions are necessary for this to happen. First, the relative quality has to increase as a response to an increase in the relative supply. If R&D is modeled as a profit driven activity, and profits are increasing in the demand, then this is a natural result. The second necessary condition is that the quality in the high-skilled sector is not too high compared to the quality in the low-skilled sector. If the initial relative quality is also the result of some form of optimization, and the supply of high-skilled workers is low, then again, this is a natural result of a profit maximizing R&D sector.

Based on numerical solutions to the transition path Figure 2 shows how the type of transition depends on the initial values.[9] The AOB curve shows the border where the direction of change in the relative quality, Q, changes. Below the curve, relative quality is increasing, whereas above the curve, relative quality is decreasing. The left panel in Figure 2 shows a strongly biased technology, while the right panel shows a

[7]I take β, λ and r to be the same as in the previous paper. The parameter-values are:

	ρ	γ	η	$\overline{q}$	B	μ	σ	β	λ	r
weak bias	0.5	1.15	0.04	2.08	0.3	1	6	2/3	8/9	1.05^5
strong bias	0.7	1.15	0.04	2.08	0.16	1	6	2/3	8/9	1.05^5

[8]Of course, *reasonable* is not an easily judgeable concept here, since I assume that none of the advanced economies are in their steady state yet.

[9]See Section B.2 of the Appendix for Matlab graphs.

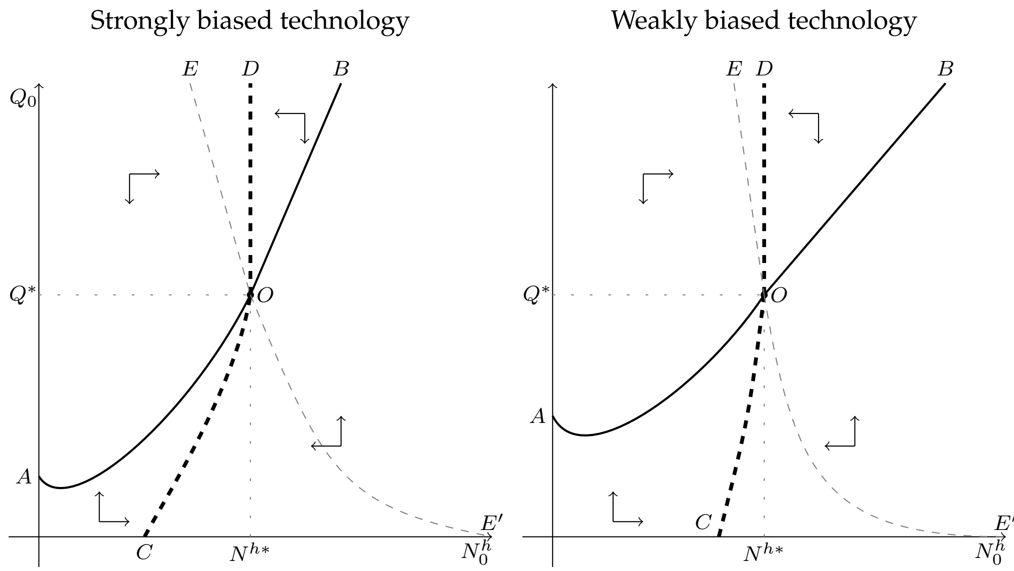

Figure 2: Phase diagram

weakly biased technology. The AOB curve is much steeper in the left panel, i.e. for higher ρs, indicating that for low values of N^h, a lower relative quality is desirable if ρ is higher, and the converse is true for above steady state values of N^h. If the elasticity of substitution is higher, relative prices are less sensitive to the relative output of the two sectors (see (1)), which translates into less sensitivity of monopolist profits. Hence, with higher ρ it is less worthwhile to invest into the high-skilled sector if N^h is low, while it is more worthwhile if N^h is high. Therefore, since the steady state is almost the same in the two cases depicted on Figure 2, and the AOB curve is bound to be less steep for lower ρs, the relative quality decreases for more initial values below N^h and for fewer initial values above N^h.

The dashed curve COD shows the border where the direction of change in the skill supply, N^h, changes. To the left of the curve, N^h is increasing, while to the right, it is decreasing. Comparing curves COD, which determine the movement of N^h, the implication is that for a given value of Q a lower supply of high-skilled labour is desirable. This can be understood from (17), which shows that for a higher ρ, the skill premium is more sensitive to the relative quality than to the relative supply of skills. Therefore for a higher ρ with the same Q a lower N^h is necessary.

The steady state, denoted by O, is at the intersection of curves AOB and COD, where neither Q nor N^h changes. Numerical solutions show that there are only two ways to reach the steady state, O: either from above right, where both Q and N^h are decreasing, or from below left, where both Q and N^h are increasing. From the left side of the EOE' curve, the economy transitions to the steady state from below, whereas from the right of this curve the economy transitions from above.

If the economy starts from the area bounded by AOE, then while the economy stays in this area the relative quality continuously decreases, while the supply of high-skilled workers increases. This is due to the fact that the initial relative quality is too high compared to the relative supply of skilled workers, therefore it is more worthwhile to invest in improving the quality in the unskilled sector, leading to a decline in the relative quality. On the other hand, the skill premium is quite high, therefore the new cohorts keep acquiring more education than previous ones, and the supply of skills increases.

If the economy starts from the COE' area, then the supply of skills decreases, while the relative quality increases as long as the economy stays in this region. Here the relative supply of skills is too high compared to the relative quality, therefore the new cohorts acquire less education than previous ones, and the supply of skills decreases. Meanwhile, since the supply of skills is high, the R&D sector focuses investment into the skilled sector, and the relative quality continuously increases.

From both of these regions, the economy eventually moves into the AOC area. Here, the relative quality is neither too high, nor too low compared to the supply of skills, and hence both the supply of skills and the relative quality increase together to the steady state, O.

In the $E'OB$ area the supply of skills is too high compared to the relative quality. Therefore, the new cohorts acquire less education, and the skill supply decreases, while the R&D sector invests into the high-skilled sector, and the relative quality increases.

If the economy starts in the EOD region, then the relative quality is too high compared to the supply of high-skilled workers. Therefore, the relative quality decreases, as there will be more investment into the unskilled sector, while the supply of skills increases, as the skill premium is high, and new cohorts acquire more education.

From both the $E'OB$ and EOD area the economy moves into the DOB region, where the relative quality is neither too high nor too low compared to the supply of skills, and both decrease together to the steady state, O.

This shows, that if the supply of high-skilled workers approaches its steady state from below, then the transition path can only feature decreasing relative quality at the beginning of the transition, but as the economy gets closer to the steady state, then eventually the relative quality increases. Therefore, technological change can only be *unskill-biased* at the beginning of the transition, and it is necessarily *skill-biased* while approaching the steady state. On the other hand, if the supply of skills reaches its steady state value from above, then the relative quality decreases for most of the transition, apart from some of the initial periods. Thus technological change is *unskill-biased* for most of the transition. Therefore the skill supply and the relative quality tend to move together during the transition. This is due to the positive dependence of these two variables on each other. If the relative quality increases in the future, the skill premium increases as well, which leads to an increase in the skill supply. If the skill supply increases in the future, then there are more gains to be made from investing in high-skilled machines, and hence relative quality increases as well. Therefore the joint increase of the skill supply and the relative quality should not be surprising. However, this does not automatically imply that the skill supply and the skill premium should move hand-in-hand as well.

There are two aspects of the skill premium that are of interest: its change in the short run, and its change in the long run. The long run change in the skill premium is the change between the initial skill premium and the final, steady state skill premium. From (23) the skill premium increases in the long run if:

$$Q_0 \leq \left(\frac{N_0^h}{N^{h*}} \right)^{\frac{1-\rho}{\beta\rho}} \left(\frac{1 - N^{h*}}{1 - N_0^h} \right)^{\frac{1-\rho}{\beta\rho}} Q^* \equiv s(N_0^h).$$

The function $s(N_0^h)$ is depicted in Figure 3 by the blue curve, and the above inequality implies that if the initial point is below the blue curve, then the skill premium increases in the long-run. There are two things to note from this inequality. First, that $s(N_0^h)$ is upward sloping. If the initial skill supply is higher, the initial relative quality can be higher, and the skill premium still increases in the long run. The intuition for this is that the relative supply and the relative quality have opposite effects on the skill premium: while the former decreases it, the latter increases it, thus leaving it unchanged. Second, that the blue curve in the strongly biased technology case (left panel) is flatter: for low initial skill supplies it is higher and for high initial skill supplies it is lower. If ρ is higher, then the effect of the relative quality on the skill premium is larger, and the effect of the relative supply is lower. Therefore, a very low skill supply does not imply such a high skill premium if ρ is larger, while a very high skill supply does not imply such a low skill premium. This implies that for higher ρs the $s(N_0^h)$ curve is flatter.

In Figure 3 the different shades of gray represent the different paths the skill premium can take throughout the transition. The two lighter colours represent the initial points from which the skill premium increases in the long-run, while the two darker grays represent the initial points from the skill premium decreases in the long-run.

The short-run change in the skill premium depends on the magnitude of the change in the relative quality and the relative supply of skilled workers. From (17), we get:

$$\frac{\frac{w_t^h}{w_t^l}}{\frac{w_{t-1}^h}{w_{t-1}^l}} = \left(\frac{Q_t}{Q_{t-1}} \right)^{\frac{\beta\rho}{1-(1-\beta)\rho}} \left(\frac{\frac{N_t^h}{N_t^l}}{\frac{N_{t-1}^h}{N_{t-1}^l}} \right)^{-\frac{1-\rho}{1-(1-\beta)\rho}} . \tag{30}$$

An increase in the relative skill supply reduces the skill premium, while an increase in the relative quality increases it. The greater the increase in the relative quality compared to the increase in the relative supply, the more likely it is that the skill premium also increases. From this equation it is easy to see that a higher ρ makes the skill premium more responsive to changes in the relative quality and less responsive to changes

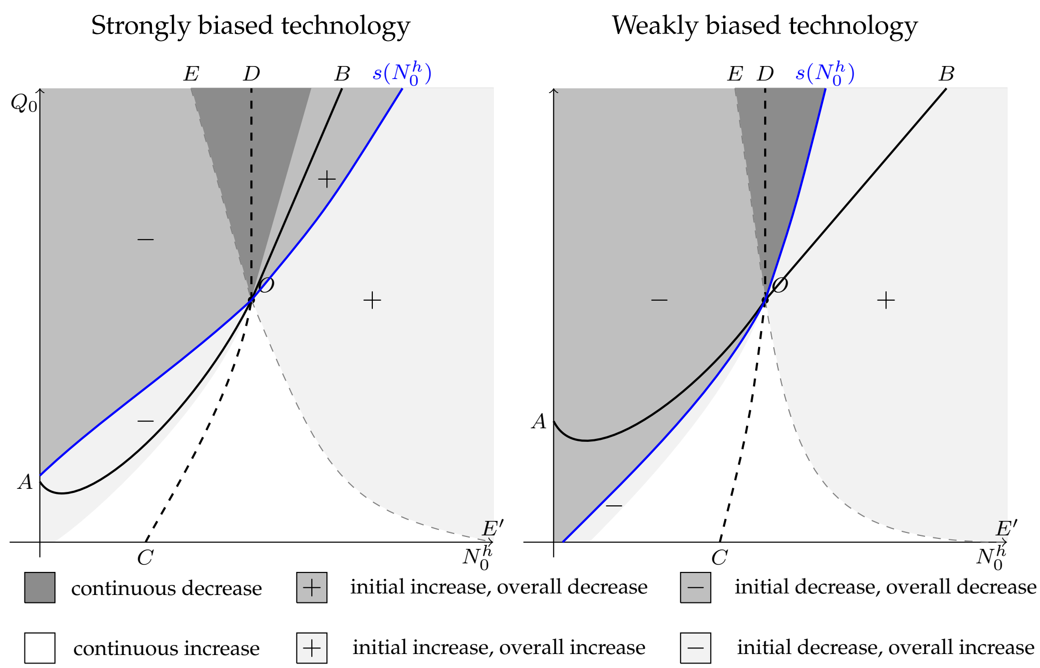

Figure 3: The path of the skill premium

in the relative skill supply. Intuitively, this is because for more substitutable intermediates, as the price effect is smaller, the effect of the relative quality on the skill premium is stronger than the effect of the relative supply. Hence, when ρ is higher a smaller increase in Q leads to a greater increase in the skill premium.

If the economy is in the DOA area, the relative quality decreases, while the relative supply increases (from Figure 2). From (30), this leads to an unambiguous decrease in the skill premium. The opposite holds for an economy that is in the BOC area, leading to an unambiguous increase in the skill premium. This is depicted by the $+$ and $-$ signs in Figure 3.

In the AOC area both the skill supply and the relative quality is increasing, hence in general, the overall effect on the skill premium is ambiguous. It is clear, that the closer is the economy to the AO curve, the less likely it is that the skill premium increases, as the relative quality hardly changes initially at these points. In the case of strongly biased technologies, as discussed earlier, the skill premium is more responsive to changes in the relative quality, and less responsive to changes in the relative supply. Therefore, for most part of the AOC area the skill premium increases (shown in white), and only for a smaller fraction does it decrease (shown in the lightest gray). The situation is different if the technology is weakly biased. In this case the skill premium only increases for a smaller part of the AOC area (again shown in white), and for the rest, the skill premium decreases.

In the case of strongly biased technologies, as the $s(N_0^h)$ curve is above the AO curve, the skill premium increases in the long-run for all initial values in area AOC. This implies that even if the skill premium decreases initially for economies in the light gray area, the transition takes the economy into the white region, where the skill premium increases continuously, finally increasing above its original value. If the technology is weakly biased, then as the $s(N_0^h)$ curve is below the AO curve, for some values (shown in the darker gray), the skill premium decreases in the long-run, and for only a smaller set of initial points does it increase in the long run after an initial decline (lighter gray area).

If the economy is in the DOB area, the change in the skill premium is ambiguous, as both the skill supply and the relative quality decreases. Again, the closer is the initial point to the OB curve, the more likely it is that the skill premium initially increases. This is due to the fact that close to the OB curve the change in the relative quality is small, and hence the decrease in the relative supply can potentially dominate its effect. In case of strongly biased technologies, from (30), the skill premium is more responsive to changes in the relative quality. Therefore, the change in the relative supply can dominate the effect of decreasing technology for a smaller set of initial points (shown in the second darkest gray), and the skill premium decreases for most values (darkest gray). In case of weakly biased technologies the skill premium increases for a larger set from the DOB area (shown in light gray), since the skill premium is more responsive to changes in the supply of high-skilled workers.

Since the $s(N_0^h)$ curve is below the OB curve for strongly biased technologies, the skill premium decreases in the long run for the entire set of initial values in the DOB region, while for a large part of the DOB area in case of weakly biased technologies the skill premium increases in the long-run (shown in the lightest gray).

To summarise, the darkest gray represents the area where the skill premium continuously decreases throughout the entire transition. In this area, the relative quality is much higher than what is profitable given the current supply of skills and the future decreasing path of skills, therefore the relative quality decreases drastically, while the relative supply of skills decreases at a slower rate (or even increases from area EOD) as the skill premium is relatively high. Therefore the skill premium decreases continuously until it reaches its steady state value. For higher ρs this area is wider, as the skill premium responds more to changes in the relative quality.

The white areas contain the initial points from where the skill premium continuously increases throughout the transition. These are points, where the relative quality is low compared to the current supply and the future increasing path of high-skilled workers. If the sub-optimality of the relative quality is sufficiently large, then it increases at such a high rate, that it dominates the slowly increasing supply of skilled workers (or for the initially decreasing supply from area COE'). Therefore from the white area the skill premium continuously increases until reaching the steady state. Again, this area is wider for higher values of ρ, as the skill premium is less sensitive to supply effects.

If the initial point is in one of the medium gray areas and $N_0^h < N^{h*}$, then the skill premium initially decreases (denoted by the $-$ sign in the area), and then increases until the steady state is reached, whereas if $N_0^h > N^{h*}$, then it increases initially (denoted by the $+$ sign in the area), and then decreases to the steady state. This suggests that the stable arm lies in the white area if the initial point is to the left of the EOE' curve, and it lies in the darkest gray area if the initial point is to the right of the EOE' curve. [10] If the initial point is in the second darkest area, then the skill premium decreases compared to its initial value in the long run. The lightest gray area contains the initial points for which the skill premium increases in the long run.

In light of this analysis, the fact that the skill supply and the skill premium have been growing together over the last few decades should not be surprising. The developed economies had to start with a sufficiently low relative quality in the high-skilled sector, and the skill supply and the skill premium had to increase together. Moreover, an unexpected increase in N^h, for example due to bigger cohort sizes or other reasons for enrollment into higher education (for example to avoid the draught), would push the economy towards the right. This would reduce the skill premium immediately, while possibly shifting the economy into the white region. An important implication of the joint analysis of skill supply and relative technologies, is that as ρ decreases, this area shrinks, but it does not disappear. Therefore, the relative bias of technology does not have to be strong in order to observe increasing skill premium and increasing skill supply.

4.2 Parameters

In this section I consider the dynamic effects of changes in various parameters of the model.[11] I analyse how the steady state changes and the characteristics of the transition path. It is important to note that the exact path of the transition, as discussed in the previous section, is determined by the region in which the old steady state falls compared to the new steady state in terms of Figure 3.

The steady state is affected by parameters in two ways. The distribution of education costs affect the steady state supply of high-skilled workers by changing the function $F(\cdot)$. All other parameters, which are either connected to the production of goods or to the R&D process affect the steady state via changing the function $h(\cdot)$.

The effects of the parameters of the R&D process are the most straightforward to assess. These parameters only affect the steady state through their effect on the growth rate. If a change in a parameter increases the growth rate of the economy, then from equation (1) the steady state gain from working as high-skilled relative to low-skilled increases. This implies an upward shift in the $F(h(N^h))$ curve, as $h(N^h)$ increases for every N^h.

Figure 4 demonstrates the effects of an upward shift of $F(h(N^h))$ on the steady states. In case of a

[10] The final increase or decrease in the skill premium is at times hardly noticeable, implying that for most of the transition the economy stays in one of the medium gray areas.

[11] The final steady state parameters are the same as in the previous section.

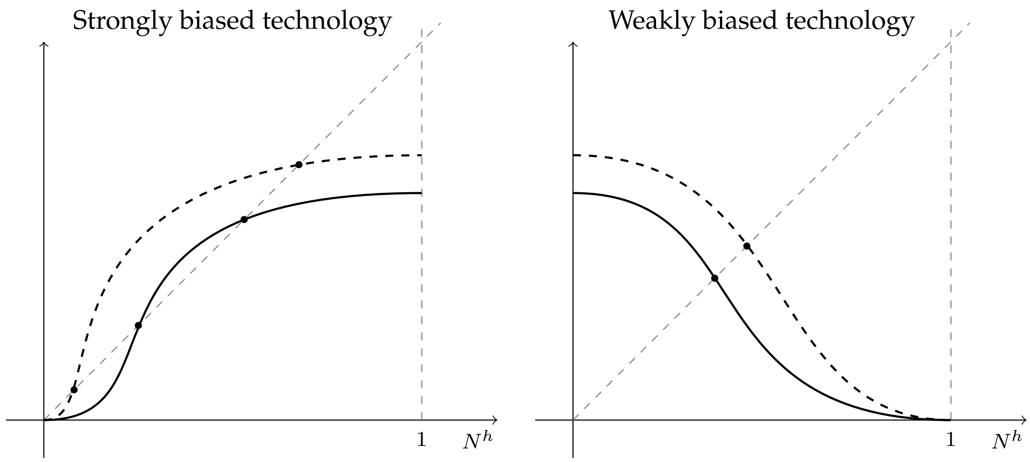

Figure 4: Higher $\overline{q}, \eta, \gamma$ or lower B, μ

weakly biased technology, the steady state N^{h*} unambiguously increases. For strongly biased technologies the situation is more complicated. Steady states where the $F(h(N^h))$ curve crosses the 45 degree line from below shift down, while steady states where the $F(h(N^h))$ curve crosses the 45 degree line from above shift up. However, the *stable* steady state is where $F(h(N^h))$ crosses from above, and in these cases, similarly to the weakly biased technology case, the stable steady state N^{h*} unambiguously increases.

Parameter η controls the effectiveness of R&D spending (through the Poisson arrival rate of innovations), and $\overline{q}$ controls the quality improvement per innovation. An increase in either of these parameters increases the growth rate. Since B is the price of investing one unit into innovation in terms of final good, B increases the cost of the R&D activity, and hence decreases the equilibrium growth rate. Therefore an increase in either η or $\overline{q}$ as well as a decrease in B increases the steady state supply of high-skilled workers unambiguously regardless of whether the technology is strongly or weakly biased. However, from (23) in case of a strongly biased technology this unambiguously implies a higher final skill premium, whereas with a weakly biased technology, this implies a lower final skill premium. Figure 5 shows the transition paths for a change in the parameter η.[12]

An increase in η increases the growth rate of the economy immediately, which increases the present value gain from acquiring education. Therefore there is a jump in the educational attainment of new cohorts, as can be seen on the top right panel for both types of technology. There is a difference though in the consequent path of $F(c_t^*)$: in case of a strongly biased technology, it continues to increase, whereas for weakly biased technologies, it declines after its initial increase. This is due to the differential response of the skill premium to the increase in the relative supply and relative quality. The initial response of the skill

[12]Since the transition paths look very similar in case of an increase in $\overline{q}$ or a decrease in B, I do not include them in the main text, they can be found in the Section B.3 of the Appendix.

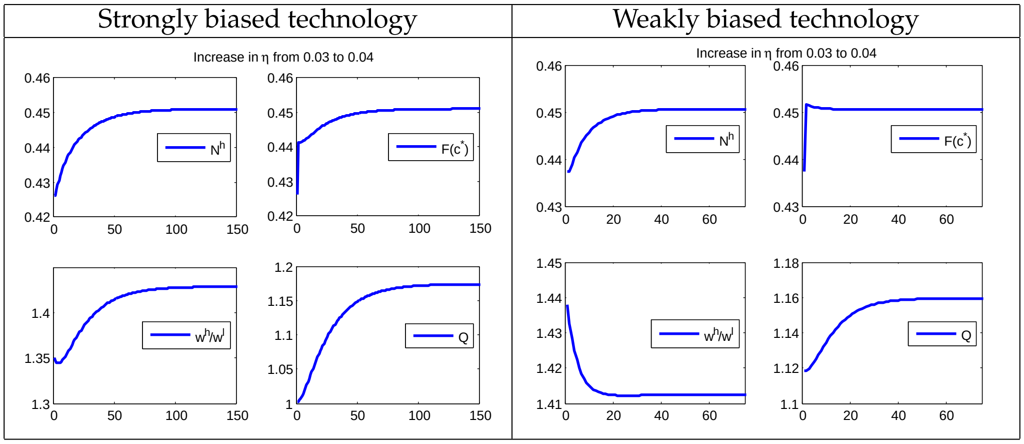

Figure 5: Change in the R&D parameters

premium in both cases is a decline, as relative quality does not change, while the skill supply increases. In case of a weakly biased technology, the skill premium continues to decrease, thereby offsetting some of the increase in the present value gain from acquiring education, whereas in case of a strongly biased technology, the skill premium starts to increase, this way further increasing the present value gain from acquiring education. The skill supply and the relative quality continuously increases for both types of technology, although the increase is more pronounced in case of a strongly biased technology.

In terms of Figure 2, this implies that the initial steady state was in the AOC region compared to the new steady state. In the case of strongly biased technologies, based on the path of the skill premium the initial point fell into the light gray region within AOC in Figure 3, and the economy almost immediately crossed over to the white region during the transition. In case of a weakly biased technology the skill premium almost continuously falls, there is just a slight increase before reaching the steady state, thus the initial point fell into the dark gray region within AOC in Figure 3, and the economy crossed over to the white region just before reaching the steady state.

The next set of parameters I consider are related to the production of the final good, γ and ρ. First consider γ, the weight of the high-skilled intermediate in the production of the final good. Intuitively an increase in this parameter increases the value of the high-skilled intermediate and thus increases the returns to acquiring education as well. This intuition is supported by equation (23), which shows that an increase in γ increases the skill premium. At the same time, γ also affects the steady state through its effect on R&D. From equation (26), an increase in γ increases the returns to investment into R&D, and hence increases the growth rate.[13] Both of these shift the $h(\cdot)$ function up, and therefore an increase in γ has similar effects as

[13] Note that an increase in γ increases the returns to R&D in both the high- and the low-skilled sector. This is the case, as γ besides measuring the relative importance of high- and low-skilled intermediates in the production of final good, also measures the absolute contribution of high-skilled intermediates. An increase in γ increases the final output for any combination of inputs, i.e. it makes production more efficient.

depicted in Figure 4.

Figure 6 shows the transition from the old steady state to the new one in case of an unexpected increase in γ. An increase in γ immediately increases the skill premium and the growth rate, thereby increasing the present value gain from acquiring education. This leads to an immediate jump in the education acquisition of new cohorts (as can be seen on the top right panels). The skill premium (bottom left panels) continues to increase in both cases, though to a much smaller extent in case of weakly biased technologies. For weakly biased technologies, the increase in the supply of high-skilled workers and in the relative quality reduces the skill premium. However, the increase in γ has a direct positive effect on the skill premium by increasing the weight of the skilled intermediate in the production of the final good (see (23)). Therefore in case of weakly biased technologies, the overall effect depends on the magnitude of the two opposing effects. In the example below, the skill premium continues to increase in the weakly biased case, even though to a smaller extent than in the case of strongly biased technologies.[14] In this case both initial steady states fall into the white region of AOC and hence during the transition the skill supply, the relative quality and the skill premium all increase together.

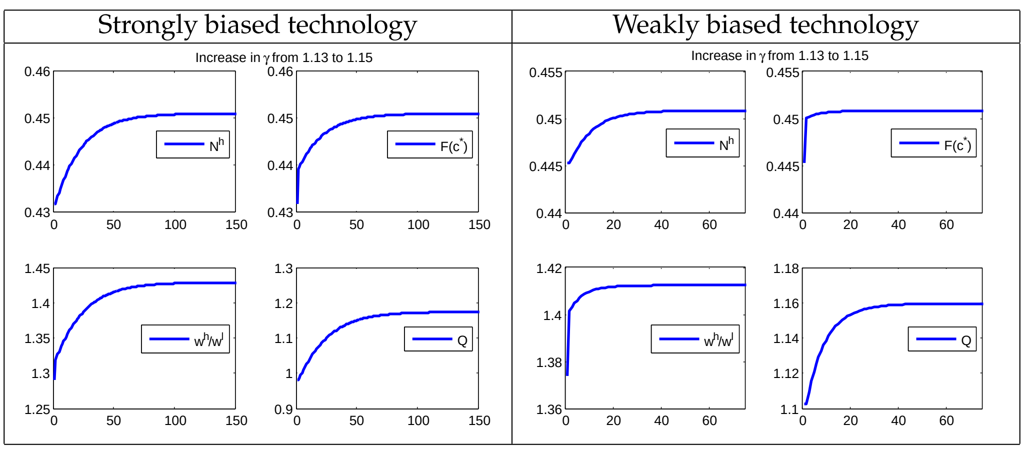

Figure 6: Increase in γ

The effects of ρ on the steady state high-skilled labour supply are more complex. This parameter controls the elasticity of substitution between the high- and the low-skilled intermediate good. This way, it affects the lifetime gain from acquiring education through both the growth rate and the skill premium. The growth rate depends negatively on ρ (from equation (26)), implying that when the intermediate goods are more substitutable with each other, the growth rate is lower. The responsiveness of the skill premium to the supply of high-skilled workers depends on the relation between ρ and $1/(1+\beta)$. When $\rho = 1/(1+\beta)$, then the skill premium does not change in response to a change in the supply of high-skilled workers, as

[14]The strength of the effect of γ on the skill premium through N^{h*} depends on ρ. The closer is ρ to $1/(1+\beta)$, the more likely it is that the direct effect of γ dominates.

the price effect and the technology effect exactly offset each other. Thus, the closer is ρ to $1/(1+\beta)$, the less the steady state skill premium responds to changes in the supply of high-skilled workers (see equation (23)). Thus, for weakly biased technologies, higher substitutability implies a flatter skill premium, one that responds less to extreme values of N^h. In case of a weakly biased technology the steady state skill premium is a decreasing function of the supply of skilled labour. Therefore a higher ρ implies a lower skill premium for low N^hs and a higher skill premium for high N^hs. Thus, the $F(h(N^h))$ curve for higher ρs goes below the one for lower ρs for low values of N^h, and goes above it for high values of N^h. In case of strongly biased technologies the steady state skill premium is an increasing function of the supply of skilled labour. In this case a higher elasticity implies a steeper skill premium: one that is lower for low values of N^h and higher for high values of N^h. The overall effect of a change in ρ thus depends on the other parameter values and on the magnitude of change. Figure 7 shows the transition path for higher elasticities of substitution, for cases where the new steady state features a lower supply of high-skilled workers.

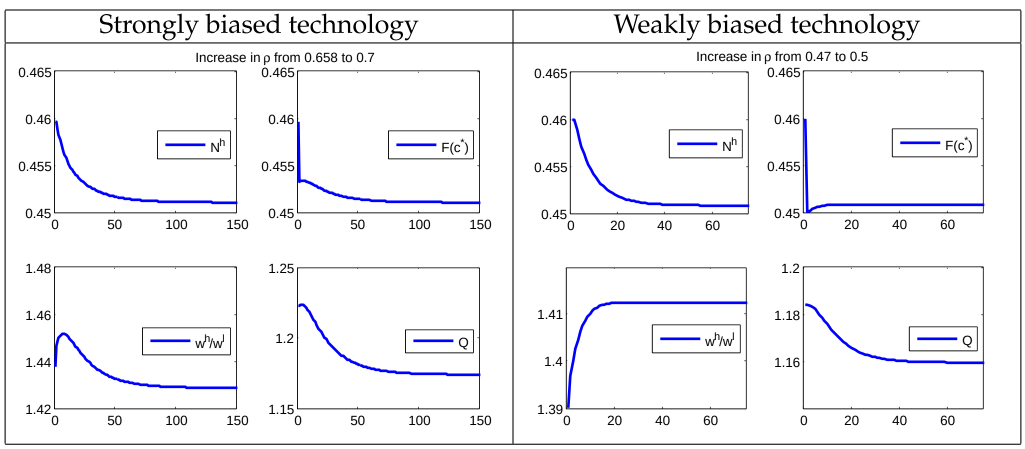

Figure 7: An increase in ρ

The increase in ρ lowers the growth rate, this way reducing the gain from working as high-skilled, and leading to the downward jump in the educational attainment of new cohorts. This reduces initially the skill premium, since the supply of skills increases, while technology stays the same. As the skill supply and relative quality decline throughout the transition, the skill premium increases further for weakly biased technologies. This leads to a slight increase in the educational attainment of new cohorts. On the other hand, for strongly biased technologies the skill premium, after its initial increase, decreases below its original level in the long run. The educational attainment of new cohorts thus continues to decrease. The initial steady state in the case of the strongly biased technology fell into the darker gray part of the BOE' region in Figure 3, as the relative quality and the skill premium slightly increase initially, and the economy crosses over to the dark gray region in DOB at the beginning of the transition. In the case of weakly biased technologies, the relative quality continuously declines together with the supply of skills. Therefore the initial steady state was in the BOE' region, and the economy crossed over to the darkest gray area of DOB just before reaching the steady state.

The last parameters to consider are the mean and the variance of the distribution of educational costs. A distribution with a lower mean, μ, can be represented by the dashed curve in Figure 4, while the higher mean distribution can be represented by the solid curve. This is due to the fact that in a lower mean distribution, there is more mass below any given point, than in the higher mean distribution. Hence, for any present value gain from acquiring education, it is worthwhile for more people to acquire education if the mean cost is lower. Therefore, the stable steady state with lower mean costs of education features higher N^h, which is quite intuitive: where education is cheaper more people acquire education in the long run.[15] Figure 8 shows the transition after an unexpected drop in the mean cost of education.

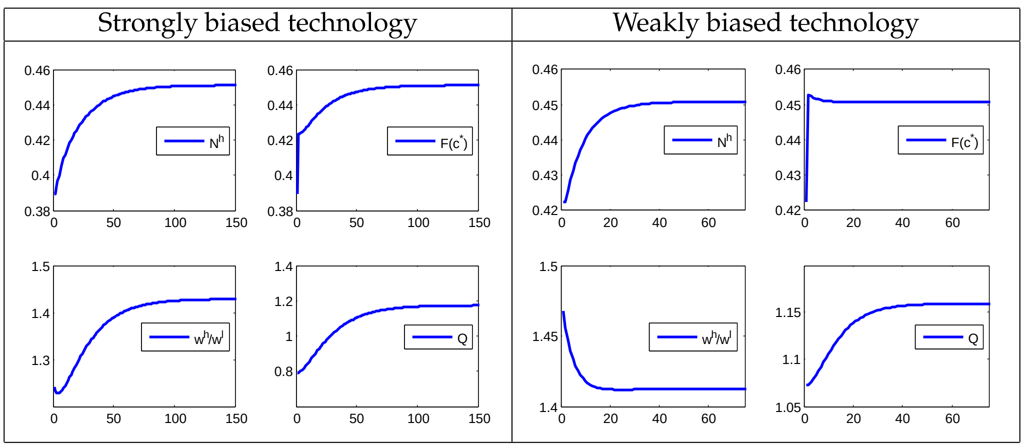

Figure 8: A decline in the mean cost of education, μ

The decline in the mean cost of acquiring education leads to a jump in the educational attainment of new cohorts, since even with the same gain from working as high-skilled, it is worthwhile for a larger fraction of the population. The skill premium in both cases decreases initially, as there is a larger supply of high-skilled workers, while technology does not adjust immediately. The skill premium continues to decline in case of weakly biased technologies, as the effects of the increase in the relative supply are not compensated by the increase in the relative quality. Therefore, the educational attainment of consecutive cohorts declines, but stays above its original level. In case of strongly biased technologies the joint increase in the relative supply and the relative quality lead to an increase in the skill premium, leading to a continuous increase in the educational attainment of new cohorts.

Finally I consider is the variance of the cost of education. A higher variance implies that there are more

[15]In general it is true that if a distribution F first order stochastically dominates distribution G, then G can be represented with the dashed curve, while F can be represented with the solid curve, and hence the steady state under G has more skilled workers.

people with low costs, up until the median costs, while for costs above the median there are more people with higher costs, i.e. fewer people with lower costs. Hence, as long as in the steady state less than half of the population acquires education, the steady state N^h is higher when the costs of education are more dispersed. Figure 9 shows the transition path after an increase in the dispersion of the costs of education.

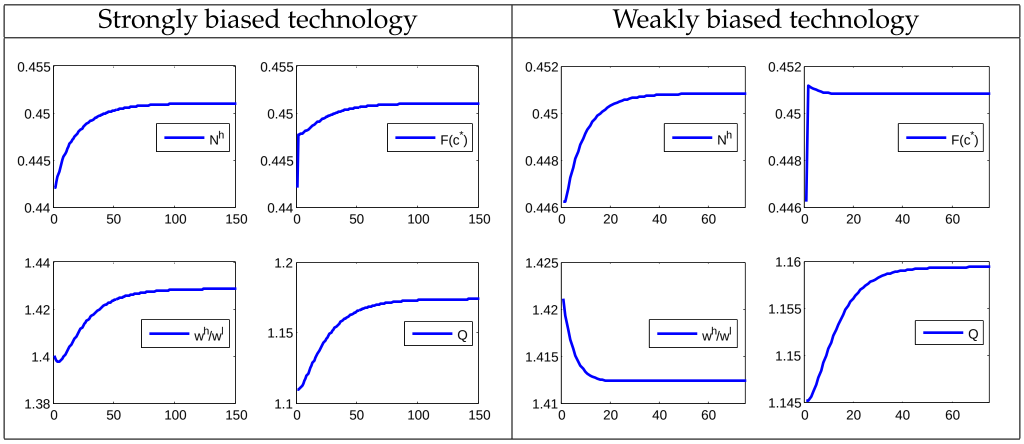

Figure 9: An increase in the dispersion of the costs of education, σ

As the variance of costs increases, a larger fraction of the new cohort acquires education (as long as the present value gain is below the median cost). The transition path and the intuition for the adjustment of the variables is exactly the same as in the case of a lower mean cost of education.

To summarise, for all of the parameter changes considered, the path of the supply of high-skilled workers and the path of the relative quality in the two sectors are similar in case of weakly and strongly biased technologies. However, the path of the skill premium and of the educational attainment of new cohorts is dramatically different for all but one parameter change. The only exception is an increase in parameter γ, where all four variables follow similar paths for the two types of technologies. Except for this case, the skill premium and the educational attainment of new cohorts always moves in opposite directions. This is due to the fact that for weakly biased technologies the increasing relative quality compensates less for the negative effect of the increasing skill supply on the skill premium. Therefore, in most cases the skill premium decreases if the skill supply is increasing, and hence the incentives of acquiring education are reduced for newer cohorts. The opposite holds for strongly biased technologies: as the skill premium continuously increases, the incentives to acquire education increases for newer cohorts.

5 Concluding remarks

In this paper I challenge the view that a strong relative bias in the technology is necessary for the simultaneous increase of the skill supply and the skill premium. Assuming, consistently with the data, that the developed economies are not in their steady state, and considering explicitly the transition to the steady state, the model shows that the joint increase in the skill supply and the skill premium can arise regardless of the bias in the technology.

I propose a model where the direction of technical change and the supply of skilled labour is endogenous. Technological change is driven by R&D firms, which invest more into developing technologies for bigger markets. Therefore when the supply of high-skilled labour increases, technology becomes more biased towards high-skilled workers. The increase in the skill-bias of technology increases the skill premium, however, this is offset to some extent by the negative effect of increasing skill supply on the skill premium. If the overall effect is an increase in the skill premium, then technology is strongly biased, whereas if the overall effect is a decline in the skill premium, then technology is weakly biased. On the other hand, the supply of skilled labour is determined by individual decisions whether to acquire education or not, therefore a higher skill premium leads to a larger supply of skilled labour. The positive dependence of these two variables on each other are crucial in understanding the dynamics.

I analyse the steady state of this model and its dependence on parameter values. This exercise shows, that for most steady state shifts that arise due to a parameter-change, a strongly biased technology is necessary to observe a long-run increase in both the skill supply and the skill premium.

I conduct a thorough analysis of the transitional dynamics, and its dependence on the initial value of the skill supply and the relative quality. The analysis shows, that if initially the relative quality is not too high compared to the supply of high-skilled workers, then the transition can feature a joint continuous increase in the supply of high-skilled labour and the skill premium. I highlight the importance of transitional dynamics by showing that this pattern can emerge independent of whether technology is weakly or strongly biased.

A Transitional dynamics

I calculate the transition using second order perturbations, for which all equations have to be defined in terms of variables that are stationary in the steady state. Two variables are not stationary in the steady state, the value of owning a leading vintage, and the present value gain from working as high-skilled rather than low-skilled. The value of owning the leading vintage, $V_t^s(q)$, is proportional to the quality of that machine. Let v_t^s denote the normalized value of owning the leading vintage in sector s at time t:

$$v_t^h = \frac{V_t^h(q)}{q} \quad v_t^l = \frac{V_t^l(q)}{q}.$$

In the steady state the discounted expected present value of working as high-skilled rather than low-skilled starting from period t is proportional to the wages in period t, which is proportional to the average quality. Let Δ_t denote the normalized present value gain from acquiring education (normalized by the current quality in the low-skilled sector):

$$\Delta_t = \sum_{j=0}^{\infty} \left(\frac{\lambda}{1+r} \right)^j \frac{w_{t+j}^h - w_{t+j}^l}{Q_t^l}$$

The transitional path is fully characterized by the initial values N_0^h and Q_0 and the following equations:

$$\begin{aligned}
v_{t+1}^s &= B^s \tfrac{(1+r)\overline{z}^s}{1-e^{-\eta \overline{z}_t^s}} && s = l, h \\
v_t^s &= \beta(1-\beta)^{\frac{1-\beta}{\beta}} (p_t^s)^{\frac{1}{\beta}} N_t^s - \tfrac{e^{-\eta \overline{z}_t^s}}{1+r} v_{t+1}^s && s = l, h \\
g_{t+1}^s &= 1 + (\overline{q} - 1)(1 - e^{-\eta \overline{z}_t^s}) && s = l, h \\
p_t^h &= \left(\gamma + \gamma^{\frac{\beta\rho}{(1-(1-\beta)\rho)(1-\rho)}} \left(Q_t \tfrac{N_t^h}{N_t^l} \right)^{-\frac{\beta\rho}{(1-(1-\beta)\rho)}} \right)^{\frac{1-\rho}{\rho}} \\
p_t^l &= \left(1 + \gamma^{1-\frac{\beta\rho}{(1-(1-\beta)\rho)(1-\rho)}} \left(Q_t \tfrac{N_t^h}{N_t^l} \right)^{\frac{\beta\rho}{(1-(1-\beta)\rho)}} \right)^{\frac{1-\rho}{\rho}} \\
Q_{t+1} &= \tfrac{g_{t+1}^h}{g_{t+1}^l} Q_t \\
\Delta_t &= c_t^* \beta(1-\beta)^{\frac{1-2\beta}{\beta}} (p_t^h)^{\frac{1}{\beta}} Q_t \\
\Delta_t &= \beta(1-\beta)^{\frac{1-2\beta}{\beta}} \left((p_t^h)^{\frac{1}{\beta}} Q_t - (p_t^l)^{\frac{1}{\beta}} \right) + \tfrac{\lambda}{1+r} g_{t+1}^l \Delta_{t+1} \\
N_t^h &= \lambda N_{t-1}^h + (1-\lambda) F(c_t^*) \\
N_t^l &= 1 - N_t^h.
\end{aligned}$$

B Initial values

B.1 $dQ = 0$ and $dN^h = 0$

The Figure below shows the border where the regions where the state variables are increasing (in black) and decreasing (in white), the border between the regions is where the state variable stays constant. The intersection of the two borders is the steady state.

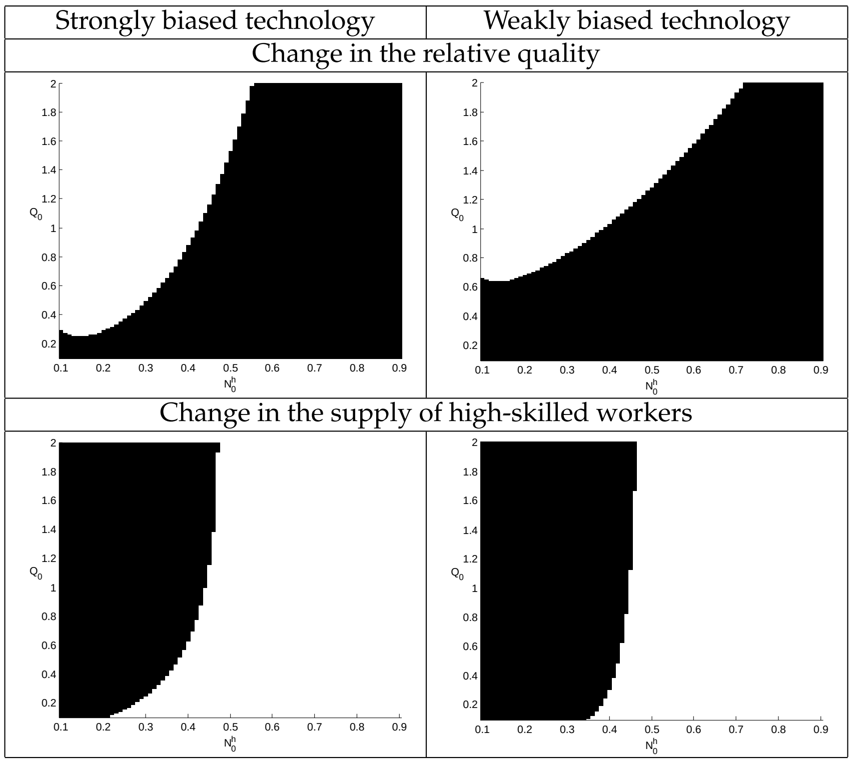

Figure 10: Phase diagram source

B.2 Short-run and long-run change in the skill premium

The Figure below shows the immediate change in the skill premium and the overall change in the skill premium for each initial point. As before black indicates an increase and white indicates a decrease.

B.3 Saddle path

The following Figure shows how the entire transition of the skill premium and the relative quality. Black shows continuous increase, darker gray shows points where there is an overall increase, but the path is not monotonic, lighter gray shows non-monotonic overall decrease, and white shows continuous decrease.

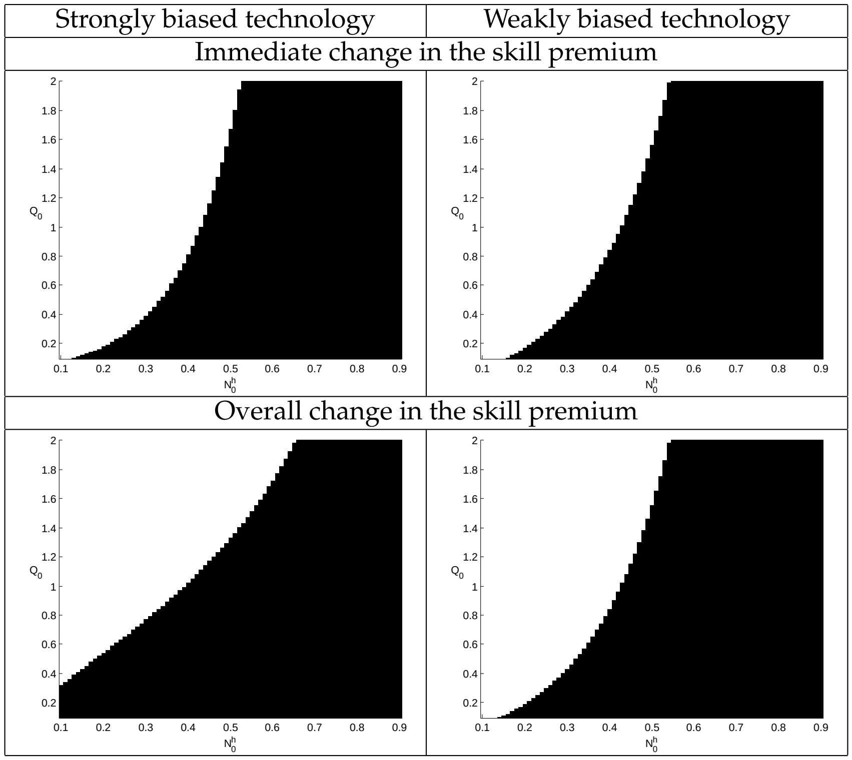

Figure 11: Skill premium change source

Since to the left of N^{h*} for most part Q decreases, the fact that Q doesn't continuously decrease to the steady state implies that the transition takes the economy into the black region in the top row of Figure 10. Similarly, for initial points above N^h there is a large part where Q initially increases, but does not increase until the steady state, as the area is in the darker gray. This implies that the transition takes the economy up into the white region in the top row of Figure 10. This suggests, that the stable arm to the steady state is a path, where either both Q and N^h increases, or they both decrease.

C Parameters

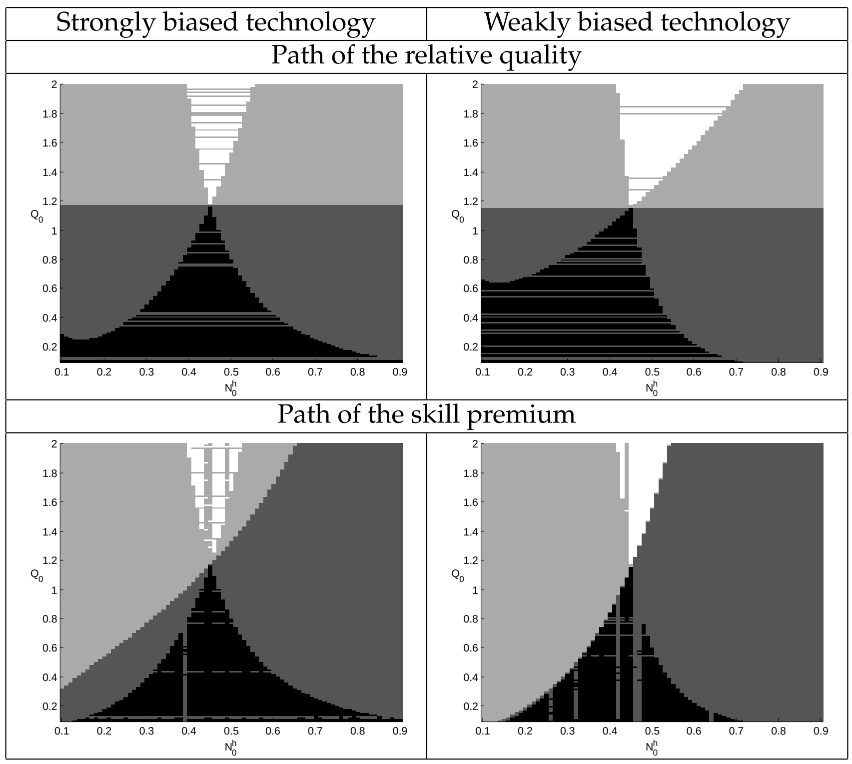

Figure 12: Skill premium change source

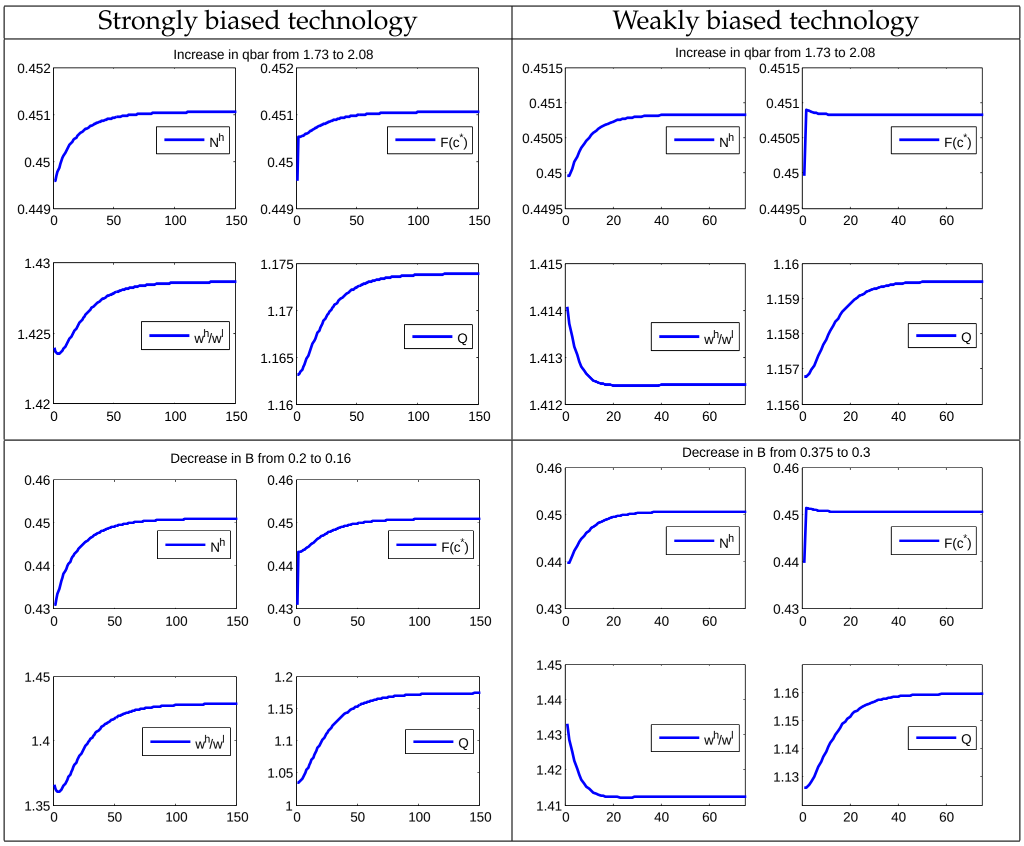

Figure 13: Change in the R&D parameters 2